Application of Magnitude Vector Fitting for approximating modal propagation function from the frequency spectrum data of underground cables

Naveen Goswamy and Ilhan Kocar

***Abstract*—Vector Fitting (VF) can be useful for approximating system equations of multi-conductor transmission lines and cables based on the Universal Line Model (ULM). However, one of the challenges posed by this technique is the additional computational logic required to evaluate the time-delays associated with the modal propagation functions, prior to arriving at a suitable system identification. This study examines Magnitude Vector Fitting (magVF) as an alternative to VF, contributing detailed implementation procedures and theoretical explanations. In addition to demonstrating the utility of magVF for simplifying the computation of the modal function magnitudes, a modification of the magVF algorithm is provided, synthesizing the approach of Weighted Vector Fitting (WVF) to produce Weighted Magnitude Vector Fitting (WmagVF). Test results are given fitting actual power system frequency spectrum data.**

***Keywords*—Electromagnetic transients, frequency response, least squares approximation, modal analysis, system identification, transmission line matrix methods.**

I. Introduction

Vector Fitting (VF) [1] has been included in EMT-type programs for a little more than a decade now. It has been utilized for the approximation of modal propagation functions based on the Universal Line Model (ULM) [2]. One of the challenges imposed by using VF in that context is the additional iterative logic required for pre-determination of the time delays associated with each modal propagation function. This study examines Magnitude Vector Fitting (magVF) [3] as an alternative to VF for the system approximation that simplifies the procedure of time-delay determination, and presents some new modifications to the algorithm. A Weighted Magnitude Vector Fitting (WmagVF) procedure is introduced employing techniques that are analogous to those used for deriving Weighted Vector Fitting (WVF) [4]. An iterative pole modification technique is also proposed for the fitting of smooth functions. Finally, this study provides and discusses experimentally derived results and opens avenues for future scientific explorations.

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A. The Universal Line Model (ULM) and modal decomposition of transmission lines and cables

Using distributed parameters, transmission line and cable voltage-current characteristics are described by the Universal Line Model (ULM) using two phase domain matrix transfer functions, namely the propagation matrix $\mathbf{H}$ and characteristic admittance matrix $\mathbf{Y_c}$, which are given as

$$\mathbf{H} = e^{-\sqrt{\mathbf{YZ}}l}, \text{ and} \tag{1}$$

$$\mathbf{Y_c} = \mathbf{Z}^{-1}\sqrt{\mathbf{ZY}}, \tag{2}$$

where l indicates the length of the line [5], [2]. The matrices $\mathbf{Y}$ and $\mathbf{Z}$ are, respectively, the shunt admittances and series impedances per unit length. These matrices are of size $N_c \times N_c$ where N_c is the number of conductors in the line.

The propagation and characteristic admittance transfer functions allow the calculation of the voltage ($\mathbf{V}$) and current ($\mathbf{I}$) at two ends s and r of a transmission line as shown below

$$\mathbf{I}_r = \mathbf{Y_c}\mathbf{V}_r - \mathbf{H}(\mathbf{Y_c}\mathbf{V}_s + \mathbf{I}_s) \tag{3}$$

$$\mathbf{I}_s = \mathbf{Y_c}\mathbf{V}_s - \mathbf{H}(\mathbf{Y_c}\mathbf{V}_r + \mathbf{I}_r) \tag{4}$$

These equations are in the steady-state frequency domain, and time-domain results can also be obtained via inverse Fourier transformations.

Furthermore, via modal decomposition, it is possible to represent the MIMO propagation transfer function $\mathbf{H}$ through its modal propagation functions H_m where $m = 1, 2, ..., N_c$. The problem then gets simplified to fitting each of the modal propagation functions as a decoupled SISO system transfer function with a delay, in the form

$$H_m = H'_m e^{-j\omega\tau_m} \tag{5}$$

where τ_m signifies the associated time delay for the modal domain equation H'_m corresponding with mode m, which is proposed to be a minimum phase system [2].

By the residue-pole form that is output from the VF method, H'_m is

$$H'_m = \sum_{n=1}^{N_m} \frac{c_n}{j\omega - p_n}$$

where N_m is the order of the approximation (number of poles) used for mode m.

In [2] it is proposed that the poles of H_m – and hence its minimum phase component H'_m – are the same as the poles used to reconstruct $\mathbf{H}$ as a MIMO system matrix. Supposing that there are N_m poles for the transfer function of mode m, and that there are N_c modes in total (which usually corresponds with the number of conductors, hence the subscript c), then, getting back to the frequency domain from the modal domain involves recomposing the MIMO propagation function as follows.

$$\mathbf{H} \cong \sum_{m=1}^{N_c} \left(\sum_{n=1}^{N_m} \frac{\mathbf{C_{nm}}}{s - p_{nm}} \right) e^{-s\tau_m} \tag{6}$$

Once a poles-residues series form of the minimum-phase H'_m with the associated time delay τ_m has been established, then finding the residues $\mathbf{C_{nm}}$ can be done using overdetermined linear systems equations with partial fraction basis functions for another set of linear least squares problems.

II. Transfer function identification from tabulated frequency response

For the uninitiated reader, it may be useful to provide some context as to how the fitting algorithms described in this study are generally used. Essentially, the goal is to provide a Laplace domain LTI transfer function represented by $\widetilde{F}(s_k)$ that can closely approximate the tabulated frequency response of a given system vector $F(s_k)$ over a frequency sequence $s_k = j\omega_k$ where $k = 1, 2, ..., K$.

Representing this system function in terms of a partial fraction series with direct (G) and proportional (E) terms, at each frequency point k, is the equation

$$\widetilde{F}(s_k) = \sum_{n=1}^{N} \frac{r_n}{s_k - p_n} + s_k E + G \approx F(s_k) \ , \tag{7}$$

with p_n as either real or one of a pair of complex conjugate poles, and r_n the corresponding residue term. Note that $\widetilde{F}(s)$ can also be expressed as in equivalent pole-zero-gain form as

$$\widetilde{F}(s) = \frac{\widetilde{N}(s)}{\widetilde{D}(s)} = F_0 \frac{\prod_{m=1}^{M} (s - z_m)}{\prod_{n=1}^{N} (s - p_n)}, \tag{8}$$

where F_0 is a constant real gain, z_m represents one of the M zeros, and p_n represents one of the N poles.

Supposing a strictly proper transfer function order such that $M < N$, then when represented by partial fractions, the E and D terms in (7) are null and can be ignored.

So, given K frequency response data points with frequency samples $s_k = j\omega_k$, (7) and (8) yield the following cost function minimization problem when trying to approximate $F(s_k)$:

$$\min \sum_{k=1}^{K} \left| F(s_k) - \frac{\widetilde{N}(s_k)}{\widetilde{D}(s_k)} \right|^2 . \tag{9}$$

Equivalently,

$$\min \sum_{k=1}^{K} \frac{1}{\left| \widetilde{D}(s_k) \right|^2} \left| F(s_k)\widetilde{D}(s_k) - \widetilde{N}(s_k) \right|^2 . \tag{10}$$

Note that (10) is non-linear and solvable within a desired margin of error using constrained optimization techniques. Such solutions, however, are computationally demanding when compared to those of linearized overdetermined least squares problems.

A. The Vector Fitting (VF) algorithm

The VF method [1] proposes a linearized method for identifying $F(s)$ in the least squares sense. Supposing that

$$F(s) \cong \frac{\widetilde{N}(s)}{\widetilde{D}(s)} = \frac{\sum_{n=1}^{N} \frac{\hat{r}_n}{s - \bar{p}_n} + sE + G}{\sum_{n=1}^{N} \frac{\widetilde{r}_n}{s - \bar{p}_n} + 1}, \tag{11}$$

given that

$$\lim_{s \to \infty} \left(\sum_{n=1}^{N} \frac{\widetilde{r}_n}{s - \bar{p}_n} \right) = 0, \tag{12}$$

the VF algorithm then transforms (11) into

$$\sum_{n=1}^{N} \frac{\hat{r}_n}{s - \bar{p}_n} + sE + G - \left(\sum_{n=1}^{N} \frac{\widetilde{r}_n}{s - \bar{p}_n} \right) F(s) \cong F(s). \tag{13}$$

Given a sufficiently large number of frequency response data points for $F(s_k)$, the unknowns to be discovered are the residue terms $\hat{r}_n$, $\widetilde{r}_n$, the poles $\bar{p}_n$, and the direct and proportional terms, G and E, respectively. Solving for these is accomplished iteratively using a series of two overdetermined linear solutions by the least-squares method. These two main stages are known as the *pole relocation* and *residue identification* steps. A set of initial poles is selected to commence the procedure. At the end of each iteration, a new set of poles and residues are provided that can be used to test for convergence. The details of the formulation for these steps under VF is provided in [1].

The VF method can converge relatively quickly and accurately depending on the function to be fitted and the initial poles selected, however improvements have been made to this algorithm since its publication, specifically to resolve issues of ill-conditioning and convergence oscillation.

B. The Weighted Vector Fitting (WVF) algorithm

The Weighted Vector Fitting (WVF) technique involves the use of an inherent weighting term that updates after each fitting iteration [4], [6]. This weighting term enforces that the linearization step is more precise. For instance, in VF, the algorithm goes from the nonlinear expression of (10) to the linear expression (11) using

$$F(s) \cong \frac{\widetilde{N}(s)}{\widetilde{D}(s)} \tag{14}$$

$$\widetilde{N}(s) - \widetilde{D}(s)F(s) \cong 0. \tag{15}$$

However in WVF, the step taken by (15) is considered to be a source of error due to numerical inaccuracies inherent in the way the approximation equality is treated. Instead, a stricter approach is suggested in order to yield a more precise approximation for the basis functions, as follows, replacing (15) with:

$$\frac{\widetilde{N}(s)}{\widetilde{D}(s)} - F(s) \cong 0 \tag{16}$$

$$\frac{\widetilde{N}(s)}{\widetilde{D}(s)} - F(s)\frac{\widetilde{D}(s)}{\widetilde{D}(s)} \cong 0 \tag{17}$$

$$\frac{1}{\widetilde{D}(s)}\left[N(s) - \widetilde{F}(s)\widetilde{D}(s)\right] \cong 0. \tag{18}$$

Note that the effect of the term $\frac{1}{\widetilde{D}(s)}$ in (21) is analogous to that of $\frac{1}{\left|\widetilde{D}(s_k)\right|^2}$ from (10). Whereas in VF that term is ignored and set to unity, in WVF it is retained and updated iteratively.

For compactness of the expression, let the proportional and direct terms – which are only necessary for non-proper transfer functions – be replaced by the function

$$\widetilde{\alpha}(s) = sE + G. \tag{19}$$

In the iterative context of the fitting procedure, let i denote the i-th iteration, with $\widetilde{W}(s_k)$ and $\widetilde{D}(s_k)$ denote weighting terms for the current and next iteration such that

$$\widetilde{W}^{(i)}(s_k) = \sum_{n=1}^{N} \frac{\widetilde{r}_n^{(i-1)}}{s_k - \bar{p}_n^{(i-1)}} + 1 = \widetilde{D}^{(i-1)}(s_k). \tag{20}$$

Then, in WVF, going from the VF equation (11) the minimization problem becomes

$$\min\left|\frac{1}{\widetilde{W}^{(i)}(s_k)}\left[\sum_{n=1}^{N}\frac{\hat{r}_n^{(i)}}{s_k - \bar{p}_n^{(i)}} + \widetilde{\alpha}(s_k) - \widetilde{D}^{(i)}(s_k)F(s)\right]\right|. \tag{21}$$

In [4] this iterative weighting technique has been demonstrated to alleviate problems of ill-conditioning and poor fitting that is otherwise present due to overemphasis on low frequency samples.

C. The Magnitude Vector Fitting (magVF) algorithm

Magnitude Vector Fitting is a more recent modification to VF that uses a symmetric partial fraction basis for the formulation of the overdetermined linear system equations to fit a magnitude-squared frequency response. The derivation presented in this section follows that of [3], in which the abbreviation magVF was first used.

Again, letting $F(s)$ be a Laplace transform of the impulse response $f(t)$ of a causal and stable LTI system. In product form, $F(s)$ can be represented by

$$F(s) = F_0 \frac{\prod_{m=1}^{M}(s - z_m)}{\prod_{n=1}^{N}(s - p_n)}, \tag{22}$$

where F_0 is the positive and real 'DC gain', z_m refers to each of the M zeros, and p_n refers to each of the N poles. For realizable bounded systems the order of the numerator (M) and denominator (N) are such that the system is either proper ($M = N$) or strictly proper ($M < N$).

Taking the Fourier Transform by resolving $F(s)$ on the imaginary axis, such that $F(s)|_{s=j\omega} = F(j\omega)$ and by complex number properties:

$$|F(j\omega)|^2 = F(j\omega)F^*(j\omega). \tag{23}$$

The magnitude-squared can also be expressed as

$$|F(j\omega)|^2 = F_0^2 \frac{\prod_{m=1}^{M}(j\omega - z_m)(-j\omega - z_m^*)}{\prod_{n=1}^{N}(j\omega - p_n)(-j\omega - p_n^*)}, \tag{24}$$

or by pulling out a negative sign, as

$$|F(j\omega_k)|^2 = (-1)^{(M-N)} F_0^2 \frac{\prod_{m=1}^{M}(j\omega_k - z_m)(j\omega_k + z_m)}{\prod_{n=1}^{N}(j\omega_k - p_n)(j\omega_k + p_n)} \tag{25}$$

Note the assumption that all poles and zeros are either purely real or present in complex-conjugate pairs. Provided that this caveat is held true, by (25) it is evident that the magnitude-squared function is composed of poles and zeros in the left hand s-plane (LHP) as well as their symmetric counterparts in the right hand s-plane (RHP).

The problem thence assumes that there is a set $|F(j\omega_k)|^2$ of tabulated data of size K frequency-domain points for which an approximated transfer function that yields $|\widetilde{F}(j\omega_k)|^2 \backsimeq |F(j\omega_k)|^2$ is desired. Given the magnitude-squared form of (25), and then under the assumption that all poles and zeros are in the left hand s-plane – as is the case for minimum phase functions by definition – it is possible to eliminate the right-hand plane poles and zeros and, taking the square root of the F_0 term, to reduce the magnitude-squared to a simple magnitude approximation such that the non-squared magnitude response can be inferred.

$$|F(j\omega_k)| = F_0 \frac{\prod_{m=1}^{M}|j\omega_k - z_m|}{\prod_{n=1}^{N}|j\omega_k - p_n|} \backsimeq |\widetilde{F}(j\omega_k)| \tag{26}$$

Furthermore, (25) can also be represented in the following symmetrical partial-fractions form:

$$|F(j\omega_k)|^2 = r_0 + \sum_{n=1}^{N}\left(\frac{r_n}{j\omega_k - p_n} - \frac{r_n}{j\omega_k + p_n}\right), \tag{27}$$

noting that $r_0 = 0$ when $M < N$ and $r_0 = F_0^2$ when $M = N$.

Applying the standard VF method to the magnitude-squared function above leads to some problems with asymmetrically perturbed poles and residues. To counter this, the use of symmetric basis functions as a modification to VF has been suggested [3], [7] and provided in the following steps.

D. Weighted magVF (WmagVF) formulation

In this section, the formulation of the proposed Weighted Magnitude Vector Fitting (WmagVF) algorithm is demonstrated.

Taking the symmetric basis functions in the numerator and denominator for the case where $M < N$,

$$|F(s)|^2 \cong \frac{\sum_{n=1}^{N}\left(\frac{\hat{c}_n}{s+\bar{p}_n} - \frac{\hat{c}_n}{s+\bar{p}_n}\right)}{\sum_{n=1}^{N}\left(\frac{\tilde{c}_n}{s+\bar{p}_n} - \frac{\tilde{c}_n}{s+\bar{p}_n}\right)+1}. \tag{28}$$

Where,

$$|F(s)|^2 \cong \frac{|\widetilde{N}(s)|^2}{|\widetilde{D}(s)|^2}. \tag{29}$$

Then,

$$\frac{|\widetilde{N}(s)|^2}{|\widetilde{D}(s)|^2} - |F(s)|^2 \cong 0 \tag{30}$$

$$\frac{|\widetilde{N}(s)|^2}{|\widetilde{D}(s)|^2} - |F(s)|^2 \frac{|\widetilde{D}(s)|^2}{|\widetilde{D}(s)|^2} \cong 0 \tag{31}$$

$$\frac{1}{|\widetilde{D}(s)|^2}\left(|\widetilde{N}(s)|^2 - |F(s)|^2|\widetilde{D}(s)|^2\right) \cong 0 \tag{32}$$

Now using an analogous approach as in WVF, including a weighting factor $\widetilde{W}(j\omega_k)$ that is composed of $|\widetilde{D}(j\omega_k)|^2$ from the previous iteration

$$\widetilde{W}^{(i)}(j\omega_k) = \sum_{n=1}^{N}\left(\frac{\bar{c}_n^{(i-1)}}{j\omega_k - p_n^{(i-1)}} - \frac{\bar{c}_n^{(i-1)}}{j\omega_k + p_n^{(i-1)}}\right)+1 = |\widetilde{D}^{(i-1)}(j\omega_k)|^2, \tag{33}$$

and

$$\frac{1}{\widetilde{W}^{(i)}(j\omega_k)}\left[|\widetilde{N}^{(i)}(j\omega_k)|^2 - |F(j\omega_k)|^2|\widetilde{D}^{(i)}(j\omega_k)|^2\right] \simeq 0. \tag{34}$$

For the first iteration let $\widetilde{W}^{(i)}(j\omega_k) = 1$ for all k. Subsequently, at the end of each iteration compute the next weighting term from $|\widetilde{D}^{(i)}(j\omega_k)|^2$, as $\widetilde{W}^{(i+1)}(j\omega_k)$. This value is used at each iteration to scale the k-th row of elements of equations represented by matrix $\mathbf{A}$ and right-hand side vector $\mathbf{b}$ from the equation $\mathbf{Ax} = \mathbf{b}$ in the pole relocation step prior to solving for the new poles. This paper proposes that in doing so, the convergence characteristics with respect to the precision of the solution will be improved.

III. Fitting of Modal Propagation Functions

A. VF and WVF approach to fitting modal propagation functions

For VF (and WVF), it is essential to select a good estimate for the modal time delay (τ_m) prior to attempting to fit the respective function. The delay must be removed from H_m of (5), and then the fitter can be run on H'_m. Fig. 1 shows an overview of this algorithm.

Without an appropriate choice of τ_m, the fit will not be successful, as illustrated by Fig. 2.

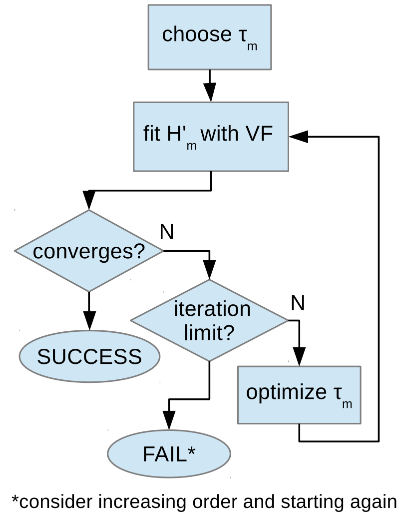


Figure 1: Algorithm for fitting H'_m using VF or WVF

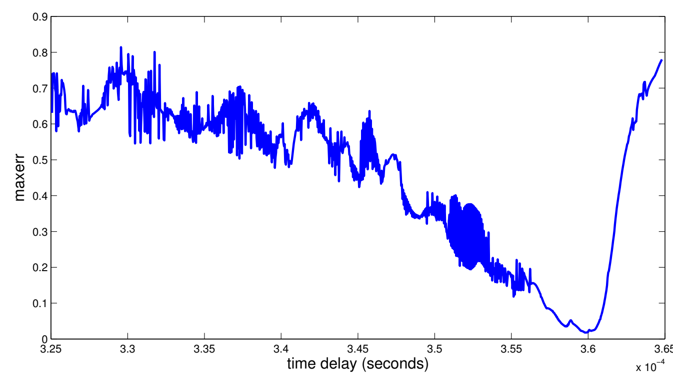


Figure 2: Typical example of VF *maxerr* with respect to choice of time delay, τ_m

In EMT-type program applications the time delay is processed in order to minimize the fitting error using Brent's method for root-finding. This involves first estimating the time delay, then applying iterative modification of the time delay in successively smaller intervals, adding the delay to the modal propagation function, performing VF/WVF, and then subtracting the delay before testing for convergence. In this study the delay is not removed and replaced, rather the magnitude is tested as an overall measure of fitness, to isolate the variations between the fitters without having to consider the delays.

B. magVF and WmagVF approach to fitting modal propagation functions

The magVF (and WmagVF) approach exploits the assumption that the propagation functions are minimum-phase to allow the fitting to proceed without prior time delay estimation. The magVF algorithm yields magnitude-squared poles and zeros which are symmetric with respect to the imaginary axis in the Laplace domain, as per (25).

To get the magnitude and phase response of a minimum-phase function from the magnitude-squared response after fitting with magVF, two approaches can be taken.

One way to get the magnitude approximation from the magnitude-squared approximation, is to take the square-root at each frequency point, as in

$$\sqrt{|H_m(\omega_k)|^2} = |H_m(\omega_k)|. \tag{35}$$

Using the fact that minimum-phase systems have a direct relationship between their magnitude and phase responses, it is possible to derive the phase response from this result [8], [9].

Alternatively, convert (27) into an equivalent poles-zeros-gain form and then select only the LHP poles and zeros. This is equivalent to going from (25) to (26). The DC gain, F_0 from (26), is determined by taking the ratio between the given response and the approximated one at any frequency point where the given response has appropriate magnitude. For this study, the magnitude responses were greatest at low frequencies, and so these values were employed for recovering F_0.

Once the phase of the minimum-phase function H'_m has been determined, letting $\measuredangle H'_m$ refer to the phase of the minimum-phase system that has been derived using magVF, and $\measuredangle H_m$ be the phase of the given modal domain function, then it is possible to employ the ULM theory from (5), such that

$$\measuredangle H_m = \measuredangle H'_m - \omega\tau_m, \tag{36}$$

$$\tau_m = \frac{\measuredangle H'_m - \measuredangle H_m}{\omega}. \tag{37}$$

Given that τ_m is theoretically constant, it can be solved as an overdetermined least squares problem using all or a select number of frequency points.

Two further modifications were made to the standard magVF, in addition to weighting and are listed below. These could be made internal to the magVF algorithm, so that they are applied automatically as needed, without additional external logic required, but for testing, they were exposed to determine their individual effects on the figures of merit.

1) Disciplined Convex Programming (DCP) to ensure non-negative definite magnitude-squared function: When using (27), magVF will occasionally return negative values for the magnitude-squared approximation at certain frequencies, particularly those where the magnitude has dropped very close to zero. Obviously, negative values are non-plausible for a magnitude-squared response. In [3] a convex optimization technique has been proposed to counter them. Similarly, Disciplined Convex Programming (DCP) methods [10] using the CVX software package [11] for MATLAB have also been implemented in this study to help to provide an optimized solution to the residues identification, where necessary.

2) A pole modification scheme based on smooth magnitude response: Since the modal propagation functions for transmission lines and cables have generally smooth magnitude responses, [12] suggests it is best to use real and logarithmically spaced values for their starting poles. In this vein, a new pole modification scheme is tested, such that at the start of each iteration, the incoming poles are modified by taking their negative absolute values. This way, all the injected poles for the subsequent iteration will be real and stable, lending themselves well to approximating smooth magnitude responses.

C. Case study: 6 conductor underground cable

Results from a 3 cable 6 conductor case study are presented. The specifications for this cable are shown in Fig. 3 and further details can be found in [13] (case CAB01).

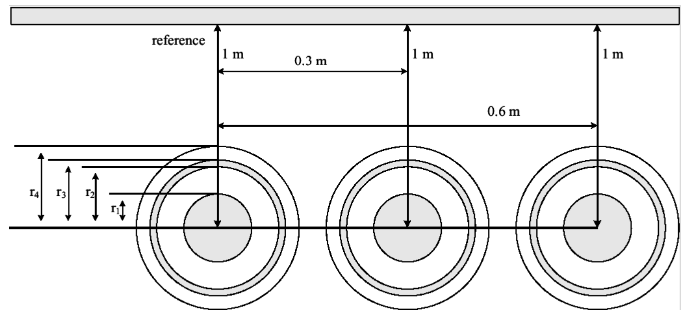


Figure 3: Configuration of underground power cables studied

Two types of tests were conducted.

1) The first involves fixing an arbitrary order, and trying to fit each modal propagation function with magVF, using different strategies (DCP optimization, weighting, pole modifications) to see the effects on convergence.
2) The second test involves a comparison of the various fitters (VF/WVF/magVF/WmagVF, with and without pole modification) to determine the lowest number of poles required to arrive at a successful fit within a defined error limit.

The figures of merit used to test the error of the magnitude fit are:

$$\varepsilon_{abs}(k) = \left| |F(s_k)| - \left|\widetilde{F}(s_k)\right| \right|, \tag{38}$$

$$maxerr = max\{\varepsilon_{abs}(k)\}. \tag{39}$$

1) Fitting using magVF with arbitrary order to see effect of incremental modifications: The algorithm used for determining the effect that the various modifications to magVF had on the final results is shown in Fig. 4. Table I gives results for the *maxerr* observed using this procedure.

Each modal propagation function was fitted with 12 poles, initially all real and distributed logarithmically. A *maxerr* of 0.0250 was defined as the threshold for successful convergence. A maximum of 300 iterations was allowed.

Examining Table I, it can be seen that modal propagation functions 2 and 3 failed to meet the desired target for maximum error using the default configuration. Functions 1 and 2 failed to meet the non-negative definite criteria, and needed the DCP constraints. After applying the constrained fitter, function 1 achieved convergence.

Next, weighting was applied to all modal domain functions. Although function 3 made a significant improvement with the application of weighting, it still remained above the desired threshold of 0.0250. Function 2 was reduced to 0.0239,

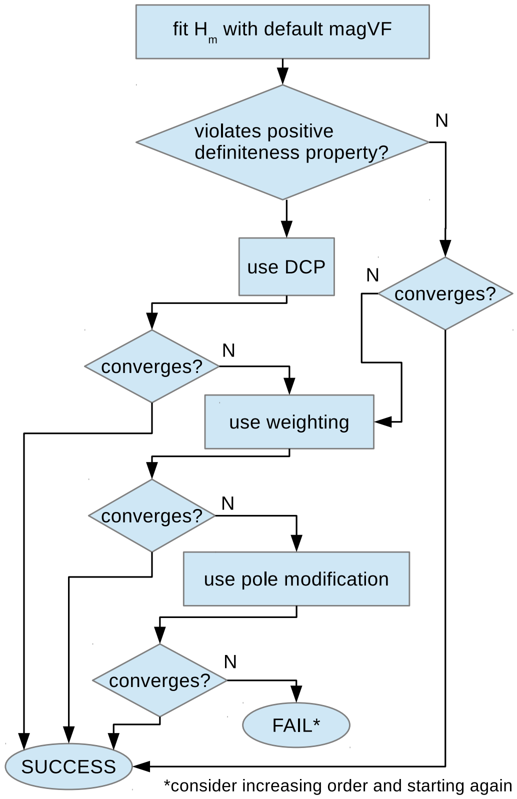


Figure 4: Testing algorithm used to determine effect of magVF modifications with fixed order

Table I: Resulting *maxerr* between given and fitted modal frequency response using magVF as per algorithm described in Fig. 4. Desired *maxerr* < 0.0250.

Modal function	**Default** *maxerr*	**Needs DCP?**	**DCP** *maxerr*	**Weighted** *maxerr*	**Pole mod.** *maxerr*
1	0.0166	Yes	0.0016	0.0016	0.0016
2	0.0272	Yes	0.0269	0.0239	0.0210
3	0.0918	No	0.0918	0.0459	0.0212
4	0.0052	No	0.0052	0.0052	0.0052
5	0.0052	No	0.0052	0.0052	0.0052
6	0.0052	No	0.0052	0.0052	0.0052

and thus achieved convergence thanks to the application of WmagVF. Since the other functions which converged had done so after the first iteration, weighting had no effect on them.

Finally, applying the pole modification scheme was required in addition to weighting to fit the remaining two functions. Function 2 converged after three iterations, while function 3 required 175 iterations before converging using this pole modification technique.

The resulting magnitude and phase plots are given in Fig. 5.

2) Minimum order required to achieve convergence using VF/WVF/magVF/WmagVF and pole modification: Another set of tests was run to see how the fitters performed with respect to minimizing the order required to arrive at a solution within a desired error. The algorithm for this is presented in Fig. 6.

The limit for the approximation order was defined as one-half of the total number of frequency samples available ($K =$

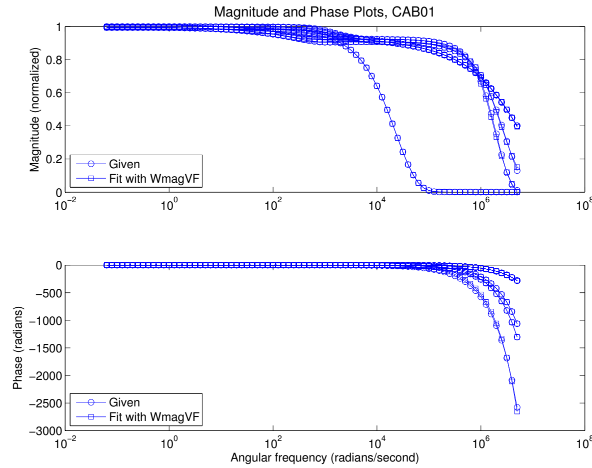


Figure 5: Magnitude and phase plots of all 6 modal propagation functions using 12 poles to fit ($Norder = 12$). Given modal data is superimposed with WmagVF approximation (see Table I for errors)

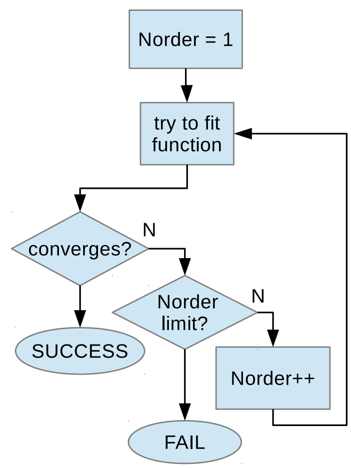


Figure 6: Algorithm for testing minimum order required to converge within desired error

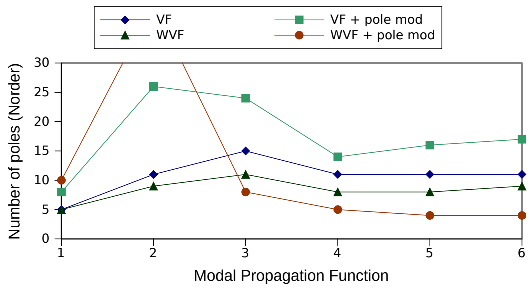
80), such that

$$Norder_{max} < \frac{K}{2} = 40 \text{ poles.} \tag{40}$$

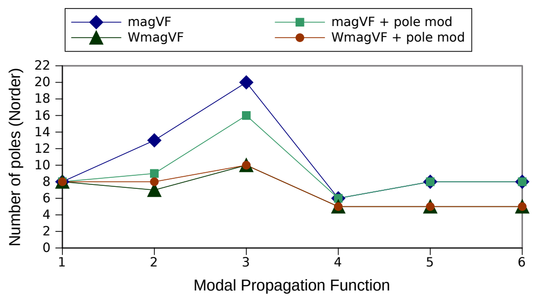
The results are summarized in Fig. 7a for VF and WVF, and Fig. 7b for magVF and WmagVF. It can be seen that in all but one case (WVF with pole modification, mode 2), the fitters were successful in finding solutions that converged within the desired maximum error limits ($maxerr < 0.0250$). Furthermore, the number of poles required by weighted fitting was consistently less than by non-weighted fitting.

3) Discussion:

a) Benefits of using weighting: It is evident that weighting showed an improvement with both fitting algorithms. This is especially true for troublesome functions – numbers 2 and 3 in this study. From both tests conducted, and with both families of fitters, weighting was able to demonstrate improved



(a) Using VF/WVF. Note that the second modal propagation function does not have a solution for WVF with pole modification. It does not appear on this figure as a result, since it never converged within the defined constraints ($Norder < \frac{K}{2}$)



(b) Using magVF/WmagVF, with/without pole modification.

Figure 7: Minimum order required for convergence with $maxerr < 0.0250$. Lower values are better.

performance.

b) Benefits of using pole modification for smooth response: A pole modification scheme of fitting based on smooth responses using real input poles was demonstrated in this paper. It proved to be beneficial in nearly all cases. It was required for modal function 3 when the order was fixed at 12 poles.

However, when employed with WVF there was one case (function 2) where this modification did not manage to provide a suitable solution. When, and how, this pole modification scheme is employed – which is effectively a perturbation of the fitter between iterations – should be examined in further detail to develop best practices accounting for the internal mechanics of the fitter and type of responses being fit.

c) Observation that W/VF does not guarantee minimum-phase systems: Taking the zeros of the approximated transfer functions provided by the VF or WVF algorithms it was observed that for many of the cases there were zeros in the right-hand s-plane. This implies that the modal approximations being returned by VF or WVF were often non-minimum phase, or mixed phase systems. The magVF and WmagVF algorithms, on the other hand, guarantee minimum-phase systems with zeros that are strictly in the left-hand s-plane. More research is needed to see the effect of pole-zero cancellation, and to study the implications of using mixed-phase approximations with the ULM.

d) Observation that magVF converges quickly: It was observed that during the second set of tests, as presented in Section III-C2, the magVF algorithm converged very quickly, often in the first iteration. As mentioned previously, this implies that weighting is often unnecessary, since it has no effect until the second iteration. Furthermore, it suggests that magVF is much quicker than the current VF/WVF method which requires an iterative approach to find a suitable value for time delay τ_m. Although, with magVF, certain modal propagation functions may require optimization of the residues to ensure non-negative definite values, this procedure is not more stringent than the iterative estimation of the time delays used in VF.

IV. Conclusion

In conclusion, this paper elaborates on the theory required for – and provides a successful demonstration of – an implementation of the magVF method for the identification of power system transfer functions. Additionally, this study provides the derivation of WmagVF, a new variation that employs iterative weighting, analogous to the WVF method. Selective modifications are also exploited as required to improve the convergence of the fit of the magnitudes for troublesome functions. It is observed that magVF can converge quicker than VF in certain cases, that magVF can implicitly guarantee minimum-phase approximations while VF cannot, that delay determination is trivial in magVF compared to VF, and that pole modification schemes and weighting can have a beneficial impact on the convergence and order of the fit. These results can be used to further improve the modelling and analysis of transmission lines and cables.

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