Alleviating Harmonic and Reactive Power Issues in Smart Grid Based on the Implementation of the Instantaneous p-q Power Theory under Unbalanced and Distorted Supply Voltages

Muhammad Shahbaz, Nadeem Jelani, Marta Molinas

Abstract— Distributed Energy Resources (DER) is emerging as the future of the electrical grid commonly known as Smart Grid or Micro grid. The electrical infrastructure of the power grid is under changes, in a sense that more and more independent power producers will supply power to the network of users that could or could not be connected to the main grid. The Micro grids are connected to the main power system grid through power electronic interface for flexible control so that the Micro grid will be crucial towards a well-functioning network. However, at the same time, the interface domain will be under high stress due to non-linear behaviour of power electronic components. Also, the use of non-linear loads at the consumers end in the form of power electronic converters, UPS, electric arc furnaces, and growing use of adjustable speed motor drives is increasing day by day. The power electronic based interface and loads inject harmonic current and reactive power into the supply grid having a significant impact on voltage and power quality, thus polluting the electric distribution network. With a network then dominated by nonlinear components (power electronics coupling for generators, and non-linear loads connected to distribution system), non-sinusoidal regimes will be a common situation. On the other hand, there is a high demand for premium electric power because of the high number of sensitive loads, which can malfunction if the supply has bad power quality. The paper suggests potential solution to the harmonic current and reactive power problems under unbalanced and distorted regimes using shunt active power filter (SAPF) with control based on Instantaneous active and reactive (p-q) power Theory.

Keywords: Active power filters, Distributed energy resources, Harmonics, Instantaneous p-q theory, Non-linear loads, Power Quality, Reactive Power, Smart Grid, Total harmonics distortion.

I. INTRODUCTION

Electrical energy is a product and, like any other product, should satisfy the proper quality requirements called power quality limits [1]. These set of limits allow the electrical systems to function in their intended manner without significant loss of performance or life [2]. From the last few decades, the tendency of connecting the power electronic (PE) equipment’s based loads to the power distribution system is increasing because of significant improvements in the voltage and current ratings of power electronic devices [3], [4]. PE loads are high source of harmonics and reactive power which results in an increased deterioration of the power system voltage and current waveforms, because of line impedance, the voltage at the point of common coupling (PCC) is no longer remains sinusoidal [3], [5]. The impacts of harmonics result in deterioration of insulation, increase in power losses, shortening life span of electrical installations, shutdowns, misoperation of sensitive equipment, capacitor failures, communication interference, overheating of transformers, overloading of neutral conductor, harmonic resonance, maloperation of electronic equipment, distorted supply voltage, system voltage dips, protection tripping’s, and AC/DC drives failure [2], [6], [7]. The higher value of reactive power causes voltage stability problems, low power factor, higher copper losses in the line conductor, and bulky equipment’s [2].

Because of these problems, the issue of the power quality delivered to the consumers end is of more interest than ever before [8]. Several International standards have been set with regards to power quality like IEEE-519, IEC-61000, EN-50160 and others [2], [8]. These standards demand that the total harmonic distortion (THD) produce by electrical equipment’s should not be higher than the defined limits of the standards.

THD is a power quality term that is used to define the amount of distortion in voltage or current waveform [2], [9]. THD in current waveform is given as:

$$THD\% = \sqrt{\sum_{n=2}^{\infty} \frac{I_n^2}{I_1^2}} \times 100$$  \hspace{1cm} (1)

where $I_1$ is the fundamental component of current and $I_n$ is the component of current with nth order harmonics.

II. ACTIVE FILTERING

In the past, tuned passive filters were used to solve the problems of reactive power and harmonics distortion [10]. However, these filters offered some drawbacks like they filter only the frequencies they are tuned for, their operation cannot
be limited to a certain load, resonances can occur because of the interaction between the passive filters and other loads with severe effects [10]. To compensate these drawbacks, recent efforts have been made on the development of an important group of power system conditioning circuits, commonly known as active power line conditioners (APLC) or simply active power filters [11]. The active power filters have gained much more attention because of excellent performance to mitigate the harmonic and reactive power problems. However, the performance of the active filters depends upon the control theory that is employed to formulate the control algorithm of the active filter controller [12].

Many control techniques have been suggested to control the active power filters and generally have two categories. The first category is based on time domain such as the p-q theory, the d-q theory, the synchronous reference current theory, direct power control, and conservative power theory [13]. The second category is based on frequency domain like Fourier Transformation, and Kalman Filter [13]. The time domain analysis is superior over the frequency domain and of great interest in recent years [13]. However, in this paper p-q Instantaneous power theory is used because of its excellent performance and simplicity to develop the control algorithm of the shunt active filter.

The voltage source inverter (VSI) based shunt active power filter (SAPF) has used to mitigate the harmonics and reactive power problems because of its low cost and higher efficiency compared to current source inverter (CSI) based SAPF [14]. However, both active filters have the same functionality: to force the converter to behave as a controlled current source in the source current after the working of the Instantaneous p-q Theory based SAPF should be according to the defined standards and it should be in phase with the distribution utility voltage.

III. ACTIVE FILTER REFERENCE CURRENT GENERATION BY INSTANTANEOUS P-Q THEORY

The Instantaneous p-q Theory is based on a set of instantaneous powers defined in time domain that bring about a good dynamic response with no restrictions are imposed on voltage and current waveforms [14]. It can be applied to a 3-phase system with or without neutral wire and equally valid for steady state and transients conditions. This theory is very efficient and flexible in designing the controllers for APCL based on PE devices [14]. The theory first transforms voltage and current waveforms from abc to αβ0 coordinates by Clarke’s transforms, then defines instantaneous powers in αβ0 and finally computes the compensating reference currents in abc by Inverse Clarke’s transforms. The theory always considers the 3-phase system as a unit, not a superposition or sum of three single-phase circuits [14].

For a 3-phase sinusoidal system the supply voltages and load currents are measured and transformed from abc to αβ0 frame using Clarke’s transform by (3) and (4):

\[
\begin{bmatrix}
    v_a \\
    v_b \\
    v_c
\end{bmatrix}
= \begin{bmatrix}
    1 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\
    -\frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{\sqrt{3}}{2} \\
    \frac{1}{2} & 0 & \frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
    v_a \\
    v_b \\
    v_c
\end{bmatrix}
\]

\[
\begin{bmatrix}
    i_a \\
    i_b \\
    i_c
\end{bmatrix}
= \begin{bmatrix}
    1 & \frac{\sqrt{3}}{2} & \frac{1}{2} \\
    -\frac{\sqrt{3}}{2} & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\
    -\frac{1}{2} & 0 & \frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
    i_a \\
    i_b \\
    i_c
\end{bmatrix}
\]

where \(v_a, v_b, v_c\) are the supply voltages and \(i_a, i_b, i_c\) are the load currents.

The instantaneous powers \(p\) and \(q\) can be written as:

\[
\begin{bmatrix}
p \\
q
\end{bmatrix}
= \begin{bmatrix}
v_a & v_b & i_a \\
v_a & v_c & i_b \\
-v_b & v_c & i_c
\end{bmatrix}
\]

From (5) we have:
\[ \frac{I_p}{I_q} = \frac{1}{v_{α} + v_{β}} \left[ v_{α} - v_{β} \right] \left[ p \right] \]

where \( p = \bar{p} + \dot{p} \) and \( q = \bar{q} + \dot{q} \). These powers are defined as:

- \( p \): Active power for a 3-phase system with or without neutral conductor in steady state or transients. It represents the total instantaneous energy flow per second between power source and load.
- \( q \): Imaginary power and proportional to the quantity of energy that is being exchanged between phases of the system. It doesn’t contribute to energy transfer between source and load.

The compensation strategy depends upon the desire purpose to achieve. Several kinds of compensation strategies are available when working with the Instantaneous \( p-q \) Theory. However, in this paper harmonics compensation (i.e. compensation of \( \dot{p} \) & \( \dot{q} \)) and reactive power compensation (i.e. compensation of \( \bar{q} \)) have done.

Therefore the compensator has to select the following powers as a reference to follow the control strategy. The instantaneous reactive power supplied by the compensator:

\[ q_c = -q \]  

Instantaneous active power supplied by the compensator:

\[ p_c = \bar{p} + p_{\text{loss}} \]  

where \( p_{\text{loss}} \) is the power drawn by the compensator from the power source to make up the switching losses of the converter and to maintain constant voltage across the dc-link capacitor at a pre-specified value.

A. Compensation Strategy and Selection of Reference Power for Compensation

The compensation strategy depends upon the desire purpose to achieve. Several kinds of compensation strategies

\[ \begin{bmatrix} i_{p}' \ 
\end{bmatrix} \begin{bmatrix} \alpha \\
\beta
\end{bmatrix} = \begin{bmatrix} \frac{1}{v_{α} + v_{β}} \left[ v_{α} - v_{β} \right] \left[ p \right] 
\end{bmatrix} \]

B. Reference Current Calculation for the Compensator

The compensator reference current in \( \alpha \beta \) will be:

\[ \begin{bmatrix} i_{cα}' \\
i_{cβ}'
\end{bmatrix} = \frac{1}{v_{α} + v_{β}} \left[ v_{α} - v_{β} \right] \left[ p_c \right] \]

and in \( abc \) by Inverse Clarke’s transformation:

\[ \begin{bmatrix} i_{cα}' \\
i_{cβ}' \\
i_{cγ}'
\end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\
\frac{1}{2} & -\frac{1}{2} & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix} i_{cα} \\
i_{cβ} \\
i_{cγ}
\end{bmatrix} \]

IV. FUNDAMENTAL POSITIVE SEQUENCE VOLTAGE DETECTOR

The designed SAF based on adopted control strategy should perfectly determine the reference current by the integration of the Instantaneous \( p-q \) Theory to mitigate the current harmonics and to make the source current in-phase.
with the utility voltage at the same time. This control works very well in the presence of ideal supply voltages. However, in case if the supply is unbalanced or highly distorted then the reference current calculated by control algorithm shown in Fig. 3 without the fundamental positive sequence voltage detector, neither completely filter the current harmonics nor compensate the fundamental reactive power [15]. Under unbalanced or distorted supply voltage conditions the Instantaneous p-q Theory must have fundamental positive sequence voltage detector for the successful operation of the active filter.

Fig. 4 drives the fundamental positive sequence signal from an unbalanced and distorted three phase voltage signal carried by power lines. The phase locked loop (PLL) control circuit in Fig. 5, described in next section, determines accurately the frequency harmonics appear in the oscillating components (α and β) powers are considered here. The average powers \( p' \) and \( q' \) are composed of fundamental positive sequence component \( V_{+1} \) as the auxiliary currents \( i'_a \) and \( i'_b \) which are used together with \( v_a \) and \( v_b \) in the Fig. 4 to calculate the auxiliary powers \( p' \) and \( q' \).

However, the average values of the real \( p' \) and imaginary \( q' \) powers are considered here. The average powers \( p' \) and \( q' \) are composed of fundamental positive sequence component \( V_{+1} \) as the auxiliary currents \( i'_a \) and \( i'_b \) which are used together with \( v_a \) and \( v_b \) in the Fig. 4 to calculate the auxiliary powers \( p' \) and \( q' \).

The instantaneous voltages \( v'_a \) and \( v'_b \) which correspond to time function of fundamental positive sequence voltage component \( V_{+1} \) are given by (11):

\[
\begin{bmatrix}
    v'_a \\
    v'_b
\end{bmatrix} = \begin{bmatrix}
    1 & 1 & 1 \\
    1 & -1 & 1
\end{bmatrix} \begin{bmatrix}
    i'_a \\
    i'_b \\
    i'_c
\end{bmatrix}
\]

By applying the Inverse Clarke’s transform, the fundamental positive sequence voltages \( (v'_a, v'_b, v'_c) \) without error in amplitude and phase angle are given in (12):

\[
\begin{bmatrix}
    v'_a \\
    v'_b \\
    v'_c
\end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix}
    1 & 1 & 1 \\
    1 & -1 & 1
\end{bmatrix} \begin{bmatrix}
    v'_a \\
    v'_b \\
    v'_c
\end{bmatrix}
\]

**A. The Design of Phase Locked Loop Circuit**

The PLL circuit shown in Fig. 5, tracks continuously the fundamental frequency of the system voltage. It works very well under unbalanced highly distorted conditions of the system voltage to quickly determine the system frequency and phase angle of the measured system voltages \( (v_a, v_b, v_c) \). The circuit uses the algorithm based on a fictitious instantaneous power given by (13):

\[
p'_{3φ} = v_a i'_a + v_b i'_b + v_c i'_c = v_{ab} i'_a + v_{cb} i'_c
\]

The expression in (13) considered that \( i'_a + i'_b + i'_c = 0 \). The \( p'_{3φ} \) is called fictitious power because it is just a variable and is not related to any instantaneous power in the power system. The fictitious current feedback signals \( i'_a(\omega t) = \sin(\omega t) \) and \( i'_b(\omega t) = \sin(\omega t + 2\pi/3) \) of the PLL circuit are built by calculating the time integral of the output \( \omega \) of the PI controller. The current \( i'_c(\omega t) \) lags at 120° by \( i'_b(\omega t) \) and both of them have unity amplitude. The PLL circuit can reach a stable point of operation only if the input \( p'_{3φ} \) of the PI controller has zero average value (\( p'_{3φ} = 0 \)) and has minimum low frequency oscillating portion in \( p''_{3φ} \), where \( p'_{3φ} = p''_{3φ} + p'''_{3φ} \).

The average three phase power \( (p'_{3φ} = p''_{3φ}) \) in term of phasors is given as:

\[
p'_{3φ} = p''_{3φ} = 3V_a I'_a \cos \phi
\]

The expression in (14) is valid only if the output of the PI controller \( \omega \) corresponds to the system frequency and the feedback signal \( i'_a(\omega t) \) becomes orthogonal to the fundamental positive sequence component of the measured voltage \( V_a \).

However, if the point where \( V_a \) leads the signal \( i'_a(\omega t) \) is reached, this is still an unstable point of operation. At this point, an event disturbance that slightly increase the system frequency (the frequency of the \( V_{ab} \) and \( V_{cb} \) in Fig. 5) will make the voltage phasor \( V_{+1} \) rotate faster than the current phasor built up by the feedback signals \( i'_a(\omega t) \) and \( i'_b(\omega t) \). Hence, the displacement angle between \( V_a \) and \( i'_a(\omega t) \), given by \( \cos \phi \) in (14), becomes greater than 90°. This results to a decreasing average input (\( p''_{3φ} < 0 \)), and consequently to a decreasing \( \omega \), making the \( \cos \phi \) even greater. This characterizes an unstable point of operation. Thus, the PLL has only one point of operation, that is \( i'_a(\omega t) \) leading by 90° the fundamental positive sequence component corresponding to \( V_a \).

Now for the same disturbance mentioned above, \( i'_a(\omega t) \) leading \( V_a \) by 90°, the \( \cos \phi \) will become less than 90° and the average power in (14) will be positive. This will make the current phasor to rotate faster, keeping the orthogonality (leading currents) between the generated \( I_{+1} \) [i'’(\omega t)] and the measured \( V_{+1} \).
This fundamental characteristic of the PLL circuit shown in Fig. 5 is exploited to form auxiliary signals $i'_{a}$ and $i'_{b}$ that are needed in the fundamental positive sequence detector of Fig. 4. Since, $i'_{a}(ot) = \sin(ot)$ leads by $90^\circ$ the fundamental positive sequence component $V_{r,a}$ of the measured system voltage. Hence, the auxiliary current $i'_{a}(ot) = \sin(ot-\pi/2)$ will be in-phase with $V_{r,a}$.

V. CURRENT MODULATOR

The performance of the active filter heavily depends on the design characteristics of the current modulator i.e method to generate the gating signals for the VSI [4]. Mostly pulse width modulation (PWM) strategies are employed in the active filter controller to modulate the current modulator. In this paper, Triangular Carrier control PWM technique is used. Fig. 6 demonstrates this method. At the input, the active filter reference current generated by Instantaneous p-q Theory and the filter actual current are compared to produce the error. To make the error steady, it is passed through PI controller. At the end, steady current is compared with the triangular wave with fixed carrier frequency to get the gate pulses for the VSI.

VI. SIMULATION RESULTS

A complete model of the shunt active filter connected to distribution power system is simulated in Matlab/Simulink and only the important results are presented. The parameters of the simulated system are shown in Table I. The unbalanced and highly distorted distribution source is supplying power to the unbalanced diode and thyristor bridge based loads shown in Table I. The expression for the source is shown in (15):

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix} = \begin{bmatrix}
\sin(ot) \\
\sin(ot-120^\circ) \\
\sin(ot+120^\circ)
\end{bmatrix} + \begin{bmatrix}
0.0909v_s \\
0.0681v_s \\
0.0318v_s
\end{bmatrix} \sin(3ot) + \begin{bmatrix}
0.0318v_s \\
0.0681v_s \\
0.0909v_s
\end{bmatrix} \sin(5ot) + \begin{bmatrix}
0.0318v_s \\
0.0681v_s \\
0.0909v_s
\end{bmatrix} \sin(7ot)
\]

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix} = v_s \sin(ot) + 0.0455v_s \sin(5ot) + 0.0318v_s \sin(7ot)
\]

Table I: Parameters of the simulated system

<table>
<thead>
<tr>
<th>Source parameters</th>
<th>$V_s=220V_{RMS}$ (Line-Ground), $R_s=10\mu\Omega$, $L_s=0.01mH$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filter parameters</td>
<td>$V_{dc1}=V_{dc2}=350V$, $C_{dc1}=C_{dc2}=1.8mF$, $R_f=10\mu\Omega$, $L_f=2mH$, $f_s=15kHz$</td>
</tr>
<tr>
<td>Load parameters</td>
<td>$R_l=10\mu\Omega$, $L_l=2mH$</td>
</tr>
<tr>
<td>I. 3-phase thyristor bridge supplying R-L load of 20Ω + 20 mH, Firing angle=15°</td>
<td></td>
</tr>
<tr>
<td>II. 2-phase diode bridge step change in load connected to phase-ab with Initial value 10A and final value 20 A</td>
<td></td>
</tr>
<tr>
<td>III. 3-phase thyristor bridge supplying R-L load of 10Ω + 30 mH, Firing angle=15°</td>
<td></td>
</tr>
</tbody>
</table>

The total simulation run time is 0.3 sec (Dell Latitude E6400 machine). Fig. 7a shows that the source voltage profile is unbalanced and distorted. The fundamental positive sequence voltage is extracted from the source (shown in Fig. 7c) for the efficient working of the control. The dynamic behavior of the active filter is checked by connecting different loads to the system during the run time of the simulation. The loads I is connected from the start ($t=0$s) of the simulation and is triggered at firing angle of 15°. The load II is connected between phase $ab$ at $t=0.1$s with step change in current from 10A to 20A. The connection of load II make the load side unbalanced. Similarly, the load III is connected at $t=0.2$s and is triggered at firing angle of 10°. Fig. 7b shows the connection of these loads to the system. Fig. 8b shows (i.e at $t=0.1$s and $t=0.2$s when load II & III are connected) that the active filter is dynamically fast and responded to the change in load within 1 power cycle.

Fig. 7b shows that load is highly unbalanced and distorted with a value of THD=20.17% shown in Fig. 9a. The SAF closely follows the reference current generated by Instantaneous p-q Theory (shown in Fig. 8b) to supply negative sequence current to make the source current balanced and sinusoidal shown by Fig. 8b. The level of THD reduced from 20.17% to 2.27% shown by Fig. 9a and 9b. However, the THD in source current after the active filtering is not exactly zero. It is because internal switching of the compensator itself generates some harmonics.

Fig. 8c shows that after the operation of the active filter the power factor of the distribution system becomes unity (source voltage and current are in phase) and source supplies only constant active power to the load.
The comparison of Fig. 9a and 9b shows that after the operation of active filter the amplitude of the fundamental source current reduced from 52.27 A to 40.55 A. It is because active filter has compensated the $\bar{p}$, $\bar{q}$ and $\bar{q}$ demanded by the load. Now, the source is supplying only active only portion of current of 40.55 A to load.

VII. CONCLUSIONS

The solution to the harmonic and reactive power problems for an unbalanced and distorted distribution supply system feeding to an unbalanced nonlinear load system is provided by shunt active power filter. The control of the shunt active power filter is developed by Instantaneous $p$-$q$ Theory. The Instantaneous $p$-$q$ Theory with the help of fundamental positive sequence voltage detector computes the compensating reference current that comprises all components that differ from the active portion of the fundamental positive sequence current of the load. The link between the control and compensator is established through gating signals issued for the semiconductor switches. The gating signals are obtained by Carrier Control pulse width modulation technique that works nicely to trigger the compensator based on Insulated gate bipolar junction transistors switches. The simulations results show that shunt active power filter has managed to control the harmonics defined by the limits, and made the source current balanced, sinusoidal, and in-phase with the supply voltage to ensure the supply of constant active power from source to load.

VIII. REFERENCES