Mode switching in modal domain models of overhead lines and underground cables

I. Zlatunić, S. Vodopija, R. Goić

Abstract- **This paper presents brief review of different methods for calculation of eigenvalues and eigenvectors for application in the modal transmission line theory. In this field of particular interest is the numerical phenomenon known as "mode switching". "Mode switching" is related to very specific frequencies and must be properly treated in modal domain models of overhead lines and underground cables. This paper summarizes the methods and identifies the differences among them. Impedance and admittance matrices of overhead lines and underground cables are calculated in wide range of frequencies in order to compare different methods.**

Keywords: **Eigenvalue, Eigenvector, Modal transmission line theory, Mode switching, Modal domain, Overhead line, Underground cable.**

I. INTRODUCTION

Underground cables and overhead lines are in EMTP-based simulation programs modeled as frequency-dependent elements known as FD (frequency-dependent) or ULM (Universal line model). A number of papers are strictly dealing with modeling electromagnetic transient behavior of overhead lines and cables [1]-[7]. Most of them use modal decomposition theory to decouple phase system in equivalent modal system as if it is consisted of single phase lines. In this process it is very important to develop suitable algorithm that will calculate eigenvalues belonging to the same set of eigenvectors. One of the most important condition that has to be fulfilled is that eigenvectors calculated from YZ are continuous and smooth throughout wide range of frequencies. If the standard routines are used for calculation of eigenvalues and eigenvectors it is found that eigenvalues and eigenvectors are not sorted properly and inevitable switching between modes can occur. For example, this can occur if we use standard Matlab function eig() or implementing basic QR or power method algorithm for calculation of eigenvalues and eigenvectors. To avoid this problem three methods which are commonly used in reported literature [1]-[3] are presented. The first one uses modified Jacobi algorithm [1], [2]. Jacobi algorithm is commonly used for solving eigenvalues and eigenvectors problems of real symmetric matrices and with certain changes it can be used for solving asymmetrical complex matrices [8]. The second one is the Newton-Raphson method [3]. This method is not often used in reported literature but can derive smooth and continuous eigenvectors and eigenvalues throughout wide range of frequencies. Since this routine utilizes the results from previous frequency as starting value for next frequency it is recommended to use another algorithm in order to get started with Newton-Raphson method. This will work efficiently only if the eigenvalues and eigenvectors do not vary widely from one frequency to the other. The third one also utilizes standard routines (power method, Jacobi method, matlab function eig and others) with correlation technique [3]. In this case mode switching is inevitable but can be recognized using correlation technique. The correlation technique checks if the eigenvectors belonging to the same set of eigenvalues are orthogonal from one frequency to the other.

Basic description and algorithm of aforementioned methods are provided in this paper. For this purpose, frequency dependent impedance and admittance matrices are calculated for underground cables and overhead lines using equations reported in [4]-[6]. The main objective of this paper is to review reported methods for solving eigenvalue and eigenvector problem in modal decomposition theory and to provide comprehensive description and implementation of them.

This paper is structured as follows: Section II outlines the problem of mode switching. Section III describes basic information of methods reported in literature for solving problem of mode switching. Section IV underlines the commonly used normalization routines for eigenvectors matrices. Section V presents simulation examples and results for simple circuits of overhead lines and cables. Section VI concludes with a discussion of the obtained results.

II. MODE SWITCHING IN THE MODAL DOMAIN MODELS

Figure 1. shows an example of multiple eigenvalue switchovers when calculating modal velocity for underground cable given in section V. Modal velocity is obtained for six modes of propagation a-f. In modal theory of underground cables, mode a in fig. 1 presents a zero sequence mode of conductor which is energized by injecting unit current into each cable conductor and extracting it from corresponding sheath. Modes b and c represent interconductor modes of propagation. Mode e is a zero sequence sheath mode and

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modes b and c are intersheath modes of propagation. Mode e has lowest modal velocity due to high inductive impedance of soil path. Modes a-c have amount of modal velocity in amount of 2/3 speed of light in cases of frequency larger than 1000 Hz.

Eigenvalues and eigenvector for every specific frequency are calculated using standard Jacobi routine. Almost the same results can be obtained when using other standard routines as matlab function eig(), QR algorithm, power methods and others. In this case when eigenvalues switching has occurred it also means that eigenvectors switch places at certain frequencies. Figure 1. shows an example of switchovers with modes a, b and c and f at frequencies near 10 Hz, 30 Hz, 40 Hz, 500 Hz, and 500 kHz.

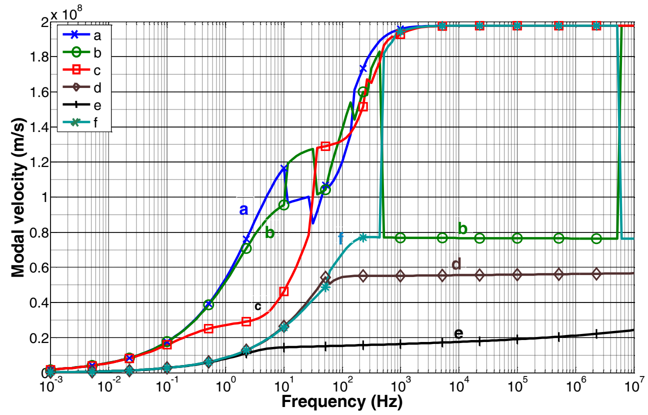


Fig. 1. Modal velocity characteristics of natural modes of propagation for the 110 kV underground cable with mode switching

According to [1], [2] two conditions have to be fulfilled in order to model FD model of underground cables and power lines in modal domain:

- The eigenvectors calculated from products YZ have to be continuous and smooth throughout wide range of frequencies.
- In order to obtain minimum-phase–shift functions, eigenvectors of YZ matrices need to be normalized so that one of their elements becomes real and constant throughout wide range of frequencies.

It is obvious that first condition cannot be fulfilled with standard eigenproblem routines. For this reason special methods are developed in order to deal with the first condition. The second condition can be easily managed using appropriate normalization routine with eigenvectors.

III. Dealing With Mode Switching

This section presents basic information about three aforementioned methods. In order to avoid undesired overflow errors properly scaling is utilized dividing each element of matrix with element ($-\omega^2\varepsilon_0\mu_0$) before calling eigenvalue and eigenvector routine. At the end calculated eigenvalues must be multiplied with same element in order to obtain good results.

A. Modified Jacobi method

Standard Jacobi method uses Jacobi rotations to diagonalize real and symmetric matrices and to find appropriate eigenvalues and eigenvectors [1], [2], [7]. The first assumption that has to be fulfilled is that matrix product YZ is diagonalizable. Our experience and data presented in literature show that in practical cases YZ for power cables and lines is always diagonalizable throughout wide range of frequencies (0.001-1 MHz). If this is not a case it is still possible to obtain eigenvector matrices using perturbation techniques in order to insure that eigenvalues are distinct [2].

In order to utilize Jacobi method adequate transformation is needed to insure that matrix is symmetric. Since matrix product YZ is asymmetric, eigenproblem (1) has to be transformed in (2).

$$YZQ = Q\lambda \tag{1}$$

$$HR = R\lambda \tag{2}$$

$$H = \sqrt{\lambda_Y}\, S^{-1} Z S \sqrt{\lambda_Y} \tag{3}$$

$$S\sqrt{\lambda_Y}\, R = Q \tag{4}$$

$$S^{-1} Y S = \lambda_Y \tag{5}$$

where:

- Y - admittance matrix,
- Z - impedance matrix,
- YZ - product of Y and Z matrices,
- Q - eigenvectors of matrix YZ,
- λ – eigenvalues of matrix YZ,
- H – symmetric matrix,
- R – eigenvectors of matrix H,
- S – eigenvectors of matrix Y,
- λ_Y - eigenvalues of matrix Y.

Eigenproblem (2) means that:

- The matrix H is symmetric throughout wide range of frequencies and the eigenvalues λ of H are the same as eigenvalues of YZ.
- Since H is symmetric, the eigenvector matrix satisfies $R^{-1}=R^T$.
- Eigenvectors Q of matrix product YZ can be obtained using equation (4).
- Eigenvectors S of Y can be easily obtained using standard routines. Admittance matrix is almost diagonal matrix and eigenproblem can be easily solved.

Using this approach it is possible to calculate eigenvalues λ and eigenvectors R applying standard Jacobi routine. After calculating eigenvector matrix R it is possible to calculate eigenvectors of matrix YZ using equation (4). In order to insure continuous and smooth eigenvectors and eigenvalues throughout wide range of frequencies it is necessary to utilize eigenvectors obtained in previous frequency step. This is done by equation (6) in which matrix H for present frequency is pre- and post – multiplied with eigenvectors calculated for previous frequency.

$$R_P^{-1} H R_P = Hn \tag{6}$$

Where:

- R_P is eigenvector calculated for previous frequency.
- Inversion of R_P is not needed since it is orthogonal and satisfies $R^{-1}=R^T$.
- Hn is new matrix which is almost diagonal and used for calculating eigenvalues and eigenvectors for present frequency.
- Since matrix Hn is almost diagonal Jacobi rotations will be almost unit matrix and this will effectively speedup calculations.

This method is tested for underground cables in [1], [2].

B. Newton – Raphson method

In Newton-Raphson method there is no need for symmetric matrix to calculate eigenvalues and eigenvectors of YZ. It utilizes equation (7) in order to find eigenvalues and eigenvectors

$$(YZ - \lambda_{kk} U)Q_k = 0 \tag{7}$$

Where:

- Q_k is eigenvector belonging to the eigenvalue λ_{kk}.
- U is unit matrix.

Equation (7) is repeated for all eigenvectors and eigenvalues in order to calculate exact values. As an example for a three phase power line or cable these equations can be written as:

$$(YZ_{11} - \lambda_{11})Q_{11} + YZ_{12}Q_{21} + YZ_{13}Q_{31} = 0 \tag{8.1}$$

$$YZ_{21}Q_{11} + (YZ_{22} - \lambda_{11})Q_{21} + YZ_{23}Q_{31} = 0 \tag{8.2}$$

$$YZ_{31}Q_{11} + YZ_{32}Q_{21} + (YZ_{33} - \lambda_{11})Q_{31} = 0 \tag{8.3}$$

In equations (8) there are four unknowns and three equations. In order to obtain solution it is possible to replace largest value at the initial frequency and set it equal to one for all frequencies. According to [3] this normalization is not good because it can sometimes lead to undesirable errors. Instead, they recommend specifying sums of the squares of the elements of the eigenvector to unity (8.4).

$$Q_{11}^2 + Q_{21}^2 + Q_{31}^2 - 1 = 0 \tag{8.4}$$

Using aforementioned four equations (8.1 to 8.4) and standard Newton Raphson method new function G(x) can be defined as:

$$G(x) = x - J(x)^{-1} F(x) \tag{9}$$

Where J(x) is the Jacobian matrix and F(x) is matrix formed by equations (8). Since Newton–Raphson is iteration method, initial values of eigenvalues and belonging eigenvectors is needed. Obviously the eigenvalues and eigenvectors obtained from previous frequency are used as starting values for present frequency. Also this method is not self-starting method and another routine must be used to calculate eigenvalues and eigenvectors for first frequency. This method is tested for overhead lines in [3].

C. Correlation technique

Correlation technique can be used to avoid multiple switchovers between eigenvectors. In order to find out "switching frequencies" it is necessary to track eigenvectors throughout the wide frequency range. The correlation technique proposed in [3] is based on the fact that the eigenvectors belonging to the same set of eigenvalue are orthogonal from one frequency to the other. The algorithm for correlation technique proposed in [3] can be summarized in five steps:

- Calculate the eigenvector matrix Q for present frequency.
- Obtain complex conjugate transpose matrix Q^{T*}.
- Calculate matrix product $(Q^{T*})Q_p$. Q_p is eigenvector matrix from previous frequency.
- In each row of the aforementioned product find the largest elements. The row number of this element defines the column number of the eigenvector from the previous frequency. Conversely, column number defines column number of the eigenvector at the present frequency. This means if mode switching has not occurred, all largest numbers will be sorted in diagonal of aforementioned matrix product.
- If eigenvector switching has occurred it is necessary to switch eigenvectors and eigenvalues in order to match previous frequency.

Our experience shows that this method works efficiently with standard eigenproblems routines (power method, matlab function eig() and jacobi method) but also shows that it cannot be applied with QR method.

IV. Eigenvector Normalization

According to [1], [2] author recommends to normalize eigenvectors of YZ matrices in order to obtain minimum phase shift functions. For underground cables author recommends to multiply each eigenvector by a factor such that one of its elements becomes constant and real throughout whole frequency range. This scaling process automatically forces all elements of eigenvector to be minimum-phase-shift-functions.

In [3] authors recommend using sums of squares of the elements of the eigenvectors to unity. In this paper author presents that forcing one element to be real and constant can sometimes take away natural variation behavior and produce undesirable errors. In this paper for the purpose of comparison in section V the first normalization routine will be utilized for all methods. In future research it is recommended to define optimal normalization routines for overhead lines and underground cables.

V. Simulation Examples and Results

For simulation purposes two examples are presented. First simulation example with results is presented for 110 kV underground cables and second one for 110 kV overhead lines. Matrices Y and Z for underground cables with sheaths are calculated throughout wide range of frequencies (0.001 Hz –

10 MHz) with 15 points for every decade in log scale. Matlab function is implemented to calculate elements of impedance and admittance matrices using well known equations [4]. Frequency dependent internal impedance of core and sheaths are calculated using full classical formulas. Earth-return impedance and mutual earth-return impedance are calculated solving Pollaczek integral with convergent series and by implementing numerical integration method using cautious, adaptive Romberg extrapolation [4]. Also calculations are implemented and compared with approximate formulas given in [4]. All results are obtained based on homogenous soil resistivity. Physical and geometry data for 110 kV three phase underground cables with sheaths and isolation are shown in figure 2. The cables used as an example in figure 2. are single core, with aluminium phase conductors, with cross–linked polyethylene insulation (XLPE) and semi-conductive layer beneath and over insulation. Outer insulation is high-density polyethylene (PEHD).

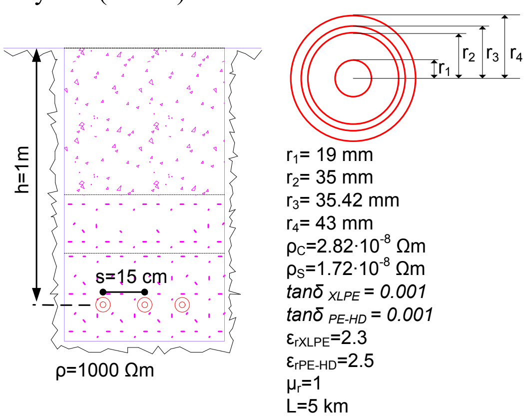


Fig. 2. Data and geometry for 110 kV three phase underground cables

For comparison purposes smooth and continuous functions of amplitude and angle of eigenvectors for cables are presented in Figure 4. Figure 3 shows modal velocity for six natural modes of propagation (a-f). Correlation technique is used with standard matlab function eig() and scaling process of eigenvectors is made according to section IV. Simulation with different kinds of cables geometry and physical data shows that all methods give good and almost identical results. Newton-Raphson method and modified Jacobi methods give reasonably good results after few iteration. Maximum relative error between these methods in this example do not exceed 0,3%, which is negligible. The differences among correlation technique and Newton-Raphson method (4 iteration used in simulations) used in this simulation are shown in figure 5. Figure 6 shows differences among correlation technique and modified Jacobi method. In this case for modified Jacobi algorithm is implemented with average 9 iteration for column 5 eigenvector.

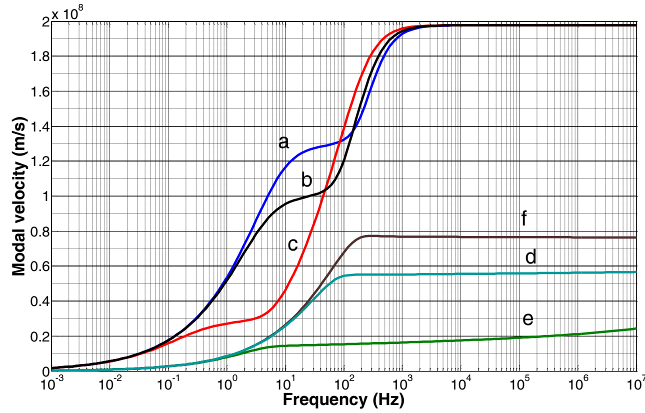
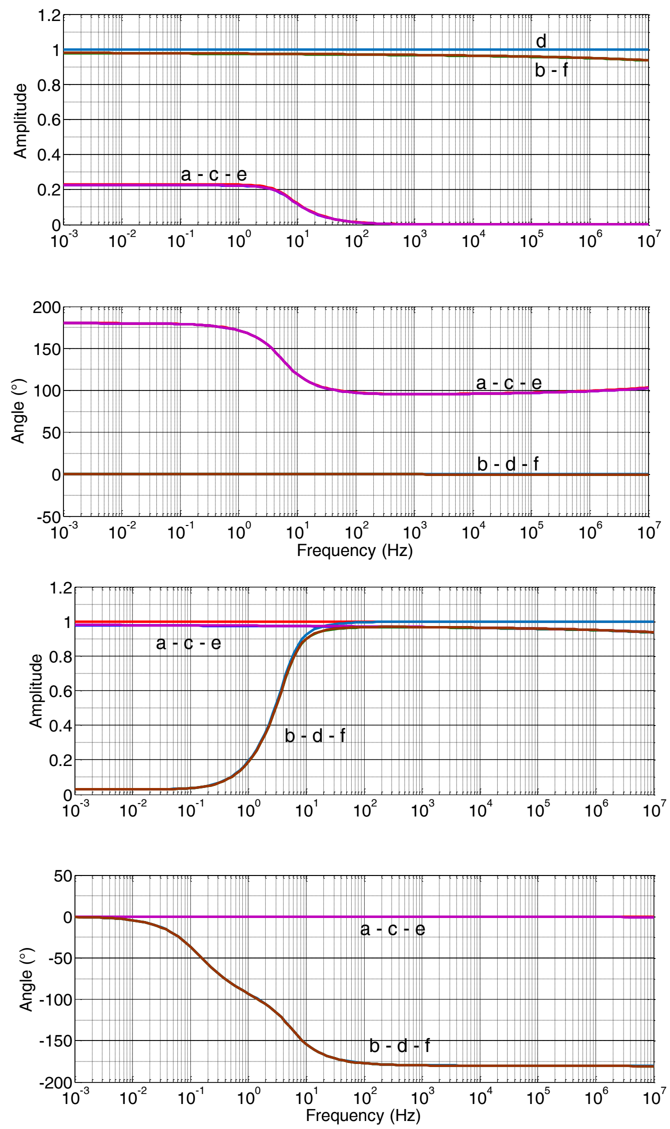
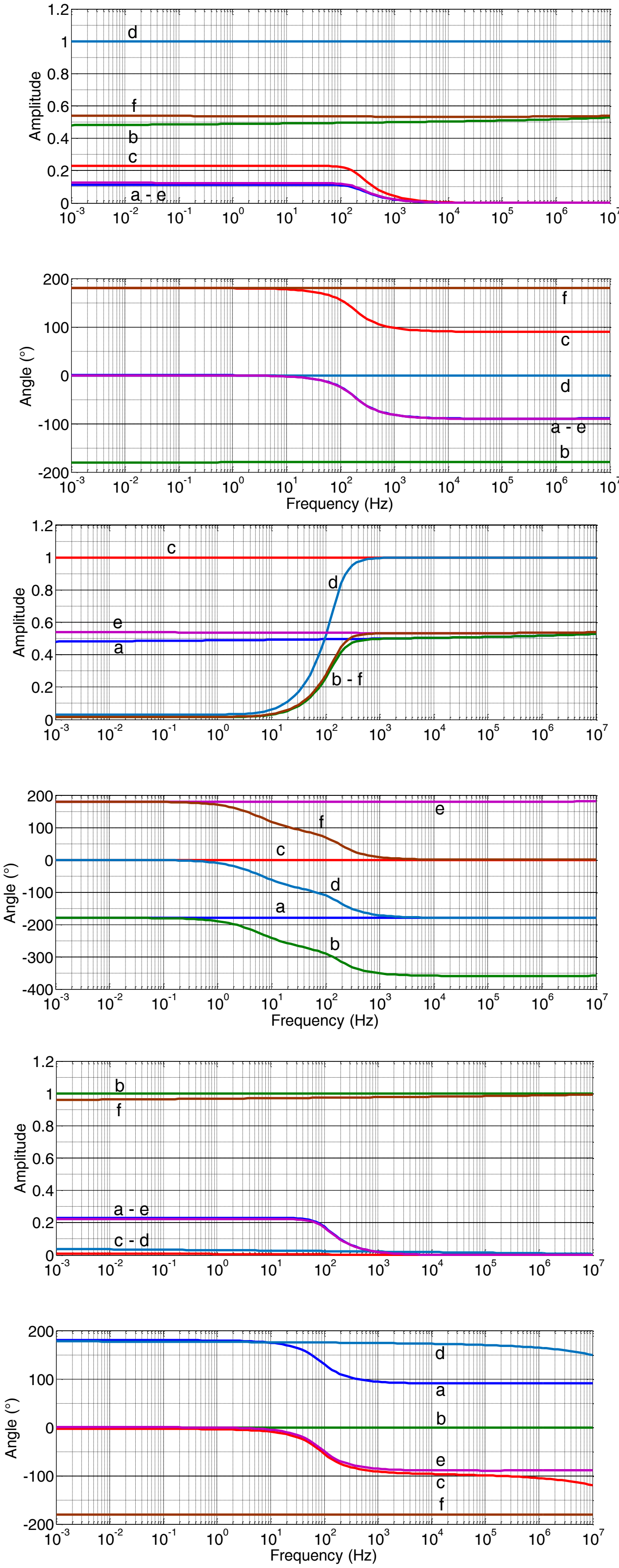


Fig. 3. Modal velocity characteristics of natural modes of propagation for the 110 kV underground cable without mode switching





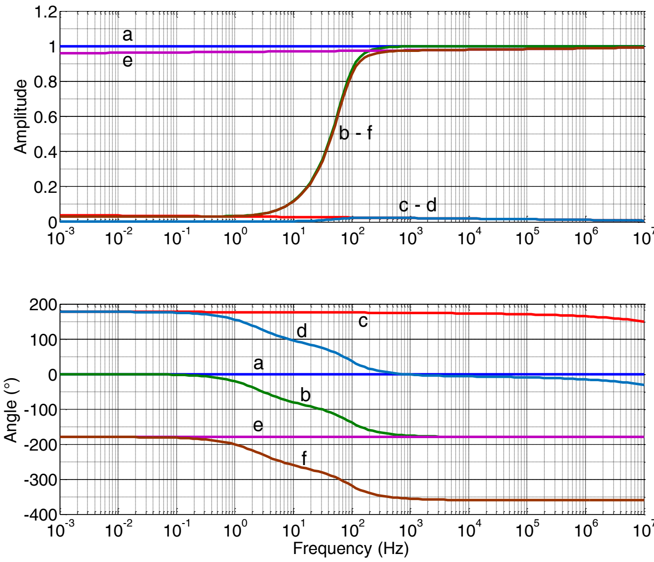


Fig. 4. Amplitude and angle of eigenvectors 1 to 6 for underground cable

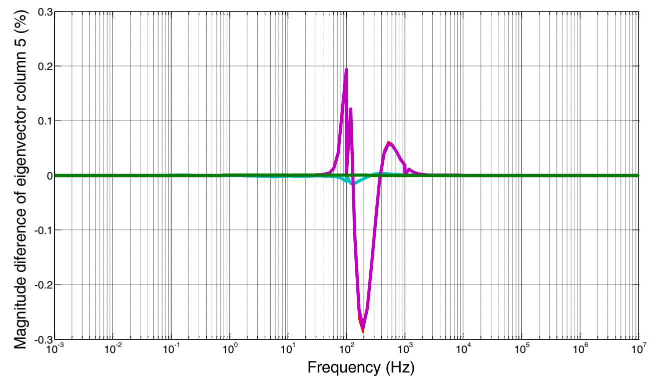


Fig. 5. Magnitude difference of Newton-Raphson (iter. 4) compared to Correlation technique

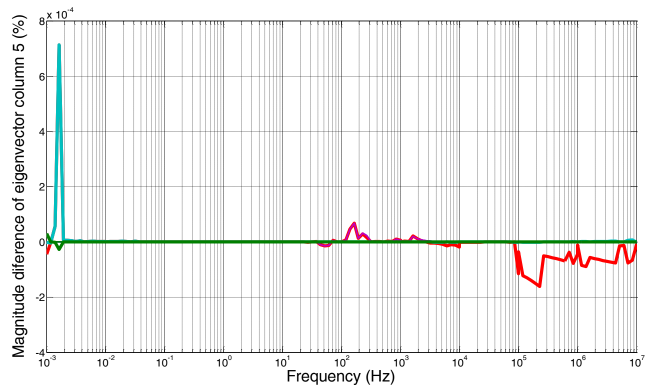


Fig. 6. Magnitude difference of Jacobi method (average iteration 9) compared to Correlation technique

Similarly, for overhead lines, calculation of matrices Y and Z are also implemented in matlab throughout same range of frequencies. For overhead lines explanations about modes of propagation are similar with cables.

Figure 8 shows continuous and smooth function of modal velocity for four natural modes of propagation (zero sequence modes and interconductor modes) for overhead lines with assumption of uniform soil resistivity. At high frequencies modal velocity for all modes has amount near speed of light.

Physical and geometry data for three phase overhead line with one sky wire is shown in figure 7. Figure 9 shows calculation functions of angle and amplitude of four eigenvectors scaled in accordance with section IV.

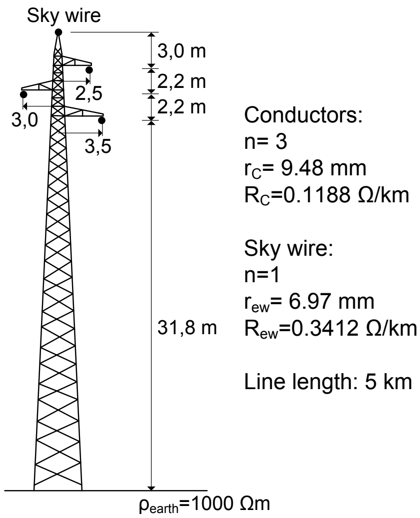


Fig. 7. Data and geometry for 110 kV three phase overhead line

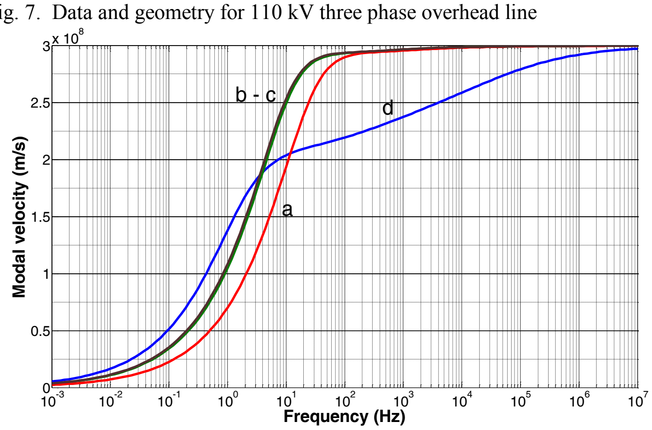
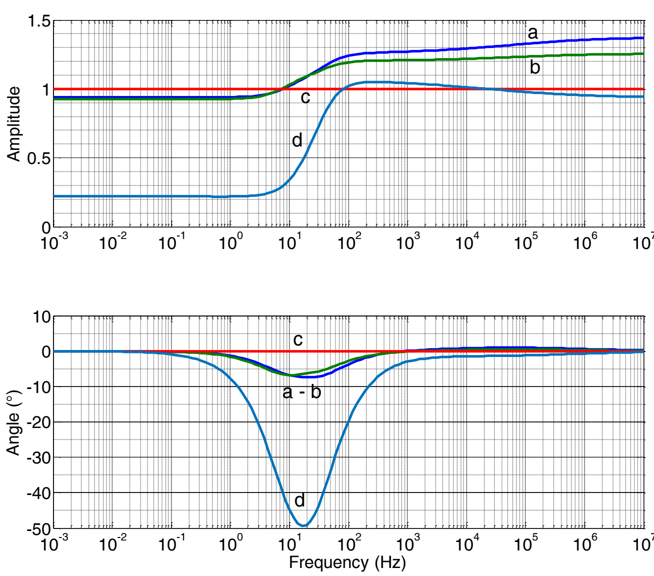


Fig. 8. Modal velocity characteristics of natural modes of propagation for the 110 kV overhead lines without mode switching



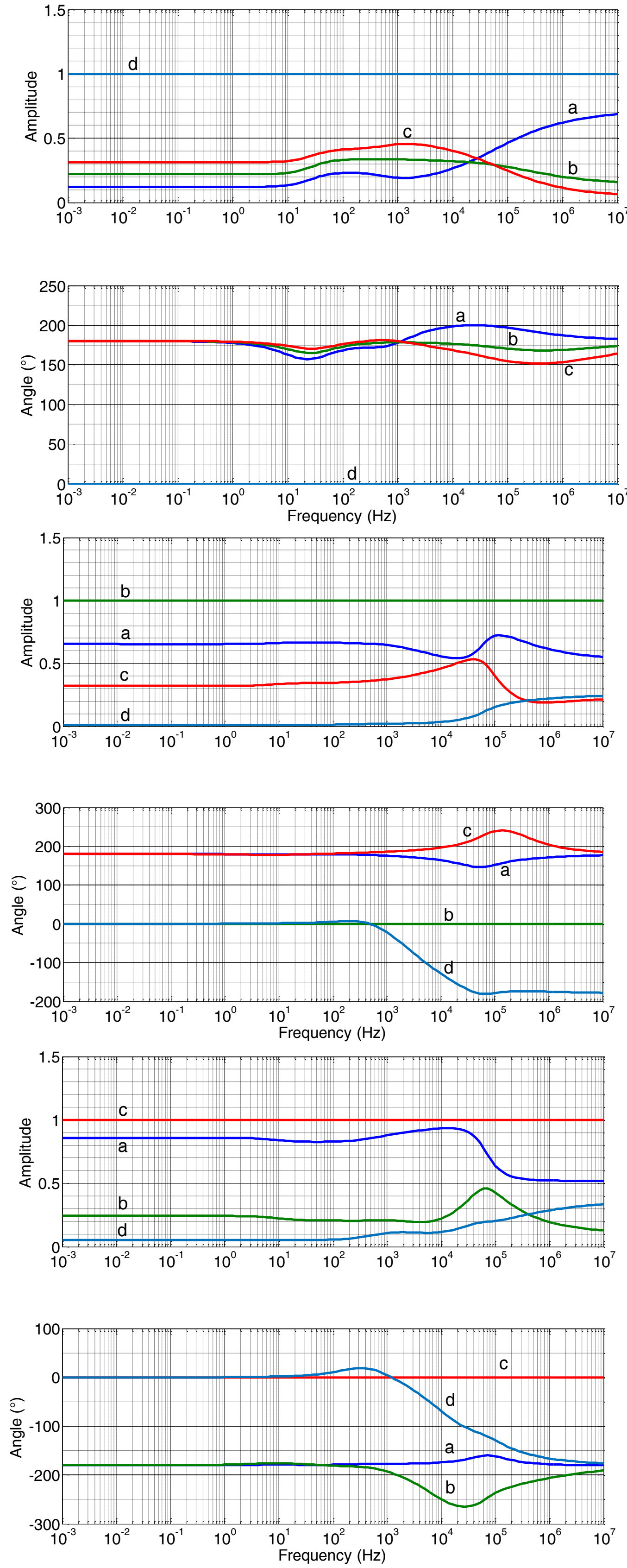


Fig. 9. Amplitude and angle of eigenvectors 1 to 4 for overhead line

VI. CONCLUSIONS

This paper provides review, implementation and simulation examples of methods for dealing with mode switching in modal transmission line theory. The methods reported in literature are: modified Jacobi algorithm, Newton – Raphson method and correlation technique. All methods can be easily implemented in EMTP programs and produce smooth and continuous functions of eigenvalues and eigenvectors throughout wide range of frequencies. The comparison between methods using aforementioned underground cables and overhead lines shows that differences are negligible but future analysis with more complicated geometry models of cables (pipe type cable and others) and overhead lines need to be investigated. According to reported literature mode switching is important to solve only in modal domains FD models because eigenvectors functions need to be continuous and smooth throughout wide range of frequencies. If phase models are used then mode switching is not important.

Newton-Raphson method and modified Jacobi method utilize eigenvectors calculation from previous frequency step and that is reason way functions are smooth, continuous and the calculation process is very fast. Correlation method uses tracking algorithm in order to recognize mode switching.

VII. References

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