Finite element calculation of leakage resistance and distributed capacitance of rail to earth in ballastless track

Li Teng, Wu Mingli, He Fan, Song Kejian

Abstract*-- The leakage resistance of rail to earth is an important parameter of electrified railways, and it directly affects the characteristics of traction return current and railway signal track circuit. High speed railways mainly adopt the ballastless track in China, whose rail leakage resistance is very different compared to the resistance in the ordinary railway ballast track condition, usually has a much larger value. Furthermore, the distributed capacitance effect of rail to earth in the ballastless track condition should not be ignored. The rail has an anharmonic cross section, and is installed on the spacing support of the concrete integrated ballastless bed. Neither the leakage resistance of rail to earth can be calculated using a practically simple theory or method, nor the distributed capacitance of rail can be accurately calculated using the model for thin circular conductor of overhead transmission lines. This paper presents the test results of the rail leakage resistance and the distributed capacitance in Jin-Qin (Tian jin city - Qin huangdao city) high speed railway of China. At the same time, according to the specific condition of the ballastless track, a finite element calculation model of the electromagnetic field has been established using Comsol Multiphysics simulation software, which includes the rail, the concrete track bed, roadbed and the earth. The leakage resistance and distributed capacitance of rail to earth are calculated using this finite element method. The calculation results and the measured results are compared with each other. Error factors between them have been analyzed. Results show that, the finite element model for calculating the leakage resistance and the distributed capacitance of rail to earth is an effective approach. The field measured data can be used in the future for the related analysis and calculation of traction networks and track circuits.

***Keywords*: Electrified high speed railway, ballastless track, leakage resistance of rail to earth, distributed capacitance of rail, finite element model, model verification.**

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I. INTRODUCTION

JIN-QIN (Tianjin-city - Qinhuangdao-city) Passenger Dedicated Line with a design speed of about 300 km/h, is an important high-speed electrified railway in northern China; its main line length is 257.429km (double track), and the ballastless track length is 162.344km, accounting for 63.1% of the total length[1].

Jin-Qin Passenger Dedicated Line adopts CRTS Ⅱ track structure of slab track[2]. Compared to traditional ballasted track, ballastless slab track has low structural height, good stability, long life, as well as advantages that ballast does not splash under the high-speed train, and the lateral movement of the track bed is small. Therefore, it has been widely used in high-speed railway in many countries[3]. However, compared to ballasted track rail, leakage resistance of the ballastless track increases, and the distributed capacitance effect of the rail to earth cannot be ignored.

Rails not only provide mechanical support and direction guidance for the electric locomotives, but also provide electrical channels for both traction return currents and the track circuit signals. With the development of the railway to high-speed heavy load direction, the traction return current correspondingly increases; track circuit signal equipments and the structure of the track bed have also been affected, including the transmission length, the sensitivity and the security of the track circuit transmission. Suppose the rail self impedance and traction load current have unchanged values, if the rail leakage resistance to ground is too large, it may cause excessive rail potential. Excessive rail potential will affect the operation of the power supply system performance, personal safety to the station passengers and maintenance personnel, and may also damage signal equipment insulation, and do harm to the safety of running trains.

China's high-speed passenger dedicated line now mostly adopts whole track bed, the high leakage resistance between rail and earth causes rail potential is much higher than the existing electrified railway. Due to the big impact of the high-speed train rolling stock wheels on the rails, the pad between the rail and the sleeper pad is generally thickened in ballastless track, which is electrically insulating; train control signal also requires good insulation between the two rails and rail-to-ground. These factors have led to the larger rail-ground leak resistance. For high-speed railway, appropriate measures must be taken to reduce the track current and rail potential.

In the construction and operation of the electrified high-speed railway, it is necessary to know the exact value of the rail leakage resistance and distributed capacitance to the ground in order to achieve good EMC between traction power supply system and train signal system[3].

II. THEORY

The rail line can be regarded as transmission line with distributed parameters[4][5][6]. The so-called distributed parameters is that the transmission line has a per unit length resistance R, inductance L, capacitance C, and leakage conductance G. Note that the rail line distribution parameters are related to the frequency. Frequency is one of the main parameters to distinguish the traction return current and track signal current. The following figure 1 shows the equivalent circuit for the track in unit length.

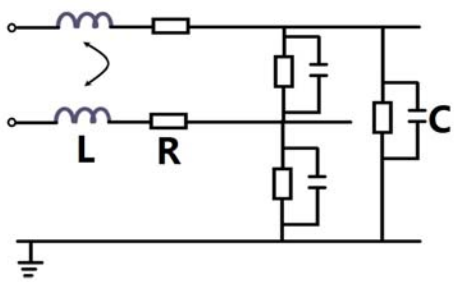

Fig. 1. Diagram of equivalent circuit for the track in unit length.

When traction current (main frequency content of 50Hz) runs through the rails, loss generates in both the internal and external rails. Rail external resistance reflects the leakage active power losses. Distributed capacitance between rails and the ground reflects the external reactive power. Note that this article does not consider the effect generated by the mutual inductance and capacitance effects between the rail line and overhead contact line.

There are a lot of literature on the calculation of the track resistance, inductance. The calculation methods include the equivalent cylindrical method, conductor subdivision scheme[7], the finite element method[4][8], and so on. But there is little literature on the track-to-ground leakage resistance and distributed capacitance for finite element analysis. Basic theoretical calculation of the distribution parameters of the rail line can be divided into the analytical method and the finite element method. In the analytical method based on simplified conditions for solving the approximate solution, many parameters haven't been taken into account, such as rail shape, the skin effect corresponding to different frequencies. The rail has an anharmonic cross section, and is installed on the spacing support of the concrete integrated ballastless bed. Neither the leakage resistance of rail to earth can be calculated using a practically simple theory or method, nor the distributed capacitance of rail can be accurately calculated using the model for thin circular conductor of overhead transmission lines. Considered the actual rail shape, finite element (FE) method subdivides conductor subdivision into many grids and then solves the corresponding physical fields using partial differential equations to calculate the parameters. However, for the consideration of limit information about the materials and the calculation consumption such as time and memory, FE method also needs a certain degree of simplification and estimation. For example, in this paper, considered physical fields are static. Further, the rail inductance is not considered. Calculation of the leakage resistance and the rail-ground distribution capacitance, and then the model verification are the main purposes of this paper, magnetic permeability is thus ignored. Therefore, the unit length rail-earth leakage impedance model is shown in the figure 2, wherein, R11, R22 means a single rail leakage resistance, R12 refers to the resistance between two rails. C11, C22 refers to a single rail-to-ground capacitance, C12 is the capacitance between two rails. Theoretically R11, R22, C11, C22 are equal. If the electric potential of two rails equals, the leakage impedance of the parallel connected C12 and R12 between two rails can be further ignored; only the rail-ground leakage impedance is considered instead.

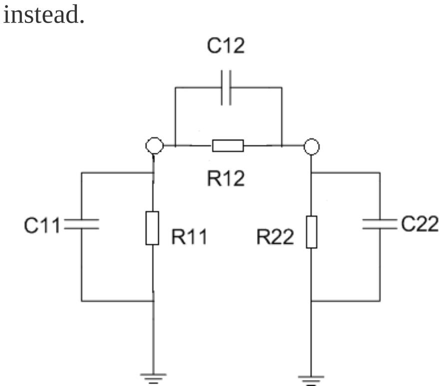

Fig. 2. Diagram of equivalent circuit for only considering the rail-earth leakage impedance in the track in unit length.

COMSOL Multiphysics is a finite element analysis and solver software package for various physics and engineering applications, especially coupled phenomena, or multiphysics. The AC/DC Module span electrostatics, magnetostatics, and electromagnetic quasi-statics phenomena[9]. The sub modules of "Electric Currents", "Electrostatics", and "Magnetic and Electric Fields" can all be used to calculate rail line parameters. In this paper, the sub module of "Electric Currents" is used to calculate leakage impedance without considering the magnetic field, i.e. the rail inductance is ignored. Only the electric field change caused by the electric potential is considered.

The partial differential equations of the "Electric Currents" include:

$$\nabla \cdot J = Q_j \tag{1}$$

$$J = \sigma E + j\omega D + J_e \tag{2}$$

$$E = -\nabla V \tag{3}$$

wherein, J is the current density(A/m^2), E is the electric field intensity(V/m), D is electric displacement field (C/m^2), V is the electric potential(V). In the model, the electric potential of the rail is set to 200V and the angular frequency ω equals to $2\pi f$, i.e. 314.159 rad/s. There is no space electric charge Qj considered, i.e. Qj is 0.

The calculation of leakage impedance of the rail to earth is:

$$R + X = \frac{V_r}{I_{ie}} = \frac{V_r}{\int J_{ie} dS} \tag{4}$$

$$R = Z_0 \times \cos\theta \tag{5}$$

$$X = Z_0 \times \sin\theta \tag{6}$$

$$R + X = \frac{r}{1 + (\omega C r)^2} - j\frac{\omega C r^2}{1 + (\omega C r)^2} \tag{7}$$

where, V_r is the total rail voltage, I_{ie} is the total current injected into the ground per unit rail length, J_{ie} is the current density in the ground over surface dS. r is the leakage resistance of the rail to earth.

The leakage resistance in Comsol can be calculated using the integration function of "Resistive losses(W)" or the "Total power dissipation density(W)"since the rail electric potential is known as 200V. After the value r and total leakage impedance R+X are calculated using (4), (5) and(6), then the capacitance C can be calculated using (7).

III. MODELING RESULTS

The structure of CRTS Ⅱ type slab track refers to lay precast track panels by cement asphalt mortar adjustment layer in the site paving concrete support layer or site-cast reinforced concrete base; it adapts to Ballastless track circuit zpw-2000 track structure[2]. This technology is the introduction of a slab technology in Germany, and now applied to the Jing-Hu, Jing-Shi, Jin-Qin, Hu-Hang passenger dedicated lines. CRTS Ⅱ type of slab track is composed of a concrete support base/layer, cement mortar adjustment layer(CA layer), precast concrete track panel, slab connection components, rail fastening clip, etc. Track structure height (internal rail top surface to the bottom of the concrete support layer) is 779mm; track panel width is 2550mm, with the thickness of 200mm and the standard length of 6450mm; CA layer theoretical thickness is 30mm; the support layer made of hydraulic material has the top surface width of 2950mm, the bottom surface width of 3250mm, and the thickness of 300mm; each track panel sets 60 transverse tendons with a diameter of 10mm, and 6 longitudinal tendons with a diameter of 20mm[2]. Figure 3 shows the general appearance of a standard of the Ⅱ type slab track. Cross-sectional details of the Ⅱ type slab track in straight line roadbed section is shown in the following figure 4.

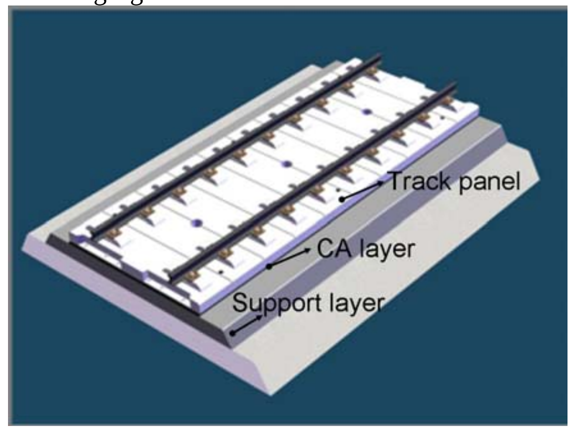

Fig. 3. Diagram of CRTS Ⅱ type slab track structure.

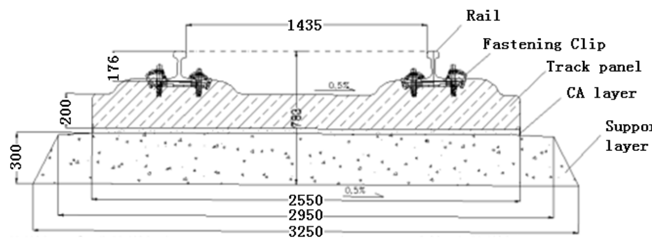

Fig. 4. Cross-sectional details of the Ⅱ type slab track in straight line roadbed section.

A. Model Geometry

Result of building a three-dimensional geometry of the II type track slab shows in the figure 5, which is close to the actual II type track slab geometry. In order to calculate the leakage impedance of the rail to the ground, a ground layer is also built under the concrete support layer with depth of 500mm, width of 3650mm. Cross-sectional details of the model is shown in the figure 6.

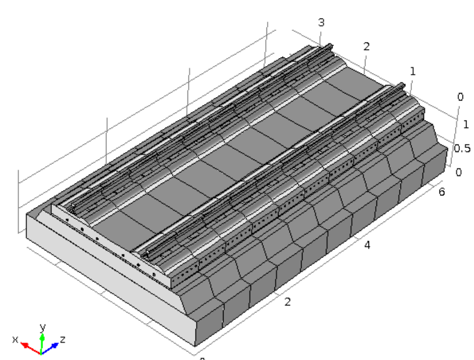

Fig. 5. a three-dimensional geometry of the II type track slab above a ground layer.

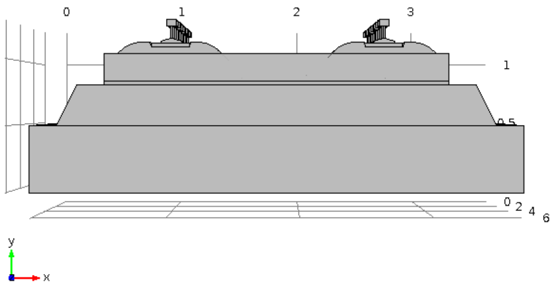

Fig. 6. Cross-sectional details of the built model.

B. *Actual Applied Model and Its Mesh*

When running above model in the Comsol, the computer is run out of memory . Since the standard lab can be divided into 10 parts with almost the same properties in longitudinal direction, the actual applied model is shown in the figure 7. The model is meshed into free-tetrahedral grids.

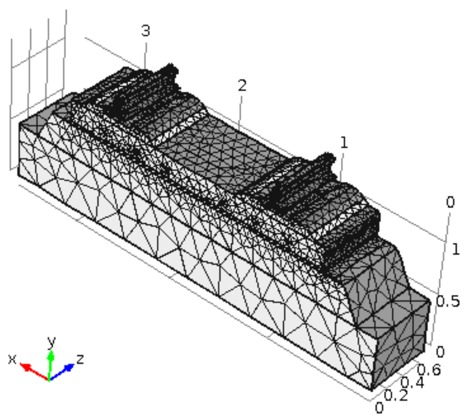

Fig. 7 The actual applied model and its mesh

C. *Material Parameters*

Table I lists applied material electrical parameters in different layers[10][11].

TABLE I
APPLIED MATERIAL PARAMETERS IN DIFFERENT LAYERS

Materials	Electrical Conductivity (S/m)	Relative Permittivity
Rail	9×10^{5}	1
Rubber cushion pad	10^{-6}	5
Track panel	0.001	4
Tendons	8×10^{5}	1
CA layer	0.002	5
Support layer	0.001	4
Ground layer	0.01	1

D. *Boundary Conditions*

Most boundary conditions in the model are set as the default values. Electric potential of ground layer surfaces is set to 0 and electric potential of rail surfaces is set to 200V.

E. *Electric Potential Modeling Results*

Figures 8 and 9 show the modeling results for the electric potential distribution in cross-sectional and lateral view.

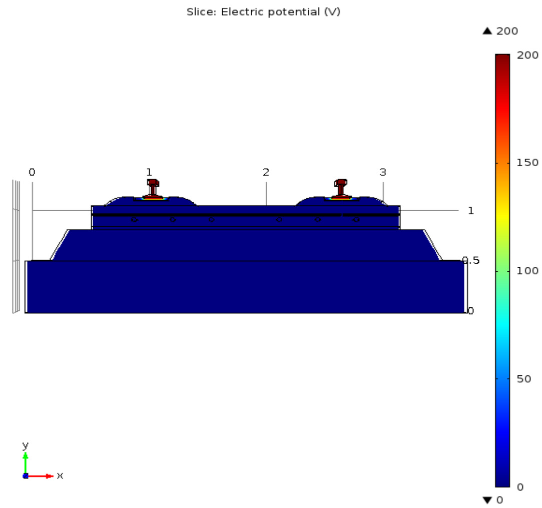

Fig. 8. Cross-sectional distribution of the electric potential

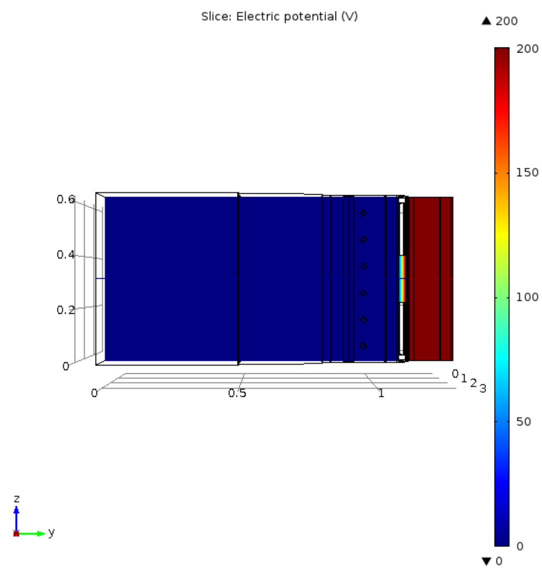

Fig. 9. Lateral view of the electric potential distribution

F. *Current Density Norm Modeling Results*

Figures 10 and 11 show the modeling results for the current density norm distribution in cross-sectional and lateral view.

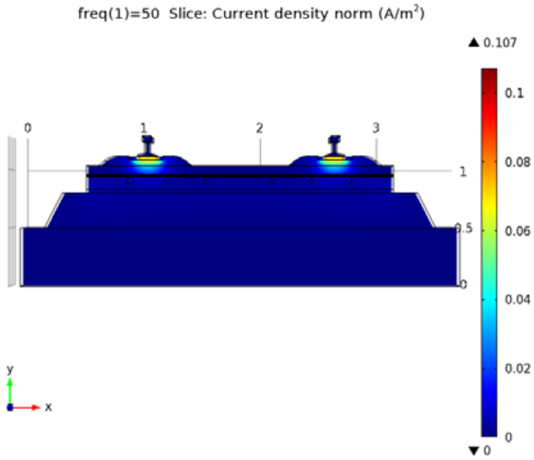

Fig. 10. Cross-sectional distribution of the current density norm

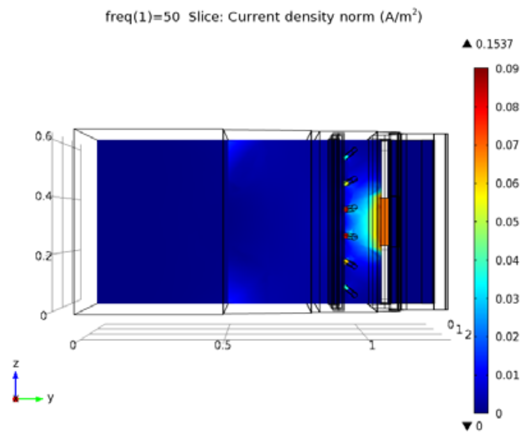

Fig. 11. Lateral view of the current density norm

G. Leakage Impedance Calculation Results

Table II shows the related modeling results and calculation results for the rail-earth leakage impedance.

TABLE II
RELATED MODELING AND CALCULATION RESULTS FOR THE LEAKAGE IMPEDANCE

Electrical Parameters	Values
Current density norm of the ground layer top surface (A)	4.9637e-4
Current density, x component of the ground layer top surface (A)	5.4773e-9 +4.5552e-11i
Current density, y component of the ground layer top surface (A)	-3.8922e-4 -3.2397e-6i
Current density, z component of the ground layer top surface (A)	6.5645e-21 +9.5953e-23i
Resistive losses (W)	0.0449
Leakage resistance (Ω • km)	1425.389
Leakage capacitance (μF/km)	7.05

IV. Measurement and Model Verification

The traditional track circuit parameter measurement method is based on transmission line theory and then subsequent on-site measurements. The main rail leakage impedance measurement methods are the "open circuit method" and "short circuit method". Depending on the test equipments, measurement methods can also be divided into "phase table method", "two voltmeter and an ammeter", "two ammeter and a voltmeter', and other methods[3]. In the actual measurement, combined with Wenner four-probe resistivity test[12] method, we take engineering simplified method, "open circuit method" to measure the rail-to-ground leakage impedance. The on-site measurement photo is shown in figure 12. The actual measured Jin Qin passenger leakage resistance and distributed capacitance values are shown in the table III.

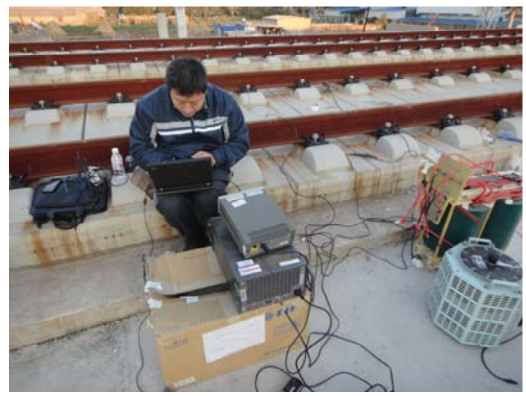

Fig. 12. On-site measurement photo

TABLE III
MEASUREMENT RESULTS FOR THE LEAKAGE IMPEDANCE

Leakage resistance at 50Hz (Ω • km)	1304
Leakage capacitance at 50Hz (μF/km)	7

Comparing the measured data and modeling data, we found that errors of the leakage impedance between them may caused by the simplification of the model, setting values of the model parameters (e.g. the leakage resistance is affected greatly by the rubber cushion pad), model geometry deviation, and measurement errors, etc. However, the comparison result shows that the model is still valid.

V. Conclusions

The calculation results and the measured results are compared with each other. Error factors between them have been analyzed. Results show that, the finite element model for calculating the leakage resistance and the distributed capacitance of rail to earth is an effective approach. The field measured data can be used in the future for the related analysis and calculation of traction networks and track circuits.

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