A New Method for the Inclusion of Frequency Domain Responses in Time Domain Codes

K. Sheshyekani, H. R. Karami, P. Dehkhoda, F. Rachidi, R. Kazemi, S. H. H. Sadeghi, R. Moini

Abstract--**The paper presents a general methodology based on the matrix pencil method (MPM) for rational fitting of frequency domain responses to be properly incorporated in time domain analysis. The method involves three distinct stages. First, the frequency domain response is approximated by a sum of exponential functions using the MPM. The time domain representation of the obtained exponentials is then directly derived by a closed form inverse Fourier transform. Finally, the MPM is used again to estimate the poles and residues from the time domain function. The main feature of the method is its direct solution, hence, avoiding any iteration in the estimation process. Moreover, the method needs no starting poles as opposed to the iterative methods such as vector fitting (VF) method. The approach is validated by comparing the results with those obtained either analytically or by the VF method for different frequency responses. Several case studies including frequency response of a distribution transformer as well as a buried coaxial cable are presented to show the generality of the proposed method.**

Keywords**: keywords. System Identification, Matrix Pencil Method, Frequency Response, Frequency Dependence.**

I. INTRODUCTION

Accurate inclusion of frequency dependent effects in time domain simulations has attracted great attention in recent years [1]-[4]. It is important in the sense that there are many cases such as transmission lines and grounding systems in which frequency dependency might arise needing to be properly investigated via time domain codes. This frequency dependency is of great importance in many power system problems including transformers, transmission line and cables as well asElectromagnetic Compatibility (EMC) problems such as transient analysis of grounding systems. The tendency towards using time domain codes for the analysis of the above mentioned problems stems from their capability to model complex networks. Moreover, nonlinear elements such as arresters are amenable to time domain solutions. Time-domain techniques can be used in a straightforward way to treat nonlinearities such as arresters. However, frequency dependency needs to be incorporated into the technique through convolution integrals, which would result in significant increase in the computation time.

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In this paper a general methodology based on the Matrix Pencil Method (MPM) for the fitting of frequency domain responses is introduced [5]. The method estimates the poles from the measured or calculated frequency domain responses. The proposed technique is well-suited for the estimation of any type of functions with a high number of resonances as well as functions contaminated by noise.Contrary to the vector fitting (VF) method; the proposed technique is a direct solution and avoids any iteration in the estimation process. Moreover, it needs no starting poles while VF is based on iteratively relocating initial poles set to better locations and its convergence might even stall in practical cases where the signal is noisy.

In order to evaluate the performance of the proposed technique, various case studies will be presented. We first use the method to approximate a known analytic frequency domain function. The method is then applied to the case of a distribution transformer as well as a coaxial cable system.

II. THEORY

Without losing any generality the frequency response of a system is supposed to be constituted by the sum of complex exponentials. This approximation is properly done in frequency domain by use of the MPM [5]. Although the detailed formulation of the MPM can be found in [6], the theory is customized here for the frequency domain application. A summary of the theory is given below for the ease of understanding. The general form of a frequency domain response can be written as,

$$y(f) = x(f) + n(f) \approx \sum_{i=0}^{M-1} R_i exp(S_i f) + n(f) \qquad (1)$$

where $y(f)$ is the frequency domain response involving two distinct terms; $x(f)$ that represents the pure response signal and $n(f)$ that accounts for the noise of the system (seen most often in measured data). In (1), R_i and S_i are complex-valued constants.

By discretizing (1), with a sample frequency of F_s, we obtain,

$$y(kF_s) = x(kF_s) + n(kF_s) \approx \sum_{i=0}^{M-1} R_i z_i^k + n(kF_s) \qquad k = 0,1,2,\dots,N-1 \qquad (2)$$

where $z_i = exp(S_i F_s), \quad i = 0,1,2,\dots,M-1$

The aim is to find the best estimatesforM, R_i and z_i, $i = 0,1,2,\dots,M$. This problem is generally treated as a nonlinear problem. We resort to the MPM to obtain the targeted aforementioned parameters [5].

A. Matrix Pencil Method

Let's ignore the noise in (2) and assume two matrices $[Y_1]$ and $[Y_2]$ as follows,

$$[Y_1] = \begin{bmatrix} y_1 & y_2 & \cdots & y_L \\ y_2 & y_3 & \cdots & y_{L+1} \\ \vdots & \vdots & \cdots & \vdots \\ y_{N-L} & y_{N-L+1} & \cdots & y_{N-1} \end{bmatrix}_{(N-L)\times L} \tag{3}$$

and

$$[Y_2] = \begin{bmatrix} y_0 & y_1 & \cdots & y_{L-1} \\ y_1 & y_2 & \cdots & y_L \\ \vdots & \vdots & \cdots & \vdots \\ y_{N-L-1} & y_{N-L} & \cdots & y_{N-2} \end{bmatrix}_{(N-L)\times L} \tag{4}$$

So $[Y_1]$ and $[Y_2]$ can be written as,

$$[Y_1] = [Z_1][R][Z_0][Z_2] \tag{5}$$

$$[Y_2] = [Z_1][R][Z_2] \tag{6}$$

where

$$[Z_1] = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_M \\ \vdots & \vdots & \cdots & \vdots \\ z_1^{N-L-1} & z_2^{N-L-1} & \cdots & z_M^{N-L-1} \end{bmatrix}_{(N-L)\times M} \tag{7}$$

$$[Z_2] = \begin{bmatrix} 1 & z_1 & \cdots & z_1^{L-1} \\ 1 & z_2 & \cdots & z_2^{L-1} \\ \vdots & \vdots & \cdots & \vdots \\ 1 & z_M & \cdots & z_M^{L-1} \end{bmatrix}_{M\times L} \tag{8}$$

$$[Z_0] = diag(z_1, z_2, \dots, z_M) \tag{9}$$

$$[R] = diag(R_1, R_2, \dots, R_M) \tag{10}$$

Now we introduce the matrix pencil as generalized eigenvalue problem [6],

$$[Y_1] - \lambda[Y_2] = [Z_1][R]\{[Z_0] - \lambda[I]\}[Z_2] \tag{11}$$

where $[I]$ is the $M \times M$ identity matrix. The rank of the left side of (11), will be M if $M \le L \le N - \mathrm{M}$ [6]. However, if $\lambda = z_i$, $i = 1, 2, \dots, M$, the rank is reduced to $M - 1$ since the i^{th} row and column of $[Z_0] - \lambda[I]$ become zero. Hence, $z_i$is found by solving the generalized eigenvalues of $[Y_1] - \lambda[Y_2]$, i.e.,

$$[Y_1][r_i] = z_i[Y_2][r_i] \tag{12}$$

where $[\mathrm{r_i}]$ is the generalized eigenvectors corresponding to $\mathrm{z_i}$, or in the equivalent form,

$$\{[Y_2]^{\dagger}[Y_1] - z_i[I]\}[r_i] = 0 \tag{13}$$

where $[Y_2]^{\dagger}$ is the Moore-Penrose pseudo-inverse of $[Y_2]$ or

$$[Y_2]^{\dagger} = \{[Y_1]^H[Y_1]\}^{-1}[Y_1]^H \tag{14}$$

and the superscript 'H' denotes the conjugate transpose operation.

We can obtain $\mathrm{z_i}$ from the eigenvalues of $[\mathrm{Y_2}]^{\dagger}[\mathrm{Y_1}]$. Hence, the complex value (S_i) obtained directly in a one step process by making use of the MPM.

Having obtained M and z_i, the complex amplitude (R_i) can be easily achieved by solving a least-square problem as,

$$\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_M \\ \vdots & \vdots & \cdots & \vdots \\ z_2^{N-1} & z_2^{N-1} & \cdots & z_M^{N-1} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_M \end{bmatrix} \tag{15}$$

B. Closed Form Inverse Fourier Transform

As described earlier, for a frequency domain response, $Y(f)$, it can be sampled at Δf and represented in the following form

$$Y(f) = \textstyle\sum_{m=0}^{M-1} r_m\, exp(S_m f) \tag{16}$$

where r_m and S_m are the complex-valued constants. If $Y(f)$ is causal and bounded, it is necessary that

$$Re\,\{S_m\} \le 0 \tag{17}$$

Then the time domain representation of $\mathrm{Y}(f)$ can be directly obtained by making use of the closed form Inverse Fourier Transform (IFT) as [7],

$$y(t) = F^{-1}\{Y(f)\} \approx \textstyle\sum_{m=0}^{M-1} \frac{c_m T_m - d_m(t-Q_m)}{\pi[(t-Q_m)^2 + T_m{}^2]} \tag{18}$$

where $T_m = -\frac{1}{2\pi}Re\{s_m\}$, $Q_m = -\frac{1}{2\pi}Im\{s_m\}$, $c_m = Re\{r_m\}$, and, $d_m = Im\{r_m\}$. Since $y(t)$ is obtained by (5), sampling time is not required to be restricted by $\Delta t = 1/2\Delta f$.

Once the time domain representation of the frequency domain response is obtained, the MPM is again utilized in time domain to calculate the poles and residues of the time domain function [7]. It is then possible to model the time domain function as a sum of complex exponentials as,

$$y(t) = \textstyle\sum_{i=0}^{K-1} R_i\, exp(S_i t) \tag{19}$$

It is well known that for a linear time-invariant system, the eigen-functions of the transfer operator are of the form $e^{s_i t}$ where S_i are the poles of the system while R_i stand for the system residues. The exponential representation of $y(t)$ can be finally cast in a rational form, thus permitting the inclusion of frequency dependency in time domain codes, *i.e.*,

$$Y(s) = \textstyle\sum_{i=0}^{K-1} \frac{R_i}{S - S_i} \tag{20}$$

It is worth noting that the MPM is also able to extract the poles and residues when the frequency response is contaminated by noise. It is done following the method described in [6].

III. Numerical Analysis and Results

To demonstrate the generality of the proposed technique, we consider different frequency responses including the frequency response of a distribution transformer and a buried coaxial cable. Prior to apply the method on these cases, we first verify the method with a known frequency response function.

A. Known Frequency Response Function

To demonstrate the validity of the proposed technique we assume a known transform function as,

$$Y(\omega) = k_1 \frac{(\alpha + j\omega)}{(\alpha + j\omega)^2 + \omega_1{}^2} + k_2 \frac{\omega_0}{(\alpha + j\omega)^2 + \omega_0{}^2} + k_3 \frac{1}{(\alpha + j\omega)^2} \tag{21}$$

where the coefficients are as follows,
$\alpha = 1,\ k_1 = 0.01, k_2 = 1.25,\ k_3 = 1.3,\ \omega_1 = 1,\ \omega_0 = 2.$

Using the Fourier transform table [8] the exact IFT of $Y(\omega)$ is

$$y(t) = F^{-1}\{Y(\omega)\} = e^{-\alpha t}\left\{k_1 \cos(\omega_1 t) + k_2 \sin(\omega_0 t) + \frac{t^2}{2}\right\} u(t) \quad (22)$$

$Y(\omega)$ is first sampled with $\Delta f = 0.02$ and N=201 points. Following the procedure described in the section II. A, we first apply the MPM to $Y(\omega)$. The obtained complex valued coefficients are listed in Table I.

TABLE I
ESTIMATED COMPLEX VALUED COEFFICIENTS FOR THE VERIFICATION EXAMPLE (21).

m	S_m	R_m
0	-13.7414 -41.6368i	0.0723 + 0.1453i
1	-9.7914 -23.9459i	0.4118 + 0.6191i
2	-7.8377 -15.6052i	0.1531 - 0.8239i
3	-5.8165 - 6.8079i	0.7690 + 0.9380i
4	-3.4340 - 2.3513i	0.4723 - 0.7540i
5	-1.0601 - 0.4316i	-0.0739 - 0.1240i

The frequency domain original data are shown in Fig. 1. Also shown in this figure is the MPM approximation of the data. It is clearly seen form the figure that the MPM results in a very good approximation of the data in frequency domain.

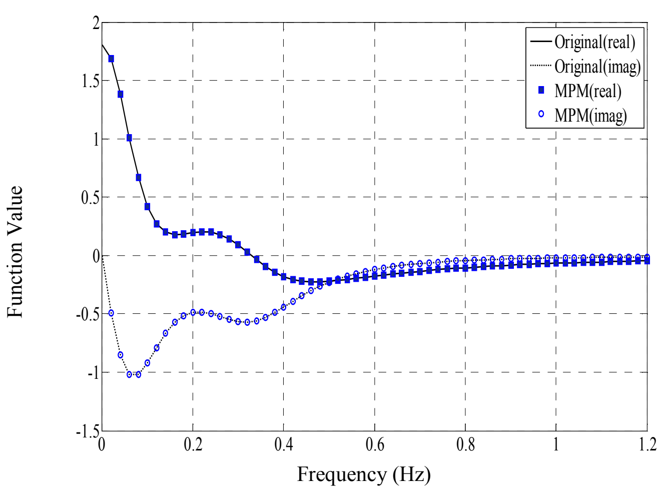

Fig. 1. Frequency domain original data together with its MPM counterpart (real and imaginary).

B. Time domain representation

Having obtained the exponential representation of the frequency domain response, we now apply the closed form IFT to the data obtaining its time domain response. The real poles and residues of the time domain signal are listed in Table. II. Results associated with the closed-form IFT are shown together with their analytical counterparts (18) in Fig. 2. It is seen that our results are generally consistent with their analytical counterparts.

C. Distribution Transformer

To further evaluate the efficiency of the proposed method, we consider the measured frequency response of a distribution transformer for which the results are available in [9]. Fig. 3 shows the measured frequency response together with the fitted curve obtained by the VF method and the MPM. It is seen that the vector fitting method accurately estimates the function. The MPM curve has very close overlay with the original data. It is noted that, the two estimation methods differ in terms of their mathematical foundation. However, the MPM is not prone to the initial poles and avoids any iteration in the approximation procedure. This might help us with the convergence of the method as well as the computational burden. Table III shows the complex valued coefficients of the transformer frequency response.

TABLE II
ESTIMATED POLES AND RESIDUES FOR THE VERIFICATION EXAMPLE (21).

m	S_m	R_m
0	-0.9038 + 2.0276i	0.0382 - 0.5129i
1	-0.9038 - 2.0276i	0.0382 + 0.5129i
2	-0.5917 + 0.3040i	-0.0345 - 1.0358i
3	-0.5917 - 0.3040i	-0.0345 + 1.0358i

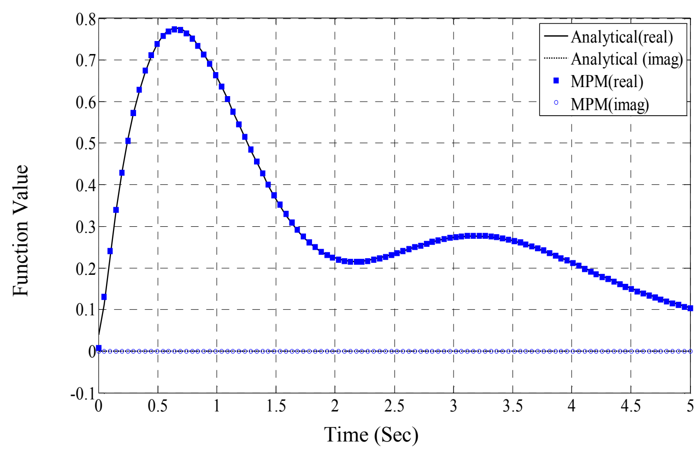

Fig. 2. Time domain representation of the frequency domain data shown in Fig. 1.

D. Cable System

In the final example, we consider a buried coaxial cable. The case contains 3 identical single-core cables, each one has two conductors. The complete layout is shown in Fig. 4. The burial depth is 0.5 m and the ground resistivity is 250 ohm-m. The cable length is assumed to be 1 km for the calculations. The geometrical and electrical data associated with this cable system are shown in Table IV. Each conductor is associated with a non-zero phase number, thus the generated model for this example will have 6 different modes (6 wires). Fig. 5 shows the magnitude of the characteristic admittance (Yc) for different modes while the propagation functions for mode 1 and 6 are shown in Fig. 6. It is seen that the technique succeeds to approximate the frequency response for different modes. Having obtained the complex values of the characteristic impedance and the propagation function its respective time domain representation could be obtained following the procedure described in section II.

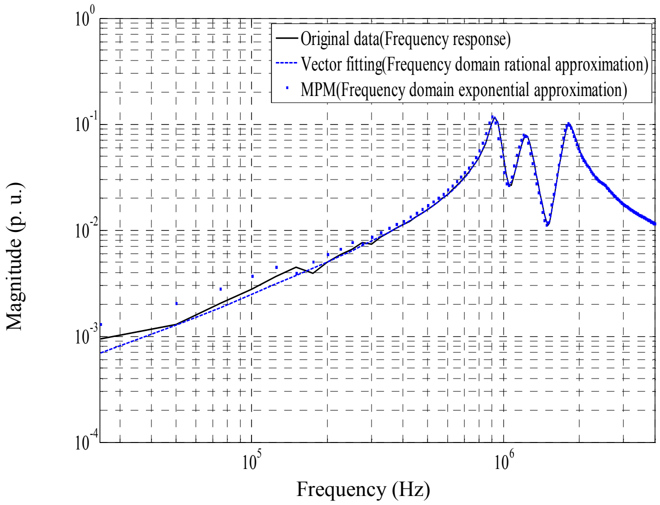

Fig. 3. Frequency domain original data together with its MPM and VF counterparts.

TABLE III
ESTIMATED COMPLEX VALUED COEFFICIENTS FOR THE FREQUENCY RESPONSE OF THE DISTRIBUTION TRANSFORMER AS SHOWN IN FIG. 3.

m	$S_m/1e\text{-}4$	R_m
0	-0.0023 - 0.2064i	0.0098 - 0.0062i
1	-0.0019 - 0.0107i	0.0062 + 0.0438i
2	-0.0024 - 0.1304i	-0.0029 - 0.0112i
3	-0.0037 - 0.0913i	-0.0182 - 0.0163i
4	-0.0064 - 0.0654i	0.0501 - 0.0129i

TABLE IV
GEOMETRICAL AND ELECTRICAL DATA ASSOCIATED WITH THE STUDIED CABLE SYSTEM.

Conductors					Insulators	
N	Inside Radius (m)	Outside Radius (m)	Resistivity (ohm-m)	Phase Number	Relative Permittivity	Loss Factor
1	0.00317	0.01254	0.17e-07	1	3.5	0.001
2	0.02273	0.02622	0.21e-06	2	2.0	0.001

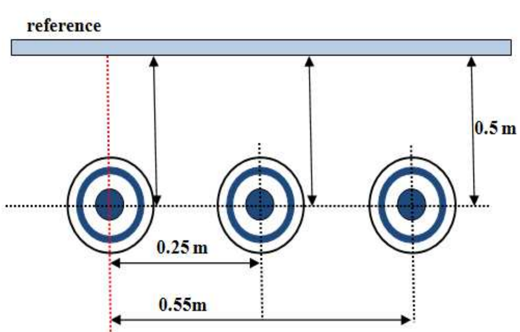

Fig. 4. Single core coaxial cable layout.

IV. NUMERICAL CONSIDERATION

As described earlier, we use the matrix pencil as generalized eigenvalue problem expressed by (11).The choice of M (dimension of the unitary matrix I in (11)), determines the precision of the method. To this aim, we take the advantage of the ratio of the singular values to the maximum singular value as,

$$\sigma_c/\sigma_{max} = 10^{-p} \tag{23}$$

where p is the number of significant decimal digits in the data. The number of singular values is then equal to M selected based on (23). p is also used to control the performance of higher order approximations. Moreover, to suppress the noise effects, the spurious singular values should be neglected. It is noted that in order to obtain the residues, R_i, a typical least-square problem such as (15) is solved.

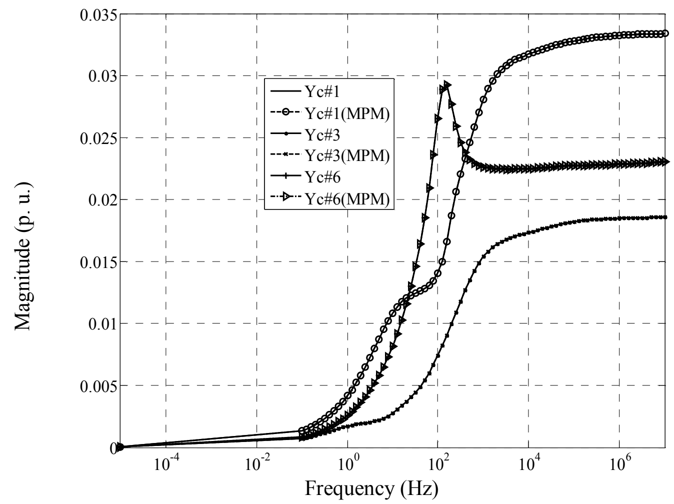

Fig. 5. Characteristic admittance Yc for the cable system shown in Fig. 4 (mode 1, 3 and 6).

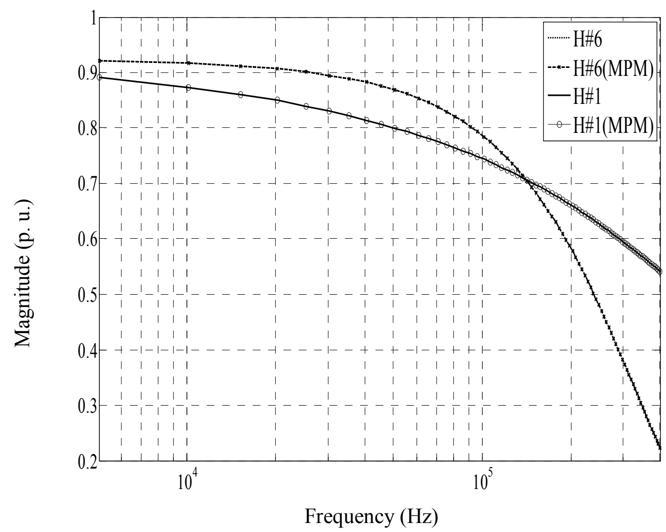

Fig. 6. Propagation function for the cable system shown in Fig. 4 (mode 1 and 6).

As to the stability of the method, it is very important to note that the real part values of S_m obtained by (16) plays the main role in the stability of the method. In cases where, there exist complex-valued constants (S_i) with positive real values, the method fails to estimate the real poles and residues. One can use a kind of normalization to circumvent this problem.

It is worth noting that the method is amenable to multiport applications as well. In fact, either a state-space model or an equivalent circuit model could be properly utilized to model multiport systems. Further discussions together with more examples will be given in the future works.

V. Conclusion

A general methodology based on the matrix pencil method (MPM) for rational fitting of frequency domain responses was proposed. The method could be properly used to incorporate the frequency dependency in time domain analysis. The method involves three distinct stages. First, the frequency domain response is approximated by a sum of exponential functions using the MPM. The time domain representation of the obtained exponentials is then directly derived by a closed form inverse Fourier transform. Finally, the MPM is used again to estimate the poles and residues from the time domain function. The main feature of the proposed method is its direct solution, hence, avoiding any iteration in the estimation process. Moreover, the method needs no starting poles as opposed to the iterative methods such as vector fitting (VF) method. The approach was validated first by a known analytical frequency domain function. In the final set of the results, the efficiency of the method was demonstrated for the case of a distribution transformer as well as a cable system. It was shown that the method could be a powerful alternative for incorporating the frequency response of different systems in time domain codes.

VI. References

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