A Modeling Methodology for Inductive and Capacitive Voltage Transformers for High-Frequency Electrical Transients Analysis

M. C. Camargo, G. Marchesan, L. Mariotto, G. Cardoso Junior, L. F. F. Gutierres

***Abstract*—A method to obtain the inductive and capacitive voltage transformers models for high-frequency electrical transients analysis is presented. These transformers are characterized by its short-circuit admittance matrix over a wide frequency band, which is obtained from a commercial sweep frequency-response analysis measurement instrument. This matrix is used as input data to the Matrix Fitting routine to get models that can be used in Electromagnetic Transients programs such as ATP/EMTP. The methodology has been applied on a 138 kV IVT and a 230 kV CVT.**

***Keywords*: inductive voltage transformer model, capacitive voltage transformer model, high-frequency transients analysis, ATP/EMTP.**

I. Introduction

INDUCTIVE and Capacitive Voltage Transformers, IVTs and CVTs, play an important role in measuring, protection and control in Electric Power Systems (EPS). Notwithstanding, it is known the IVT's and CVT´s difficulty to quickly represent the changes in primary voltage on its secondary terminal. This is due to the various types of disturbances that EPS are subjected, such as lightning and system faults. These events result in electrical transients, what may cause failure of protective devices, misoperations and delayed operations of relays [1]-[4]. In the last decades has intensified the use of numerical relays that are faster and sensitive, with this, increased the importance of the study of the high-frequency electromagnetic transients effects on IVTs and CVTs.

There are different IVTs and CVTs models available in the literature. While one line of research focuses on analyzing their responses through its Transfer Functions, another invests in its modeling by electrical equivalents circuit [5]-[9]. However, they have limitations since it is valid for a small frequency band, generally up to 10 kHz. Furthermore, it is necessary to know all the values of their electrical parameters and mutual, inductive and capacitive coupling, besides your physical and constructive features. The models designated to electromagnetic transient analysis should consider its nonlinear behavior as well as the frequency-dependent effects, and they are not available. In order to meet this gap, the present work proposes the IVT and CVT high-frequency models for electromagnetic transients analysis, up to 3 MHz.

II. Model development

The methodology relies on the "black-box" modeling, where a two-port network represents the equipment and it expresses its behavior seen by its terminal for a wide frequency band. For a better understanding, the model development is divided in two stages, described below.

A. Frequency Response Measurements

The first step towards this modelling is to apply a Sweep Frequency-Response Analysis (SFRA) to obtain the IVT's and CVT´s short-circuit Admittance Matrix. Both IVT and CVT are modeled here by a 2x2 matrices order. To accomplish this test, a commercial Sweep Frequency Response Analyzer (SFRA) with a special procedure described in [10] was carried out.

Fig. 1, Fig. 2, Fig. 3 and Fig. 4 show how to perform the measurements of the voltage ratios, which will generate the admittance matrix, using the SFRA device with a nonstandard connection type. In these figures, "Measurement", "Reference" and "Output" are the terminals available of the SFRA equipment. In the diagonal elements measurements, the shield of these cables must be connected together but not grounded, while for the off diagonal elements all the shields should be grounded. Furthermore H and X represents, respectively, the Primary and Secondary Voltage Transformer's terminals. Note that for the indexes notation of the (2x2) admittance matrix: H_1=1 and X_1=2.

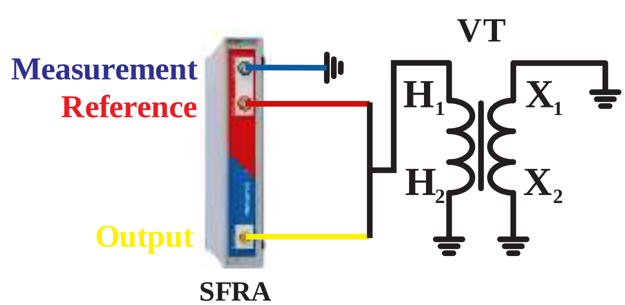

Fig. 1. SFRA connections for the measurement of the voltage ratio V11.

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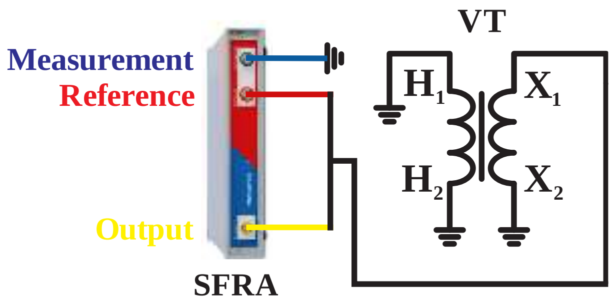

Fig. 2. SFRA connections for the measurement of the voltage ratio V22.

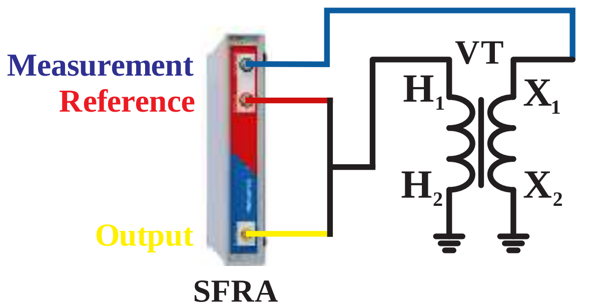

Fig. 3. SFRA connections for the measurement of the voltage ratio V21.

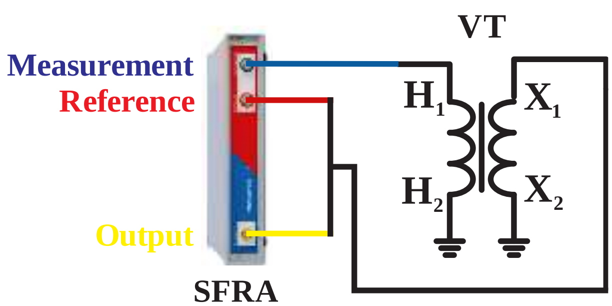

Fig. 4. SFRA connections for the measurement of the voltage ratio V12.

Note that, this equipment is designed for another purpose, to perform voltage transfer studies to assess possible mechanical deformations or internal faults in transformers. Therefore, the acquired data from measurements needs to be corrected and recalculated, through an external routine, in order to adequately represent the mutual and self-admittance values of the Matrix, as detailed in [10] and briefly described in the Appendix.

B. *Rational Approximation by Vector Fitting*

The rectified Admittance Matrix serves as input to the Matrix Fitting method, a frequency-response approximation tool by means of modified rational functions, which is part of the Vector Fitting (VF) routine [11-13].

The Vector Fitting approximates the frequency response f(s) by means of rational functions expressed by partial fractions as shown in (1), where 'd' and 'e' are optional, and 'r' and 'p' are, respectively, the residuals and poles.

$$f(s) \approx \sum_{k=1}^{N} \frac{r_k}{s - p_k} + d + se \quad (1)$$

The VF is an iterative pole relocation technique carried out by repeated solutions of linear problems until its convergence is reached. Freely available for non-commercial purposes in MATLAB®, the Matrix Fitting Toolbox latest version also enforces the function's passivity [14-17]. The results are an approximated rational function, in state-space or pole-residue model, and an RLC equivalent network proper to be used in electromagnetic transients analysis software, such as the Alternative Transients Program (ATP).

Roughly, the aim in this stage is to obtain an equivalent network, whose nodal admittance matrix matches the original transformer's admittance matrix.

III. Model's Creation and High-Frequency Time-Domain Validation

Time-domain measurements and simulations were performed for both Inductive and Capacitive Voltage Transformers in order to validate their models in high frequencies. There were performed two types of measures for each device to be studied: the SFRA procedure to get the voltage ratios and a voltage step response test. The low-amplitude voltage step was applied to the IVT and CVT high voltage terminal using a function generator and the measured was accomplished using an oscilloscope. On the other hand, for the comparisons purpose, the response models were verified through EMTP/ATP simulations under same conditions.

For both IVT and CVT modeling, the order of approximation in the Matrix Fitting was 140 poles, and the passivity enforcement has been applied.

A. *Inductive Voltage Transformer Results*

The 138 kV IVT manufacturer data available for the laboratory measurements is shown in Table I.

TABLE I
138 kV IVT Manufacturer Data

Primary Voltage (kV)	138/√3
Secondary Voltage (V)	100/√3
IVT ratio	1380:1
Frequency (Hz)	60

Fig. 5 shows the measured IVT's admittance (in blue), its approximation through VF (in red) and the deviation between both data (in green). For the SFRA procedure 150 frequencies logarithmically distributed between 20 Hz and 3 MHz were considered. As a result, from the Admittance Matrix fitting operation, the IVT's RLC equivalent model was created.

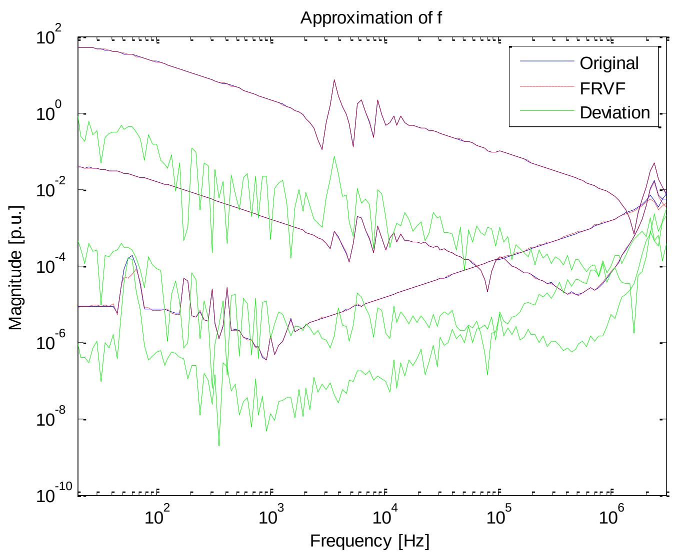

Fig. 5. IVT's Admittance Matrix approximation by Vector Fitting results.

A 20 V step was applied to the IVT's high voltage terminal. The applied and the secondary voltage are shown, respectively, in the top and bottom waveforms in Fig. 6 obtained from the oscilloscope.

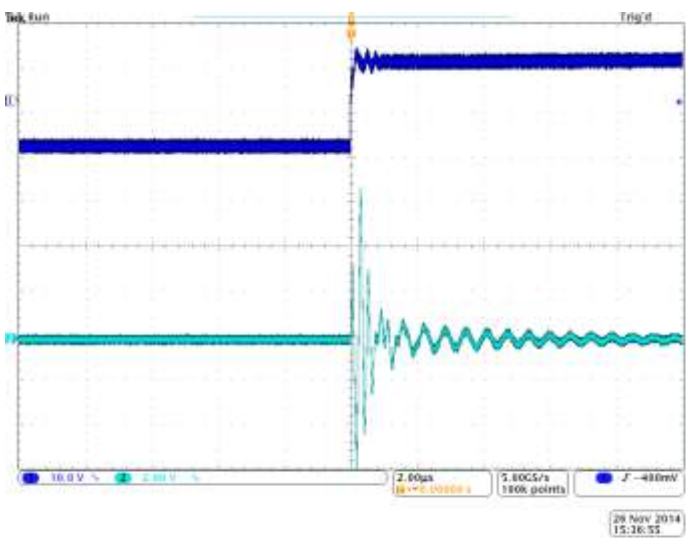

Fig. 6. IVT's 20 V peak-to-peak step applied on high-voltage terminal (top) and secondary response (bottom).

The model's response for the same step voltage test is shown in Fig. 7, where the y-axis expressed the voltages in Volts and the x-axis the time in seconds. The 20 V peak-to-peak applied to the primary terminal and the secondary response are represented, respectively, by the top and bottom waveforms below.

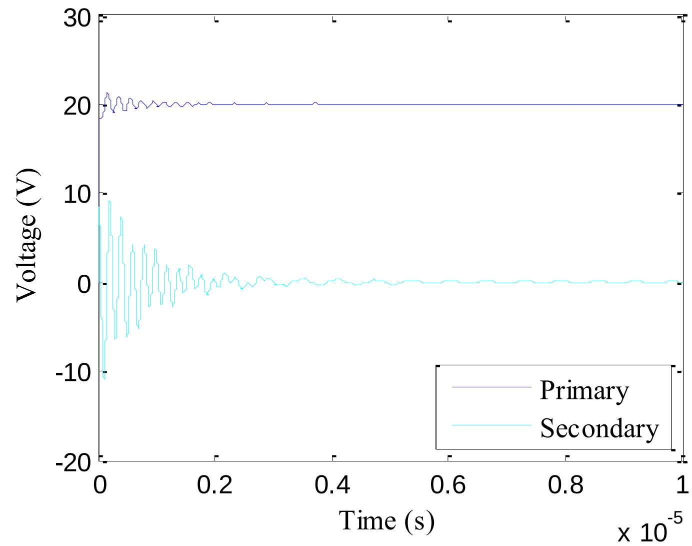

Fig. 7. IVT's 20 V step applied on model's high-voltage terminal (top) and its secondary response (bottom).

By comparing the step voltage test results from the laboratory measurements, Fig. 6, and the EMTP/ATP simulation, Fig. 7, one perceives that the waveforms are very close, whether be in amplitude, phase or frequency.

B. Capacitive Voltage Transformer Results

The 230 kV CVT used in this essay has its manufacturer data available in Table II.

TABLE II
230 kV CVT MANUFACTURER DATA

Primary Voltage (kV)	230/√3	
Secondary Voltage (V)	X1-X3	115
	X2-X3	115/√3
Intermediate Voltage (kV)	23/√3	
CVT ratio	1154.7 - 2000:1	
Capacitances (µF)	C1	0.0129
	C2	0.1160
Frequency (Hz)	60	

The measurements performed in the SFRA consisted of 400 frequencies logarithmically distributed between 20 Hz and 3 MHz. Fig. 8 presents the obtained CVT's admittance (in blue), its approximation through VF (in red) and the deviation between both data (in green). The CVT's RLC equivalent model was generated after the Admittance Matrix fitting operation.

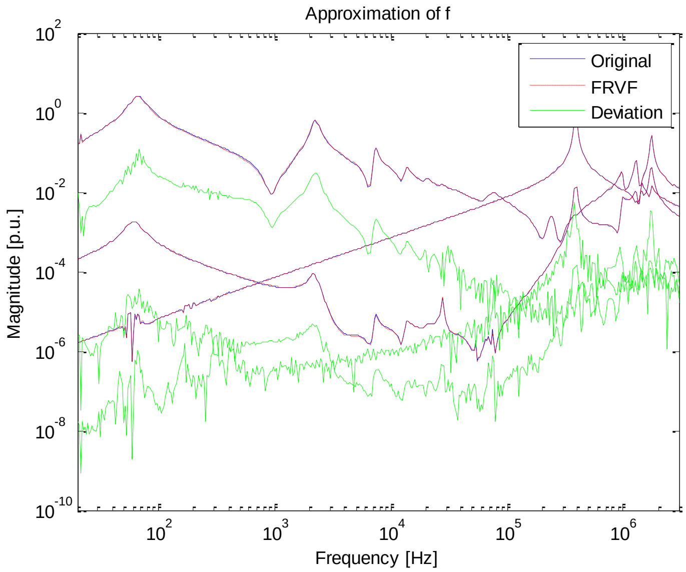

Fig. 8. CVT's Admittance Matrix approximation by Vector Fitting results.

A 20 V step was performed in the CVT high voltage side. The oscilloscope acquired the primary voltage and its secondary response. The same test conditions were applied to the CVT's model, obtained as a result from the fitting process, in EMTP/ATP simulations. Fig. 9 and Fig. 10 demonstrates the comparison between the CVT's measurement (red) and model's simulation (black) for, respectively, the applied step voltage and its response on secondary side.

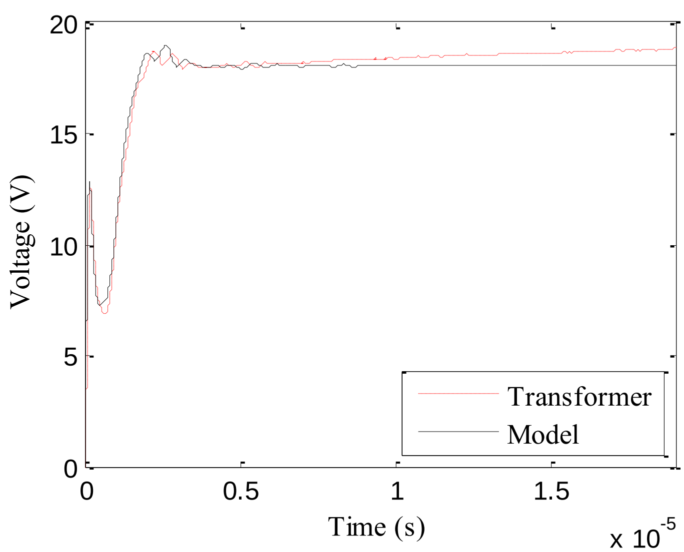

Fig. 9. CVT's input step voltage waveform, comparison between transformer measurements and simulated model.

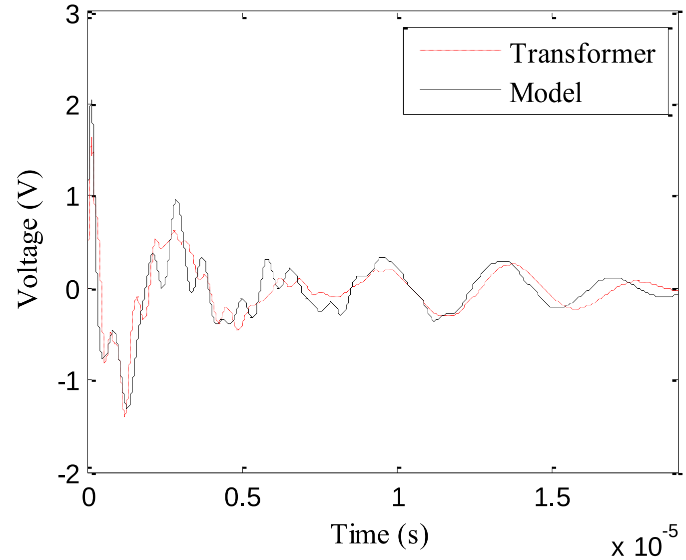

Fig. 10. CVT's secondary response voltage waveform, comparison between transformer measurements and simulated model.

As one can see, the waveforms are in a good agreement, although from 10 μs onwards there is a little discrepancy between both amplitudes and phases. It can be explained by the non-linearity present at lower frequencies in CVTs with inbuilt iron core on the secondary side of the intermediary transformer, which cannot be represented by the model.

IV. 60 Hz Time-Domain Simulations

The main objective of the proposed models it to represent VT's behavior for high-frequency studies. However, it is also important to check its validity under 60 Hz. Hence, after the high-frequency validation of the proposed IVT and CVT models, simulations through ATP Launcher were performed for each transformer in order to verify their 60 Hz performance. It should be mentioned that the amplitude value of the voltage source used in the ATP input file must be provided in its peak value per phase.

A. Inductive Voltage Transformer Simulation

As described before, the voltage transformation is a 138/√3 kV – 100/√3 V, that is 1380:1. In the simulation, the specified high-voltage was converted to its peak value and, then, applied to the IVT's model primary terminal. Fig. 11, presents the obtained secondary voltage, where the y-axis expressed the voltage in Volts and the x-axis the time in seconds.

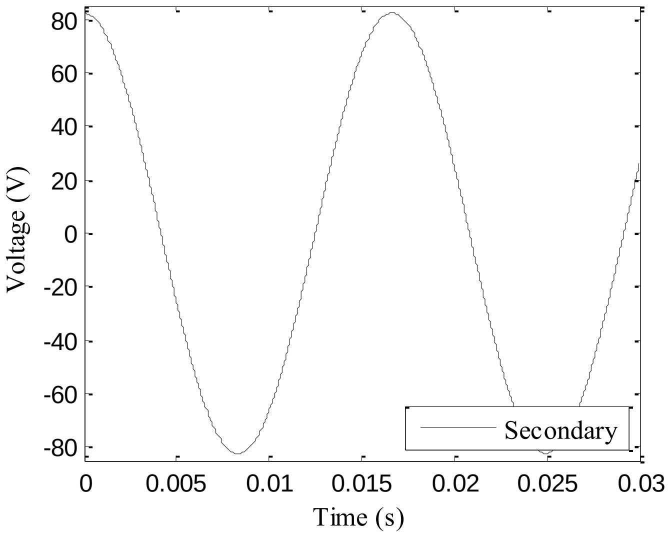

Fig. 11. Simulation result of IVT's model secondary voltage.

Instead of the ideal 57.735 V (100/√3 V), the measured voltage on the model's secondary terminal was around 58.231 V (82.35/√2 V), which results in a 1368:1 ratio. The error is minimum, over 0.86%. As can be seen, there is a very good match between the IVT's and its model.

B. *Capacitive Voltage Transformer Simulation*

The CVT measurements for its modeling were developed between its high-voltage H1-H2 and low-voltage X1-X3 windings. Thus, for this configuration, the corresponding relation is 230/√3 kV – 115 V, that is 1154.7:1. After its conversion to peak value, the specified high-voltage was applied in the model's input. Fig. 12, where the y-axis expressed the voltage in Volts and the x-axis the time in seconds, shows the measured secondary voltage.

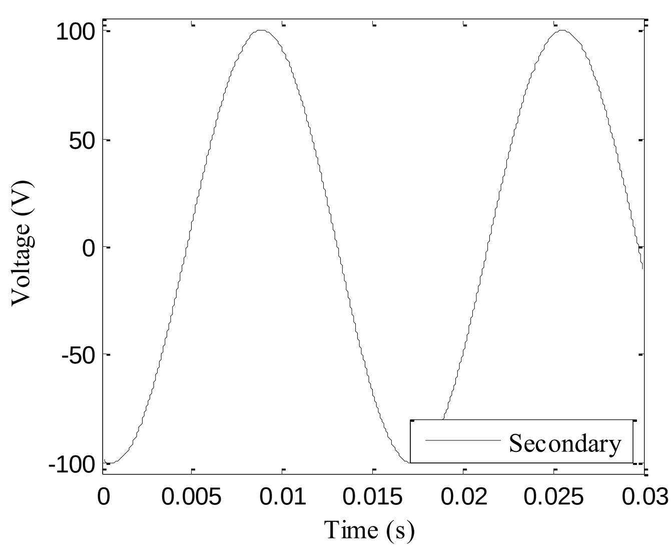

Fig. 12. Simulation result of CVT's model secondary voltage.

As one can see, there is some error in the result. The obtained RMS secondary voltage is 70.853 V (100.2/√2 V), which is deviated of 38.39% from the expected 115 V. Studies are being conducted to verify the reason of this mismatch.

V. Conclusions

In this paper a modelling method for Inductive and Capacitive Voltage Transformer for assessing its transient performance has been developed. The models were validated for high frequencies through the comparison between the laboratory step response and the ATP/EMTP simulations.

The IVT model is very precise for high-frequency components and it can accurately reproduce its real behavior in 60 Hz, as it is proved in the simulations on ATP Launcher. Then, this IVT model can be readily used for very fast transient studies.

As it is shown in the voltage step response, the CVT model is accurate for the high frequencies up to 3 MHz. On other hand, the CVT model needs more accuracy for 60 Hz components. As there are no records of CVT's modeling for high frequencies, this model is still relevant.

VI. Appendix

Often used for industrial offline transformer diagnostics, the SFRA measurement equipment are designed to perform voltage transfer studies on transformers and thus, are not able to measure current directly. However, this measurement is possible through an internal shunt resistor of 50 Ω [10]. Then, the acquired data from the SFRA needs to be corrected. The procedure's details for applications in Voltage Transformers, modeled here by a 2x2 matrices order, are presented below.

The SFRA measures the voltage ratio between the terminals Measurement and Reference, $R(s) = V_{MEAS}(s) / V_{REF}(s)$, by injecting voltage on its Output terminal. This measurement shows the magnitude and phase relation between the terminals, and it is presented on the polar form that is, composed by magnitude of voltage relations "Vji" (in decibels) and phase "Fji" (in degrees). Therefore, despite the conversion of the magnitude from logarithm to linear, it is also necessary to convert the data to the rectangular form (called here as "Tji"). This conversion is done for the diagonal ("Tii") and off diagonal ("Tji") elements respectively by (2) and (3).

$$T_{ii} = 10^{(V_{ii}/20)} e^{i \cdot (F_{ii} \cdot \pi / 180)} \tag{2}$$

$$T_{ji} = 10^{(V_{ji}/20)} e^{i \cdot (F_{ji} \cdot \pi / 180)} \tag{3}$$

After the conversion, it is possible to obtain the correct elements of the admittance matrix Y(s). The corrected self-admittance of terminal *i*, which corresponds to the diagonal elements of Y(s), is defined by (4) [10].

$$Y_{ii}(s) = \frac{V_{MEAS}(s)}{Z_{in}\left(V_{REF}(s) - V_{MEAS}(s)\right)} \tag{4}$$

Dividing both numerator and denominator by the reference voltage, "V_{REF}", (4) becomes:

$$Y_{ii}(s) = \frac{V_{MEAS}(s)/V_{REF}(s)}{Z_{in}\left(1-\left(V_{MEAS}(s)/V_{REF}(s)\right)\right)}. \quad (5)$$

Considering that the voltage transfer "Tii" is equal to $V_{MEAS}(s)/V_{REF}(s)$, and the internal impedance of the instrument, "Zin", is 50 Ω, (5) becomes:

$$Y_{ii}(s) = \frac{T_{ii}(s)}{50(1-T_{ii}(s))}. \quad (6)$$

On other hand, the mutual admittance between terminals *j* and *i*, corresponding to the off diagonal elements of Y(s), is

$$Y_{ji}(s) = -\left(\frac{V_{MEAS}(s)}{Z_{in}\cdot V_{REF}(s)} + Y_{jj}\cdot\frac{V_{MEAS}(s)}{V_{REF}(s)}\right). \quad (7)$$

In a similar manner, from (4) to (5) and substituting the value of "Zin", (7) becomes:

$$Y_{ji}(s) = -\left(\frac{T_{ji}}{50} + Y_{jj}\cdot T_{ji}\right). \quad (8)$$

VII. ACKNOWLEDGMENT

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