A Study of Current Waves Propagating Along Vertical Conductors and Their Associated Electromagnetic Fields

Yoshihiro Baba, Vladimir A. Rakov

Abstract—Using the finite-difference time-domain (FDTD) method, we have analyzed current waves propagating along vertical conductors of different type, including a cone, an inverted cone, and a parallelepiped (uniform-thickness conductor), each located above ground and excited at one of its ends by a lumped current source, as well as their associated electric and magnetic fields. A current wave suffers neither attenuation nor dispersion in propagating from the apex of cone to its base, and the resultant field structure is transverse electromagnetic (TEM), as expected from the theory. On the other hand, a current wave suffers significant attenuation and dispersion in propagating from the base of cone to its apex, and the resultant field structure is non-TEM. In propagating along a parallelepiped, a current wave suffers attenuation and dispersion particularly near the source region, which become more pronounced as the thickness of parallelepiped increases. The resultant field structure away from the source region is close to TEM. Further, we have shown that the field structure around a vertical phased current source array, which simulates unattenuated propagation of current wave at the speed of light, is TEM. In the case of vertical in-phase current source array placed between two horizontal perfectly conducting planes, a cylindrically expanding TEM wave is formed, and the input impedance seen from the terminals of the array varies with time as $60\ln(h/c)t$, where $h$ is the separation between the planes, and $c$ is the speed of light.

Keywords: Current attenuation, electromagnetic field structure, FDTD method, grounding impedance, lightning, transmission-line tower, traveling waves.

I. INTRODUCTION

In calculating transient voltages on power transmission lines due to direct lightning strikes using a circuit-theory-based approach, a vertical transmission-line tower has been widely represented by a uniform lossless transmission line terminated at its bottom end in the tower grounding impedance [1]-[3]. Several formulas for the characteristic impedance of transmission-line tower [1][2], where the tower is approximated by a cone or a cylinder, each located above ground, are found in the literature. It is, therefore, important to know the propagation characteristic of current waves along such vertical conductors.

In this paper, using the finite-difference time-domain (FDTD) method [4] for solving Maxwell’s equations, we will investigate the characteristics of current waves propagating downward along vertical perfect conductors of different type, including a cone, an inverted cone, and a parallelepiped, each placed between two horizontal perfectly conducting planes and excited at its upper end, and their associated electromagnetic fields. The inverted-cone configuration is used to investigate characteristics of current waves, which are reflected at the base of conical-shape tower and then propagate up from its base to apex. Additionally, we will analyze the field structure around a vertical phased current source array, which simulates unattenuated propagation of current waves at the speed of light, and that around a vertical in-phase current source array located between two horizontal perfectly conducting planes.

II. METHODOLOGY

Fig. 1 (a) shows a vertical perfectly conducting cone of base radius 8 cm placed between two horizontal perfectly conducting planes 40 cm apart, to be analyzed using the FDTD method. A current source, having a height of 1 cm and a cross-sectional area of 1.5 cm x 1.5 cm is inserted between the cone apex and the top perfectly conducting plane. The source produces a Gaussian pulse having a magnitude of 1 A and a half-peak width of 0.33 ns. This current wave propagates downward along the surface of the cone, away from its apex, until it encounters the bottom plane. Fig. 1 (b) shows a vertical perfectly conducting inverted cone of base radius 8 cm placed between the same two horizontal planes. A current source having a height of 1 cm and an approximately circular cross-sectional area whose radius is 8.5 cm is inserted between the cone base and the top perfectly conducting plane. In this case, a current wave propagates downward along the surface of the inverted cone toward its apex, until it encounters the bottom plane. Fig. 1 (c) shows a vertical perfectly conducting parallelepiped having a cross-sectional area of 1 cm x 1 cm or 3 cm x 3 cm placed between the same two horizontal planes. A current source having a height of 1 cm and a cross-sectional area of 1.5 cm x 1.5 cm or 3.5 cm x 3.5 cm is inserted between the parallelepiped top and the top perfectly conducting plane. In this case, a current wave propagates downward along the surface of the parallelepiped until it encounters the bottom plane.

The current source in the FDTD simulation is implemented by imposing magnetic field vectors along the closest possible loop enclosing the current source [5]. Currents and fields are

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calculated up to 2.5 ns with a time increment of 0.01 ns. The working volume of 2 m x 2 m x 0.4 m, shown in Fig. 1, is divided into 0.5 cm x 0.5 cm x 1 cm rectangular cells. Due to such rectangular discretization, the cone in Fig. 1 has a staircase surface. The lateral dimensions of the volume are limited by perfectly conducting planes, which do not influence a current wave propagating on the cone for about 6 ns after the current injection at the top of the conductor. This configuration is similar to that used in the small-scale experiments carried out by Chisholm et al. [2] and by Chisholm and Janischewskyj [6].

In small-scale experiments with a conical conductor placed between two horizontal conducting planes, Chisholm and Janischewskyj [6] and Bermudez et al. [3] have detected a lower than expected current at the apex of the conical conductor and ascribed this current deficit to a fictitious non-zero (about 60 Ω, constant or decreasing with time) grounding impedance to the bottom conducting plane. In reality, this phenomenon is a result of attenuation of current waves reflected from perfect ground at cone base and propagating toward the apex.

Fig. 3 (a) shows FDTD-calculated waveforms of vertical and horizontal electric fields at two points 40 cm away from the center point of the current source for different angles \( \theta \), \( \pi/4 \) and \( \pi/2 \), relative to the axis of the cone, shown in Fig. 1 (a). Fig. 3 (b) shows those for the inverted cone shown in Fig. 1 (b). Figs. 3 (c) and (d) show those for the parallelepiped having a cross-sectional area of 1 cm x 1 cm and 3 cm x 3 cm, respectively [see Fig. 1 (c)].

The electromagnetic field structure around an ideal biconical antenna, excited by a source connected between the cone apexes, is spherical TEM [7]. The theta-directed electric field \( E_{\theta} \) of the spherical TEM wave which can be viewed as produced by an unattenuated current pulse \( I \) propagating away from the excitation point of the biconical antenna is given by

\[
E_{\theta}(r,\theta,t) = \frac{1}{2\pi\epsilon_0 cr\sin\theta} I(0,t-r/c)
\]

where \( \epsilon_0 \) is the permittivity of vacuum, \( c \) is the velocity of light, \( r \) is the radial distance from the excitation point to the observation point, \( \theta \) is the angle between the antenna axis and a straight line passing through both the excitation point and the observation point (\( \theta \) should be equal to or larger than the half-cone angle), and \( I(0,t) \) is the source current. Equation (1) applies to the configuration presented in Fig. 1 (a) until the current pulse arrives at the cone base. Equation (1) also applies to a zero-angle inverted cone above a conducting plane, that is, to an infinitely thin wire above ground provided that \( \theta \leq 0 \) [8].

For the configuration shown in Fig. 1 (a), the vertical and horizontal components, \( E_z \) and \( E_h \), of the electric field can be evaluated by multiplying (1) by \( \cos(\pi/2-\theta) \) and \( \sin(\pi/2-\theta) \), respectively. For a spherical TEM wave, \( E_z(r=40 \text{ cm}, \theta=\pi/4), E_z(r=40 \text{ cm}, \theta=\pi/2) \), and \( E_h(r=40 \text{ cm}, \theta=\pi/4) \) should be the same. For a source current wave having a peak of 1 A, the magnitude of these electric fields should be 150 V/m. This theoretical prediction for the configuration shown in Fig. 1 (a) is to be compared with the corresponding electric field waveforms, calculated using the FDTD method and shown in Fig. 3 (a). All three waveforms in Fig. 3 (a) are very similar, which is consistent with the theoretical prediction, and the magnitudes of these electric fields are only 7 to 10% less than the theoretical value (150 V/m). Therefore, the electromagnetic field structure around the cone excited at its apex is essentially spherical TEM until a reflection from the bottom perfectly conducting plane arrives at the observation point.
Fig. 2. (a) Current waveforms at different vertical distances from the source at the apex of the cone shown in Fig. 1 (a), (b) current waveforms at different vertical distances from the source at the base of the inverted cone shown in Fig. 1 (b), current waveforms at different vertical distances from the top of the parallelepiped having a cross-sectional area of (c) 1 cm x 1 cm and (d) 3 cm x 3 cm excited at the top as shown in Fig. 1 (c), calculated using the FDTD method.

Fig. 3. FDTD-calculated waveforms of vertical and horizontal electric fields at two observation points located (a) 40 cm away from the excitation point for (a) the cone shown in Fig. 1 (a), (b) the inverted cone shown in Fig. 1 (b), (c) the parallelepiped having a cross-sectional area of 1 cm x 1 cm shown in Fig. 1 (c), and (d) same as (c), but having a cross-sectional area of 3 cm x 3 cm.
On the other hand, as seen in Fig. 3 (b), electric field waveforms at the same observation points, calculated for the cone excited at its base and shown in Fig. 1 (b), differ considerably from each other. This indicates that the electromagnetic field structure around a cone excited at its base is non-TEM. This implies that in the configuration shown in Fig. 1 (a) a current wave reflected from the bottom plane also produces a non-TEM electromagnetic field structure. Shown in Figs. 3 (c) and (d) are electric field waveforms at the same observation points, calculated for the configuration shown in Fig. 1 (c). Differences among three electric-field waveforms shown in Fig. 3 (c) are small. The corresponding current wave [Fig. 2 (c)] attenuates only slightly beyond 5 cm from the excitation point, but it attenuates significantly within 5 cm. These facts indicate that the electromagnetic field structure around a vertical parallellepiped having a cross-sectional area of 1 cm x 1 cm is close to TEM except for the immediate vicinity of the excitation point. Since differences among field waveforms shown in Fig. 3 (d) are small and the corresponding current wave [see Fig. 2 (d)] attenuates little beyond 5 cm from the excitation point, the electromagnetic field structure around a vertical parallellepiped having a cross-sectional area of 3 cm x 3 cm is also close to TEM at points distant from the excitation point.

IV. DISCUSSION

In the preceding section, we have shown that a current wave suffers neither attenuation nor dispersion in propagating from the cone apex to its base and the resultant field structure is TEM, while a current wave suffers significant attenuation and dispersion in propagating from the cone base to its apex. In this section, we examine the field structure around a vertical phased current source array, which simulates unattenuated propagation of current wave at the speed of light. Also, we examine the field structure around a vertical in-phase current source array, each placed between two horizontal perfectly conducting planes. The behavior of impedance seen from the input terminals has been employed by Chisholm and Janischewska [6] in order to represent the apparent time-varying grounding impedance.

Fig. 4 shows a vertical phased array of 40 current sources placed between two horizontal perfectly conducting planes 40 cm apart, to be analyzed using the FDTD method. Each current source produces a Gaussian pulse having a magnitude of 1 A and a half-peak width of 0.33 ns, which is turned on in a manner such that to simulate a wave moving downward at the speed of light.

Fig. 5 shows FDTD-calculated waveforms of vertical and horizontal electric fields at two observation points located (a) 40 cm away from the top of the vertical phased current source array shown in Fig. 4.

Fig. 6 shows a vertical in-phase array of 40 current sources placed between two horizontal planes 40 cm apart. Every current source produces a Gaussian pulse having a magnitude of 1 A and a half-peak width of 0.33 ns. In contrast with the phased current source array shown in Fig. 4, all sources are turned on simultaneously. As a result, the array shown in Fig. 6 generates a cylindrically expanding wave between the horizontal planes.

Figs. 7 (a) and (b) show FDTD-calculated waveforms of voltage between the horizontal planes (evaluated integrating vertical electric field between them) and current propagating radially along the upper plane (evaluated using Ampere’s Law and azimuthal magnetic field just below the upper plane) at radial distances of 20, 40, 60, and 80 cm from the vertical in-phase current source array. Table 1 shows values of characteristic impedance of two parallel planes evaluated as the ratio of the FDTD-calculated voltage and current at radial distances of 20, 40, 60, and 80 cm from the array.

The inductance \( L \) and capacitance \( C \) of two parallel rings whose vertical spacing is \( h \), inner radius \( r \), and radial width is \( \Delta r \) for a current wave propagating radially from excitation terminals (see Fig. 6), are given by

\[
L(r) = \frac{\mu_0 h \Delta r}{2 \pi r}, \quad C(r) = \frac{\varepsilon_0 \pi r \Delta r}{h}. \tag{2}
\]

The square root of the ratio \( L(r) / C(r) \) is given by

\[
\sqrt{L / C} = 60 h / r \quad [\Omega] \tag{3}
\]
which is equal to the characteristic impedance $Z_c$ of two parallel planes derived in [10]. Note that $Z_c$ decreases with increasing radial distance $r$ from the excitation terminals.

Values of $Z_c$ evaluated for $h=40$ cm, and $r=20, 40, 60,$ and $80$ cm are given in Table I. It is clear from Table 1 that FDTD-calculated values of characteristic impedance agree well with corresponding theoretical values.

![Fig. 6](image)

**Fig. 6.** A vertical in-phase array of 40 current sources in air placed between two horizontal perfectly conducting planes 40 cm apart, to be analyzed using the FDTD method. Every current source produces a Gaussian pulse having a magnitude of 1 A and a half-peak width of 0.33 ns. All current sources are turned on simultaneously.

**TABLE I. VALUES OF CHARACTERISTIC IMPEDANCE OF TWO PARALLEL PLANE EVALUATED AT RADIAL DISTANCES OF 20, 40, 60, AND 80 CM FROM THE VERTICAL IN-PHASE CURRENT SOURCE ARRAY (SEE FIG. 6), OBTAINED FROM THE FDTD-CALCULATED VOLTAGES AND CURRENTS SHOWN IN FIG. 7, AND THOSE CALCULATED USING (3).**

<table>
<thead>
<tr>
<th>$r$, cm</th>
<th>FDTD, $\Omega$</th>
<th>Eq. (3), $\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>108</td>
<td>120</td>
</tr>
<tr>
<td>40</td>
<td>57</td>
<td>60</td>
</tr>
<tr>
<td>60</td>
<td>39</td>
<td>40</td>
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<tr>
<td>80</td>
<td>29</td>
<td>30</td>
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In small-scale experiments with a conical conductor placed between two horizontal conducting planes, Chisholm and Janischewskyj [6] have detected a lower than expected current at the apex of the conical conductor. They ascribed this current deficit at the cone apex to a fictitious non-zero (initially about 60 $\Omega$ and decreasing with time) grounding impedance to the bottom perfectly conducting plane, and represented this time-varying grounding impedance using (3), in which $r$ was replaced by $ct$, with the initial value of $r$ being set at $h/c$.

Fig. 8 shows a perfectly conducting cone placed between two horizontal conducting planes 40 cm apart, which simulates a small-scale time-domain reflectometry (TDR) experiment that was carried out by Chisholm and Janischewskyj [6] and is to be analyzed using the FDTD method. A step voltage of 200 V generated by the source divides equally between the 50-$\Omega$ series resistor and the 50-$\Omega$ characteristic impedance of the coaxial cable connecting the source to the apex of the cone. As a result, a step voltage of 100 V is applied to the apex of the conical conductor. The voltage between the center conductor and the outer shield of this coaxial cable is monitored near the source.

Fig. 9 shows the FDTD-calculated waveform of voltage (thinner solid line) and the corresponding measured waveform (thicker solid line). The FDTD-calculated waveform agrees fairly well with the measured waveform. The first reflection...
from the junction between the 50-Ω coaxial cable and the conical conductor arrives at the voltage measurement point around 3.3 ns, and the second reflection from the bottom conducting plane arrives there around 6 ns. The magnitudes of measured and calculated voltages in Fig. 9 are almost constant from 0.2 ns to 3.3 ns because of the constant characteristic impedance of the coaxial cable. The constant magnitude of voltage from 3.4 ns to 6 ns indicates that the characteristic impedance of the cone is constant until the wave propagating downward from the cone apex encounters the bottom conducting plane. The latter characteristic impedance value is estimated to be about 140 Ω from either FDTD-calculated or measured results and equal to the theoretical value (140 Ω) of characteristic impedance of biconical antenna given [7] by

\[
Z_C(\alpha) = 60 \ln \left( \cot \frac{\alpha}{2} \right)
\]

(4)

where \(\alpha\) is the half-cone angle. Since \(\alpha = 11.3^\circ = (\tan^{-1}(8/40))\) for the cone shown in Fig. 8, its characteristic impedance, according to (4), is equal to 140 Ω.

Chisholm and Janischewskij [6] have modeled their small-scale experiment by a 50-Ω lossless uniform transmission line (representing the coaxial cable) connected in series with a 140-Ω lossless uniform transmission line (representing the conical conductor) either short-circuited at its bottom end (\(Z_g=0\)) or terminated in lumped impedance, \(Z_g=60 \, h/(ct)\), with the initial value of \(r\) being set at \(h/c\) (broken- and gray-line curves, respectively, in Fig. 9). The waveform calculated assuming the apparent grounding impedance to follow 60 \(h/(ct)\) reproduces both measured and FDTD-calculated waveforms quite well. However, it is important to note that the apparent grounding impedance discussed in this paragraph is fictitious and constitutes an engineering approximation to account for neglected attenuation of waves propagating upward along the conical conductor from its base.

V. CONCLUSIONS

We have analyzed current waves propagating along vertical conductors of different type. A current wave suffers neither attenuation nor dispersion in propagating from the apex of a cone to its base, and the resultant field structure is TEM. On the other hand, a current wave suffers significant attenuation and dispersion in propagating from the base of cone to its apex, and the resultant field structure is non-TEM. In propagating along a vertical parallelepiped above ground, a current wave suffers attenuation and dispersion near the source, which become more pronounced as the thickness of parallelepiped increases, while the field structure away from the source region is close to TEM. The field structure around a vertical phased current source array, which simulates unattenuated propagation of current wave at the speed of light (possible only in an unrealistic case of zero-thickness conductor), is TEM. In the case of vertical in-phase current source array placed between two horizontal perfectly conducting planes, a cylindrically expanding TEM wave is formed, and the input impedance seen from the terminals of the array varies with time as 60ln\((h/ct)\). This time-varying impedance has been employed by Chisholm and Janischewskij [6] to represent the apparent grounding impedance of conical conductor located above conducting plane, which is an engineering approximation to account for neglected attenuation of waves propagating upward along the conical conductor from its base. These results are useful in developing engineering or equivalent-circuit models of transmission-line tower for lightning surge studies.

VI. ACKNOWLEDGMENT

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VII. REFERENCES


VIII. BIOGRAPHIES

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