Abstract—Series compensation is used in power systems when transmission distances are great and where large power transfers over these distances are required. Traditionally with turbine-generators, Subsynchronous Resonance (SSR) has been identified as a problem due to the use of series compensation. Although SSR is a well-documented phenomenon for conventional turbine-generator systems, not many publications have addressed subsynchronous oscillations in the context of wind farm systems. The first aspect of this paper is to examine the occurrence of subsynchronous oscillations between wind generation and series-compensated power systems under transient conditions. The second aspect of this paper is to quantify the frequency and damping content within the resulting transients. Fourier analysis and Modified-Prony methods are implemented to demonstrate their effectiveness for countermeasure detection of such uncontrolled oscillations. The Alternative Transients Program (ATP) is used for time-domain modeling and simulation of transient conditions.

Keywords: Subsynchronous Oscillations, Series-compensated wind farm, Electrical self-excitation, Alternative Transients Program (ATP), Fourier analysis, Modified-Prony method.

I. INTRODUCTION

IND power based generation has been rapidly growing world-wide during the recent past. In the United States, this is expected to reach a penetration level of 20% by 2030, [1]. In order to transmit large amounts of wind power over long distances, system planners may often add series compensation to existing transmission systems for enhanced power transfer capability. Subsynchronous Resonance (SSR) has been identified as a problem for conventional turbine-generators due to the application of series capacitors [2], [3]. Wind turbine generators may also be susceptible to subsynchronous oscillations or other resonant interactions [4], [5]. However, not much information is available in literature about such interaction possibilities.

The motivation for this paper is an actual event that occurred in a system with a typical ring-bus configuration as shown in Fig. 1. Breakers 5 and 6 were opened up as part of a regular system switching procedure, placing the generators (both wind (W) and combustion turbine (CT) unit) in a radial connection with series compensation. This resulted in undamped current oscillations causing the CT generator unit to trip. Further investigation and testing has indicated that the interaction could be due to the wind generators, which were online during the event.

The organization of this paper is as follows: Section II provides a conceptual overview of subsynchronous oscillations and highlights important differences in the case of induction machine based wind generators. Section III describes the system configuration and modeling approach. Section IV presents ATP simulations and results for three different transient scenarios such as, starting transients, series capacitor insertion and a transient 3-cycle, 3L-G fault. Section V shows the application of Discrete Fourier Transform (DFT) and Modified-Prony method for countermeasure detection. Section VI provides conclusions and recommendations.

Fig. 1. System configuration showing the possibility of wind generation being left in a radial connection with series compensation due to switching of breakers 5 and 6. BP indicates bypass breaker for the series capacitor bank.

II. SUBSYNCHRONOUS OSCILLATIONS AND RESONANCE

A. SSR in conventional turbine-generators

Application of series capacitors in a power system network will result in a natural electrical frequency, $f_e$ given by:

$$f_e = f_o \frac{X_c}{X_\Sigma}$$

where, $f_o$ is the nominal system frequency, $X_c$ is the reactance of the series capacitor and $X_\Sigma$ denotes the sum of all system inductive reactance such as the transmission line, transformer and generator sub-transient reactance. The ratio of capacitive reactance of series capacitors to the transmission line inductive reactance is called degree of compensation,
usually given in percentage value. Typical values of compensation lie in the range of 20-80% [6]. Application of series capacitors in ac networks will result in transient currents. Transient currents flowing in the generator stator when reflected mathematically on to the rotor by Park’s transformation will yield modulated frequencies of the form, \( f_o \pm f_e \). The difference frequencies are called \( f_o \pm f_e \) sub synchronous frequencies. If the sub synchronous frequency coincides with any of the shaft natural modes then, shaft oscillations of high amplitude will result. This network resulting condition in the exchange of electro-mechanical energy is in general referred to as SSR. There is also a possibility of pure electrical self-excitation based resonance which is independent of the shaft torsional modes. Uncontrolled self-excited current and electromagnetic torque oscillations can occur if the rotor resistance at a particular sub synchronous frequency exceeds the network resistance. In synchronous generators, this tendency can be decreased by using pole-face damper windings. Fig. 2 gives a summary of the two distinct mechanisms through which sub synchronous interactions can occur.

B. SSR in series-compensated wind farms

Subsynchronous interaction in series-compensated wind farms are also subject to a similar mechanism. However, some important differences exist with respect to conventional turbine-generators. Firstly, there are a number of wind turbine-generators in a wind farm as against one large turbine-generator. Secondly, wind generators are predominantly based on induction machine (either squirrel cage type, wound rotor or doubly fed type). Thirdly, the torsional frequencies associated with wind turbine-generator systems are very low, usually in the range of 1-4 Hz.

Each of the above mentioned differences, have a bearing on subsynchronous interaction in series-compensated wind farms. The presence of a large number of wind machines can be accounted for by using a single machine equivalent representation, details of such a representation appears in the next section. Secondly, use of an induction machine will result in a natural resonant circuit as shown in Fig. 3. The slipping \( s_e \) corresponds to the sub synchronous slip, and is usually negative owing to the higher rotor speed. The relationship between the actual rotor speed, power frequency slip and subsynchronous slip can be written as:

\[
\omega_{\text{rotor}} = (1-s)\omega_{\text{syn}} = (1-s_e)2\pi f_e
\]  

(2)

For a particular value of sub synchronous slip, the apparent rotor resistance in Fig. 3 becomes negative and exceeds the network resistance. Under such conditions, any transient disturbance can trigger undamped current and electromagnetic torque oscillations. Last in consideration is the possibility of torsional interaction due to series compensation in wind turbine-generators. Torsional interaction does not pose a serious threat in wind turbine-generators since very high compensation levels are needed to induce synchronous complement frequencies in the range of 1-4 Hz. Such high levels of compensation are not practically used. Moreover, at such high levels of compensation instability due to electrical resonance is already under play. Hence, in series-compensated wind farm systems self-excited electrical resonance is a greater concern than torsional electro-mechanical interaction.

![Fig. 2. Conceptual overview of electrical self-excitation and torsional interaction with principal variables involved in interaction [6].](image)

![Fig. 3. Natural resonant circuit of an induction machine with stator and rotor quantities, series capacitance and magnetization inductance. Subsynchronous slip is denoted by \( s_e \).](image)

III. SYSTEM CONFIGURATION AND MODELING

A wind farm system in radial configuration with a 500-kV series-compensated transmission line has been used for investigation. Parameters used for the wind turbine-generator and transmission network are reported in the Appendix.

A single per turbine equivalent representation has been used to represent the aggregate effect of several identical wind turbine-generator units. This per turbine equivalent is based on the fact that, aggregation of several identical wind turbine-generator units causes the aggregated inertia to scale up; however, the base electrical power also gets scaled. Hence, the inertia constant remains the same. Similar argument can be extended to other machine parameters and hence, an equivalent aggregate in per unit quantity can be obtained. This can further be scaled on to a per turbine base, leaving the individual machine parameters unchanged.

While this per turbine equivalent representation may not be a very realistic reproduction of the wind farm internal conditions, it is sufficient to capture the worst case system behavior assuming all machines behave coherently. In [7], the dynamic response of a group of wind generators following a transient event was shown to be much smoother when the machine parameters were not exactly identical. This is expected since, the natural frequencies of the machines will be different and as there is no synchronizing torque in the case of induction machines, the electrical and mechanical modes oscillate at different frequencies which in a short while tend to eliminate each other. Hence, a statistical variation such as a normal distribution in the operating conditions or variation of
wind turbine parameters within a wind farm tends to have a benign effect in exciting electrical or electro-mechanical resonant modes.

To verify the fidelity of this per-turbine equivalent representation, its transient response during starting conditions has been compared against the simulation of five individual machines. Fig. 4 is a representation of the five machine system and Fig. 5 is the per-turbine equivalent representation. Fig. 6 compares the speed and generated power in each of the above configurations. As expected, the results show insignificant error. Series compensation remains bypassed in both cases.

The typical starting sequence [8], [9] for a fixed-speed wind machine is given in Table I. Simulations in Fig. 6 use a similar starting sequence. Simulations have been performed using the ATPDraw interface for the ATP program [10], [11].

Each wind machine is represented using UM-3 component for the generator and a two-mass network representation for the generator rotor and turbine inertia; all of which is referred to the high speed shaft. The mechanical torque is considered constant and fine tuned to operate each wind turbine at nominal slip and power.

![Fig. 4. Five machine system – representing five machines in operation.](image)

![Fig. 5. Single per-turbine equivalent representation of five machine system.](image)

### IV. ATP SIMULATION CASES AND RESULTS

Three different transient scenarios, such as starting transients, insertion of series capacitor and the application of a transient 3-cycle, 3L-G fault have been used to explore the possibility of subsynchronous oscillations.

#### A. Electrical resonance during starting sequence

In order to investigate the possibility of pure electrical resonance, a single lumped mass representation was used for the turbine-generator system instead of the standard two-mass representation in this particular simulation (case A). This is required for de-coupling the effect of any torsional modes during electrical self-excited resonance. A typical starting sequence as outlined in Section III is simulated; while, series capacitors corresponding to 70% compensation level are kept in service. An aggregation level of 400-MW (200 x 2-MW) is used for the simulation.

Fig. 7 shows the electromagnetic torque and speed responses. Subsynchronous oscillations in the electromagnetic torque tend to reach up to 60 p.u. Currents and voltages are the principal interacting variables during electrical self-excitation and as a result, the generator electromagnetic dynamics is influenced. Electromagnetic torque and speed responses in Fig. 7 indicate that the wind turbine-generator can experience severe oscillations from which it never recovers. Section V uses the results (Phase currents and phase voltages) from this simulation case for the application of Fourier analysis and Modified-Prony estimation.

#### B. Insertion of series capacitor

By opening the bypass breaker (Fig. 5) a series compensation of 70% was inserted with the wind turbine-generator operating in steady state. The response of the electromagnetic torque is shown in Fig. 8. There is a sudden change in torque at 16 sec when the series capacitor is inserted; but the resulting oscillations damp out quickly. Increasing the compensation level will increase the amplitude of oscillations. However, electrical instability does not occur, as was illustrated in case A. Fig. 9 shows the response of the shaft torque at 16 sec. There is a sudden change in the shaft torque; however, the shaft oscillatory modes do not experience any sustained oscillation. A two-mass representation was used for the generator rotor and turbine inertia with shaft stiffness, all of which is referred to the high speed side.

Fig. 10 shows the change in generator electromagnetic torque due to series capacitor insertion for varying compensation and aggregation levels. It can be observed that, higher series compensation level leads to a greater change in electromagnetic torque response. Secondly, as the number of machines aggregated increases, the amplitude of torque response at higher compensation levels increases drastically.

### TABLE I

<table>
<thead>
<tr>
<th>Sequence step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Kinetic energy of wind used to accelerate the machine to a speed of approximately 5-30% below the synchronous speed</td>
</tr>
<tr>
<td>2</td>
<td>Generator circuit breakers close</td>
</tr>
<tr>
<td>3</td>
<td>If equipped with soft-starter, it is bypassed within 2-3 sec</td>
</tr>
<tr>
<td>4</td>
<td>Power factor correction capacitors are switched on in single or multiple steps</td>
</tr>
</tbody>
</table>
Fig. 6. Comparison of mechanical speed and real power during startup transient conditions, lower two plots (in blue) refer to a single per-turbine equivalent of the five machine case. (a) Mechanical speed for five machines; (b) Real power for each turbine; (c) and (d) corresponding plots for single per-turbine equivalent.

Fig. 7. Mechanical speed and electromagnetic torque during starting with 70% series compensation in service, illustrating the possibility of electrical resonance. Lower two plots are a zoom-in version of the above plots, (a) Mechanical speed; (b) Electromagnetic Torque; (c) and (d) indicate corresponding zoom-in.
C. Application of a transient 3-cycle, 3L-G fault

With 70% series compensation in service, a transient 3-cycle, 3L-G fault was applied at 19 sec to the thevenin bus in Fig. 5. Fig. 8 shows the electromagnetic torque response and Fig. 9 shows the response of the shaft torque. As in the case of series capacitor insertion, there is a sudden change in shaft torque upon removal of the fault at the end of 3-cycles. The resulting oscillations however, damp out quickly. Response of the shaft torque as illustrated in Fig. 9 for case B and C simulations clearly demonstrates that, torsional interaction due to application of series capacitors is not a serious concern in wind turbine-generator systems at realistic levels (20-80%) of series compensation.

V. COUNTERMEASURE DETECTION – APPLICATION OF DFT AND MODIFIED-PRONY METHOD

A. Fourier analysis – DFT

Fourier analysis (DFT) of the electromagnetic torque, and stator phase-A current for the case of electrical self-excited resonance (Section IV- Case A) is presented in Fig. 11. A dominant frequency of 46 Hz can be seen in the electromagnetic torque and a complementary frequency of 14 Hz is present in the stator current. This is consistent with the theory of Park’s transformation during the occurrence of self-excited resonance.

The frequency of 14 Hz in the stator current can also be derived analytically by the application of (1). However, instead of using the blocked rotor reactance of the induction machine (Fig. 3), the sum of stator and magnetization reactance should be used in the calculation of $X_{Σ}$. Application of (2) will yield the actual resonant speed corresponding to the resonant frequency of 14 Hz, during electrical self-excitation. A characteristic dip in the mechanical speed can be observed in Fig. 7, during self-excited resonance. This characteristic dip in the mechanical speed during subsynchronous oscillations indicates that the machine is getting attracted towards a lower resonant speed beyond which it never recovers [12].

B. Countermeasure Detection

Simulations in Section IV have demonstrated that, resonance due to electrical self-excitation is a concern in wind turbine-generators even at realistic compensation levels. Countermeasure detection is proposed in this section for identifying electrical self-excitation condition.

The countermeasure detection logic is shown in Fig. 12. First part of the detection logic uses phase currents and phase voltages as input to obtain a modulated signal. This modulated signal retains all the frequency characteristics of the electromagnetic torque. The second part of the logic performs a DFT and modified prony operation on the modulated signal. A parallel combination of DFT and Modified-Prony is required to ensure sufficient sensitivity to resonant conditions. The Modified-Prony method used in this paper is based on [13] and Appendix II gives key equations related to the basics of Prony and Modified-Prony methods. Application of Prony estimation will yield amplitude, frequency, damping factor, $ξ$ and phase. If the frequency is within 0-60 Hz, damping factor $ξ ≥ 0$ and using the ratio of the amplitude with respect to the dc component, it is possible to send a logic 1 to the OR gate for tripping. Similar constraints can be applied to the DFT results. However, in the case of DFT, this information needs to be complemented with the speed signal. It can be observed from Fig. 7 that, electrical self-excitation condition is accompanied by a characteristic dip in the mechanical speed. This information will help improve selectivity of the countermeasure detection logic.

Fig. 8. Series capacitor is inserted at 16 sec and a transient 3-cycle, 3L-G fault is applied at 19 sec, with 70% compensation in service. (Negative scale denotes generating electromagnetic torque)

Fig. 9. Response of the mechanical shaft torque due to series capacitor insertion at 16 sec and application of a transient 3-cycle, 3L-G fault at 19 sec.

Fig. 10. Change in the amplitude of electromagnetic torque response due to series capacitor insertion, with varying compensation and aggregation levels.
Fig. 13 shows the modulated signal derived from the simulations in Section IV (Case-A). Fig. 14 shows the application of DFT and Modified-Prony estimation to the modulated signal. Table II provides results from the application of Modified-Prony method. For this case, it can be inferred that the damping ratio associated with 45.29 Hz is positive, indicating the presence of undamped subsynchronous oscillations.

![Modulated Signal](image1)

![Zoom-In Modulated Signal](image2)

Fig. 11. Fourier analysis of electrical resonance simulations, showing complementary frequency relationship between stator phase-A current and electromagnetic torque. (a) Electromagnetic Torque; (b) DFT of Electromagnetic Torque; (c) Stator phase-A current; (d) DFT of stator current.

![Countermeasure Detection Logic](image3)

Fig. 12. Countermeasure detection logic for electrical resonance and self-excited conditions.

![Modulated Signal](image4)

Fig. 13. Modulated signal (left) and its zoom-in version (right). Modulated signal derived using phase currents and phase voltages from Section IV - Case A.
Fig. 14. Application of Modified-Prony method (on left) and DFT (right) to the modulated signal. DC component and linear trend has been filtered out, prior to the application of DFT and Modified-Prony estimation.

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>Damping Factor, ξ [1/s]</th>
<th>Amplitude [p.u]</th>
</tr>
</thead>
<tbody>
<tr>
<td>284.57/(2π) = 45.29</td>
<td>0.65</td>
<td>0.47</td>
</tr>
</tbody>
</table>

VI. SUMMARY AND CONCLUSIONS
ATP simulations have been performed to investigate the occurrence of subsynchronous oscillations between series compensation and wind turbine-generator systems. The resulting subsynchronous frequency content has been quantified using Fourier analysis and Modified-Prony method.

Following conclusions have resulted:

1. Electrical self-excitation and resonance with series compensation in service is a concern during generator breaker switching operation or application of starting torque in wind turbine-generator systems. Even realistic compensation levels of 70% poses a serious threat.

2. Several machines acting coherently has the effect of reducing the equivalent network resistance as seen by each machine. As a result, there is higher probability of self-excitation and electrical resonance with increasing levels of aggregation.

3. The amplitude of electromagnetic torque response to transient events increases with the compensation level. Higher compensation levels also lead to greater possibility of electrical self-excited resonance.

4. Torsional interaction due to series compensation is not a concern at realistic levels of series compensation [20-80%] in wind turbine-generators; whereas, subsynchronous interaction due to electrical resonance is a concern even at realistic levels of compensation.

5. Phase currents, phase voltages and speed signals are sufficient to determine if the induction generator is experiencing sustained subsynchronous oscillations.

6. Countermeasure detection logic based on the application of DFT and Modified-Prony method to the case of electrical self-excitation has been demonstrated to be effective.

7. The countermeasure detection logic proposed in this paper can be included as part of the protection functionality in wind turbine-generators. It can also yield vital information for performing real-time system identification.

VII. APPENDIX I: SYSTEM DATA

A. Wind turbine-generator parameters

Data for a 2-MW induction generator based wind turbine [14],

<table>
<thead>
<tr>
<th>Rated power</th>
<th>Nominal slip</th>
<th>Stator reactance</th>
<th>Gen. rotor inertia constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 MW</td>
<td>0.015</td>
<td>0.075 p.u</td>
<td>0.5 s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rated voltage</th>
<th>Magnetizing reactance</th>
<th>Rotor resistance</th>
<th>Turbine inertia constant</th>
<th>Shaft stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>690 V</td>
<td>3.8 p.u</td>
<td>0.018 p.u</td>
<td>4.5 s</td>
<td>0.55 p.u</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rated frequency</th>
<th>Stator resistance</th>
<th>Rotor reactance</th>
<th>Shaft stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 Hz</td>
<td>0.048 p.u</td>
<td>0.12 p.u</td>
<td></td>
</tr>
</tbody>
</table>

Each wind turbine-generator interfaced with:
0.5-MVAR, 690 V No-load compensating capacitors and 0.69 / 34.5 kV, wind turbine-generator transformer

B. 500-kV Transmission system parameters [15]

Positive sequence transmission line data:
R = 5.60286 Ohm, X = 140.0715 Ohm,
Capacitive reactance = (k) X; k being varied (0.2-0.8)
Positive sequence system thevenin:
X thevenin = 16.808 Ohm
34.5 / 500 kV, wind farm step-up transformer
VIII. APPENDIX II: PRONY AND MODIFIED-PRONY BASICS

The classical Prony algorithm determines a linear prediction model for a set of sampled data as a linear combination of exponential functions [16]. Assuming signal data \(x[n]\) has \(N\) complex data samples, the Prony method will fit the data with a sum of \(q\) complex exponential functions.

\[
\hat{x}[n] = \sum_{k=1}^{q} A_k \exp\{-(\xi_k + j2\pi f_k)(n-1)T_s + j\phi_k\} \tag{3}
\]

For, \(n = 1, 2, \ldots, N\) and \(k = 1, 2, \ldots, q\)

The objective is to estimate the frequencies \(f_k\), amplitudes \(A_k\), damping factors \(\xi_k\) and phases \(\phi_k\). The estimation problem is based on the minimization of squared error over the \(N\) data values; the squared error is defined as:

\[
\delta = \sum_{n=1}^{N} |\epsilon[n]|^2 \tag{4}
\]

Where, \(\epsilon[n] = x[n] - \hat{x}[n] = x[n] - \sum_{k=1}^{q} h_k z_k^{-n-1}\)

The classical Prony algorithm, addresses this by solving a set of linear equations for the co-efficients of the recurrence equations that the signal satisfies. The Modified-Prony method, as proposed in [13], will estimate for fixed \(q\), any function \(\mu\) that solves a constant co-efficient differential equation:

\[
\sum_{k=1}^{q} v_k D^{k-1} \mu = 0 \tag{5}
\]

For real-data sets with equispaced time samples, the function,

\[
\mu(t) = \sum_{j=1}^{q} \alpha_j e^{j\beta_j t} \tag{6}
\]

The Modified-Prony algorithm proposed in [13] is shown to perform better than several standard non-linear regression algorithms. It also offers greater practical advantage in that the function \(\mu(t)\) is chosen to best fit the observed data. For further details readers are referred to [13].

IX. REFERENCES

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