Improving the Performance of the Lumped Parameters transmission Line Model by Using Analog Low-Pass Filters

A. R. J. Araújo, R. C Silva, S. Kurokawa

Abstract—This paper proposes a simple and efficient method to improve the performance of the lumped parameters transmission line model used to simulate electromagnetic transients in electric power systems. The lumped parameters model presents in its simulations numeric spurious oscillations that do not represent the real value of the transient. To mitigate these oscillations, an analog low-pass filter will be designed and inserted in the traditional lumped parameters transmission line model. To verify the proposed mitigating procedure for spurious oscillations, a three-phase transmission line will be decomposed in 3 single-phase transmission lines using modal decomposition. Each single-phase line will be represented by lumped parameters transmission line model with and without analog filters inserted. It will be used the distributed transmission line parameters model, that uses algebraic equations in frequency domain to evaluate voltages and currents and it is considered the ideal response, to compare the results. Once obtained the voltages at receiving end for each single-phase line, one can obtain the three-phase voltages at the receiving end. The proposed filter is an efficient tool to mitigate the spurious numeric oscillation and the results are accurate and reliable for the analysis in electromagnetic transients in power systems.

Keywords: Analog filter, lumped parameters model, distributed parameters model, electromagnetic transients, transmission line.

I. INTRODUCTION

It is known that an efficient representation of transmission lines must takes into account the fact that longitudinal and transversal parameters are distributed along the length line [1]. Due to this consideration, the currents and voltages along the line are described by differential equations whose solutions are not easily found in time domain. Since 1960 several researchers have been dedicating efforts to develop and to improve transmission line models for simulating electromagnetic transients in electric power systems [2-4].

Lumped parameters transmission line model (LPM) has been frequently used for representation of transmission line. It considers that a segment of single-phase transmission line, whose parameters are distributed along its length, can be represented by lumped resistor, inductor, capacitor and conductance association constituting a π-circuit [2-5] and, consequently, a transmission line is approximated represented by a π-circuits cascade [1]. The LPM can be used as an efficient model to represent single-phase and multiphase lines [1-5]. This model can be used for analyzing transients resulting from sudden changes in the network configuration, like as faults and opening/closing of circuit breakers.

For representing multiphase lines, modal decomposition theory is used and a n-phase transmission line can be represented in modal domain as being n uncoupled single-phase lines [5,6].

The LPM is an efficient representation for transmission lines in simulations of electromagnetic transients resulting from faults switching on/off breakers and because it is developed directly in time domain, this model is fully compatible with Electromagnetic Transient Programs such as EMTP and ATP and it has the advantage of considering the nonlinear components inserted in the line, the loss for corona effect and fault arcs [7,8].

However, simulations with LPM is characterized by intrinsic high-frequency components associated with spurious oscillations that produces a general distortion of the wave form and exaggerated magnitude peaks [9]. These characteristics are not present if distributed parameters transmission line model are used. These models are obtained directly from solutions of the differential equations, written in frequency domain, and consequently, are more adequate to represent transmission lines but cannot be inserted in EMTP or ATP programs.
because they are not developed directly in time domain. When the differential equations in time domain are converted to hyperbolic algebraic equations in frequency domain, the distributed transmission line model are known as Universal Line Model (ULM) [10,11]. Once obtained the solutions in the frequency domain, it can be applied the Inverse Laplace Transform implemented by numeric methods and then the solution for the voltages and currents in the time domain [12]-[13].

To mitigate the spurious oscillations obtained in the simulations using LPM, in [14] is proposed a digital filtering. The simulation of an electromagnetic transient involving transmission lines is separated in two steps: First the transient simulations are obtained and keep up. The second step consist on apply the digital filter on it.

This work proposes to substitute the digital filter by analog low-pass filter. The advantage of this substitution consist on that analog filters can be directly inserted in the LPM and all the filtering process is done together with electromagnetic transient simulations and the second step is not requested. Thus all the simulations are performed in real-time. The LPM with analog filters results in an efficient and friendly tool to represent transmission lines during the electromagnetic transients on power systems.

II. LUMPED PARAMETERS TRANSMISSION LINE MODEL

The LPM considers that a small line segment can be represented, as being a π-circuit constituted by lumped circuit elements [1]. Then a generic single-phase line can be represented by a \( n \) π-circuits cascade as shown in Fig. 1.

In the Fig. 1, the series parameters \( R \) and \( L \) are the longitudinal resistance and inductance, respectively. The shunt parameters \( G \) and \( C \) are the transversal conductance and capacitance respectively being expressed by:

\[
R = R' \cdot \frac{d}{n}; \quad L = L' \cdot \frac{d}{n}; \quad G = G' \cdot \frac{d}{n}; \quad C = C' \cdot \frac{d}{n}
\]  

(1)

In (1) the \( R' \) and \( L' \) are the per-unit length longitudinal parameters and \( C' \) and \( G' \) are the per-unit length transversal parameters. The term \( d \) and \( n \) are the length line and the number of π-circuits cascade respectively. For the circuit shown in Fig.1, it is possible to write (2):

\[
\begin{bmatrix}
\frac{dx(t)}{dt} \\
\end{bmatrix} = [A][x(t)] + [B]u(t)
\]

In (2) the \([x(t)]\) is a vector that contains the longitudinal currents and transversal voltages in each π-circuit and \([A]\) and \([B]\) are state matrices. In the transmission line representation shown in Fig. 1, the currents and voltages along the line can be easily evaluated, directly in time domain, by any numerical integration method. However LPM introduces spurious oscillations in the simulations that do not represent the real waveform peaks and its frequency spectrum contains high frequency components. These peaks do not correspond to the real value of the transient, and can cause errors to the analysis if considered, as discussed further. To reduce them, an analog low-pass filter will be designed and directly inserted in the LPM as present following.

III. INCLUDING AN ANALOG FILTER IN THE LPM

Spurious numeric oscillations and exaggerated magnitude peaks resulting from lumped parameters transmission line model can be mitigated by digital filters [14]. In this case, simulation results obtained with the lumped parameters line model were filtered by a FIR digital filter and results obtained showed, after the filtering process, the simulation results obtained with LPM were very similar to results obtained with classic distributed parameters line model. However, the procedure proposed in [14] has as negative aspect the fact that the filter was not included in the line model and the use of the resulting model (line and digital filter system) requires to steps: First electromagnetic transient simulations, using the traditional lumped parameters line model, are evaluated and after that, the filtering procedure is evaluated. Therefore, the filtering procedure proposed in [14] increases the complexity of the simulation process and this fact can discourage the use of the lumped model with digital filter.

This work proposes to replace the digital filter by an analog low-pass filter. This way the electromagnetic transient simulations and the filtering process are evaluated on real-time .making this procedure easier than the previous one.

In order to project an analog low-pass filter to mitigate the high frequencies inherent to lumped parameters line model it is necessary to know the behavior of the spurious oscillations resulting from lumped parameters line model. Considering a 100 km single-phase line submitted to energization procedure by a 1 p.u., 60 Hz, cosinusoidal voltage and resistive load at receiving end as it is shown in Fig. 2.
To simulate the energization procedure of the line shown in Fig. 2, the single-phase transmission line was represented by lumped parameters model (LPM) using 100 π-circuits in cascade and by Universal Line Model (ULM) that is a classic distributed line parameters model developed in frequency domain that usually is used to verify the performance of others transmission line models. The per-unit length parameters R and G are from the [15]. The L and C were calculated considering a rectilinear conductor 24 m about the ground and the radius is 2 cm. At the receiving end is connected a 2000 Ω resistive load. The results are in per-unit value (pu) and the base value is 100 kV. The electrical parameters are shown in table I.

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>VALUE/KM</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>0.05 Ω</td>
</tr>
<tr>
<td>L</td>
<td>1.7 mH</td>
</tr>
<tr>
<td>G</td>
<td>0.556µS</td>
</tr>
<tr>
<td>C</td>
<td>6.58 nF</td>
</tr>
</tbody>
</table>

The voltage \(V(t)\) obtained from the two models is shown in Fig. 3.

The both curves are similar but the LPM, curve (1), presents numeric spurious oscillations. These oscillations occur due to a segment of transmission line, whose parameters are distributed along its length, are represented by lumped parameters of circuit and they are independent from the numeric method used to solve the state equations. These oscillations do not represent the real value of the voltage at end receiving. For instance, considering the first oscillation, red curve, its peak is 30% higher than the blue one (considered the exact response). This peak, if is considered in the analysis, can result in improper performance from the protection system or overestimation of isolator for the transmission line during its design. To mitigate these spurious oscillations, it will be proposed a low-pass filter inserted directly in the LPM as shown in Fig. 4.

![Proposed analog low-pass filter.](image)

Defining the travel time \(\tau\) to get from one end of the line to other as being, as is shown in Fig. 3, it is possible to write:

\[
\tau = \frac{d}{\nu}
\]

In (3) \(d\) is the length of line and \(\nu\) is approximately the speed of light. In this work the period of the electromagnetic transient as being \(2\tau\), as shown in Fig. 3. From this definition, it is possible to obtain the frequency of the electromagnetic transient \(F_0\) as being:

\[
F_0 = \frac{\nu}{2d}
\]

From Fig. 3, it is possible to consider that frequency of the high frequency spurious oscillations is multiple of the frequency of the electromagnetic transient. This way, defining the frequency of the spurious oscillations as being \(F_1\), it is possible to write the following relationship:

\[
F_1 = k \cdot F_0
\]

In equation (5) \(k\) is an integer number. From (4) and (5), the frequency of spurious oscillations can be written as function of the length of the line as:

\[
F_1 = k \cdot \frac{\nu}{2d}
\]

Therefore, from equations (3)-(6), the cutoff frequency \(F_c\) of the low-pass filter must be set in the following frequency range:

\[
\frac{\nu}{2d} < F_c < k \cdot \frac{\nu}{2d}
\]

From the circuit of the filter shown in figure 4, it is
possible to calculate its cutoff frequency. Using the definition of cutoff frequency, algebraic manipulation leads to the cutoff frequency of the filter given by:

\[ \omega_c = \sqrt{\frac{(R^2_j C^2_j - 2L_j^2 C^2_j)}{2L_j^2 C_j^2}} \]  

(8)

Taking into account the simulation results shown in Fig. 3, it is reasonable to consider \( k = 30 \), that is the number of spurious oscillations in the period \( 2\tau \). From this consideration, and using equations (7) and (8), the low-pass filter parameters were defined and are shown in table II.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_f )</td>
<td>900 Ω</td>
</tr>
<tr>
<td>( L_f )</td>
<td>5.4 mH</td>
</tr>
<tr>
<td>( C_f )</td>
<td>8.4 nF</td>
</tr>
</tbody>
</table>

Then the cut-off frequency \( f_c \) of filter is equal to 47.3 kHz. After to specific the low-pass filter, it was inserted in the lumped parameters transmission line model. It was observed that good results are obtained when filters are inserted at the two ends of the line represented by a \( \pi \)-circuit cascade as it is shown in Fig. 5. The low-pass filter was inserted in the LPM at the two ends of the line represented by a cascade of \( \pi \)-circuits as it is shown in Fig. 5.

![Fig. 5. Low-pass filter inserted in the lumped parameters model.](image)

The LPM, with two low-pass filters was used to simulate the energization of the line shown in Fig. 2. The results obtained with LPM with filters and with ULM are shown in Fig. 6. Fig.6 shows the voltages at the receiving end of the line. Curves 1 and 2 show, respectively, results obtained with ULM and with LPM showed in fig. 5. It is possible to observe that lumped parameters line model with two analog filters mitigated the high frequency oscillations and reduced the magnitude peaks, making then similar to the peaks observed in ULM.

Then it is possible to conclude that the analog filter was adequately projected and that proposed model is more efficient than lumped parameters line model. The proposed model will be used to study electromagnetic transients in three-phase transmission line during the switching procedure.

**IV. USING THE PROPOSED MODEL FOR REPRESENTING THREE-PHASE LINES**

To validate the proposed transmission line model with the analog low-pass filter inserted, it will be used a 440 kV three-phase transmission line whose transmission tower is presented in Fig. 7.

![Fig. 7. Three-phase transmission line used for simulation.](image)
The Fig. 8 shows a three-phase transmission line with a three-phase synchronous generator at sending end and balanced three-phase load $Z_{load}$ at the receiving end.

The modal decomposition theory was used for representing the three-phase transmission line shown in Fig. 8. This way, the three-phase line can be decoupled in its three uncoupled single-phase transmission lines (propagation modes) that are independent from each other. The voltages and currents are evaluated in modal domain for each phase and once obtained the modal currents and voltages, they are converted to the three-phase currents and voltages in phase domain.

This way, taking into account the modal decomposition, the proposed lumped parameters model also can be used to represent three-phase lines as it is shown in Fig. 9.

In Fig. 9, phase-mode transformation matrix decouples the three phase line into its uncoupled propagation modes. Then, each propagation mode is represented in modal domain as being a single-phase transmission line and currents and voltages in modal domain are calculated. After that, by using a mode-phase transformation matrix, currents and voltages in three-phase domain are calculated.

It was used the Clarke’s matrix for the decomposition method [1],[6]. Each propagation mode (described by propagation 1, 2 and 3 in Fig. 9) was represented by a cascade of 100 $\pi$-circuits cascade, energized by a AC voltage source at the sending end and with a resistive load at the receiving end. The Clarke’s matrix is given by (9). The tests were realized considering the line represented by the traditional and proposed LPM (with and without the low-pass filter) and ULM. The transient voltages at the receiving end are shown in Fig. 10 to Fig. 12

\[
[Z_{line}] = \begin{bmatrix}
0.6738 + j1.2263 & 0.0580 + j0.3430 & 0.0580 + j0.3430 \\
0.0580 + j0.3430 & 0.6740 + j1.1531 & 0.0581 + j0.3245 \\
0.0580 + j0.3430 & 0.0581 + j0.3245 & 0.6740 + j1.1531
\end{bmatrix}
\]

\[
[Y_{line}] = \begin{bmatrix}
0.0580 + j0.5514 & -j0.1027i & -j0.1027 \\
-j0.1027 & j0.7467 & -j0.0672 \\
-j0.1027 & -j0.0672 & j0.7467
\end{bmatrix}
\]

\[
T_{clarke} = \begin{bmatrix}
\frac{2}{\sqrt{6}} & 0 & \frac{1}{\sqrt{3}} \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\
\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}}
\end{bmatrix}
\]

The Fig. 10 shows the voltage obtained from the ULM. From Fig. 10 all the responses do not present spurious oscillations because this model uses the hyperbolic equations in the frequency domain. The Fig. 11 presents the spurious oscillations that are intrinsic to the model and do not represent the real value for the three-phase voltages. Using the proposed filter, the spurious oscillations were significantly mitigated by the filters and the responses become similar to the ULM.

The Fig. 11 shows the voltage obtained from the traditional LPM (without filter) where the spurious oscillations are intrinsic to LPM.
Fig. 11. Three-phase voltage $V_B(t)$ for the traditional LPM.

The Fig. 12 shows the voltage obtained from the proposed LPM (with filter).

![Figure 12](image)

Fig. 12. Three-phase voltage $V_B(t)$ for the proposed LPM.

Other practical test performed consists on to insert a disturbance at the transmission line during its stead state, as it is shown in Fig. 13.

![Figure 13](image)

Fig. 13. Three-phase transmission line with balanced load.

A procedure based on more practical application in electromagnetic transient’s analysis is performed. A 440 kV synchronous generator is connected at the sending end of the three-phase line and at the receiving end there is a three-phase load as shown in Fig. 13. In the phase 1 is connected a DC voltage source equal to 2 p.u. that will be used to generate an impulsive voltage during an instant for the analysis.

Initially the switch $S$ is closed and the three-phase line will be energized by the synchronous generator and a resistive load $Z_0$ equal to 2000 Ω. After the response reaches the steady state, $v(t)$ is connected in series at $t_1=16.85 ms$ and stays until $t_2=17.15 ms$. After $t_2$, $v(t)$ is equal to zero and stays in this condition permanently.

This operation generates an impulse on the voltage waveform at the phase 1. Approximately 0.33 ms after the impulse reaches the receiving end resulting the second transients on the line. The three-phase voltage will be calculated using the ULM, traditional and proposed LPM for evaluate the transient three-phase voltage at the receiving end. The Fig. 14 shows the three-phase voltage for ULM.

![Figure 14](image)

Fig. 14. Three-phase voltage $V_B(t)$ for the ULM.

The Fig. 15 shows the three-phase voltage for the traditional LPM. Even in the second transient there are the spurious oscillations. So using the proposed model, fig. 16, one can observe the reduction of spurious oscillations.

![Figure 15](image)

Fig. 15. Three-phase voltage $V_B(t)$ for the traditional LPM.

The Fig. 16 shows the three-phase voltage for proposed LPM.

One can observe that the spurious oscillations have been mitigating when low-pass filters are inserted in LPM and this response is similar to ULM. One disadvantage consist on that inserting low-pass filter in the LPM, it causes a small shift in the responses in time domain that should be considered in the analysis, but all the responses converge in steady state.
This article is a initial study about the spurious oscillations present in the lumped parameters line transmission line model, not considering the frequency effect in the longitudinal parameters. In the regular transmission line, as shown in this article, the frequency has not outstanding effect in low frequencies. Then the lumped parameters line model, rejecting the frequency effect on the line parameters, is appropriate model when the simulations does not take account high frequencies phenomena as for example, the energization procedure [15]. Including analog low-pass filters in the LPM can be done, considering the frequency effect on the longitudinal parameters of the line, because the frequency effect is easily introduced in the LPM [16]. However the authors have not performed studies on frequency effect in zero sequence and do not have the conditions to evaluate it.

V. CONCLUSIONS

This work has presented the lumped parameters transmission line model where the transmission line was represented by lumped parameters of circuit. This model is commonly used to study the electromagnetic transient in power system because all the simulations are performed directly in time domain. The spurious oscillations are intrinsic to the LPM and they are independent of the numeric method used to solve the state equation, intrinsic of the LPM simulations. An alternative model for mitigating the spurious oscillations consists on inserting an analog low-pass filter in the lumped parameters transmission line model. It was propose using 2 low-pass filters connected as shown by the Fig.5. The spurious oscillations were completely mitigated, producing similar responses to the ones obtained for the distributed parameters model using the ULM. The low-pass filter proposed has the advantage of being inserted directly in the lumped model. Thus the filtering process is performed in real time, resulting the main advantage when the proposed filter is compared with digital filter. Another advantage consists on those parameters of the low pass filter are adjustable, so the spurious oscillations can be more reduced. Hence the proposed lumped parameter model consists on a valid alternative to reduce the spurious oscillations and can be used to study the electromagnetic transient in the power systems.

VI. REFERENCES

VII. BIOGRAFIES

**Anderson Ricardo Justo de Araújo** received the degree of Electrical Engineer and Master degree from Faculdade de Engenharia de Ilha Solteira, UNESP, Brazil in 2012 and 2014 respectively. He is currently a Ph.D. student at the same university. His main areas of interest are: Electromagnetic transients in power systems using numerical methods, tower transmission modeling and transmission lines models for simulation of electromagnetic transients on power systems.

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