Calculation of Underground Cables Frequency-Dependent Parameters Using Full-Wave Modal Analysis

Shahnoor Habib and Behzad Kordi

Abstract—**In this paper a new method has been proposed for calculating the frequency-dependent parameters of underground cables. The method uses full-wave formulation for calculating the modal electromagnetic fields and corresponding voltages and currents and then extracting frequency-dependent per-unit-length parameters of underground cables. The proposed method can be used for any cross-sectional shape of cables. Single coaxial cables and sector shaped cables are studied in this paper and the calculated per-unit-length parameters are compared with those obtained from PSCAD/EMTDC and other methods available in the literature.**

***Keywords*: underground cable, frequency-dependent parameters, modal analysis, finite element method.**

I. Introduction

The accurate calculation of propagation characteristics of buried cables embedded in lossy earth is required for power system transient analysis. Most simulation methods utilize quasi-TEM based formulations [1], [2] that are reasonably valid at power frequencies, however, they become inaccurate at higher frequencies. Further, measurements on cable systems show that the proximity effect strongly influences the wave propagation in cable systems, and commonly-used models are incapable of reproducing measurement results. In [3], it is shown that simulation results are not consistent with measurements for the inter-sheath mode excitation of a system of three cables. The ground mode is also very important for underground cables that has to be evaluated accurately at higher frequencies.

The approach of this paper is based on a full-wave 2D modal analysis of cables buried in lossy ground using finite element method. Solving an eigenvalue problem determines the electromagnetic field distribution over the cross section of the cable, as well as the attenuation and phase constants for all possible modes. Line voltages and currents are calculated by integrating the electric and magnetic fields, respectively. Once the modal voltages and currents are determined, we calculate the per-unit-length parameters of the cable system under study at the frequency of interest.

In this paper, we present a comparison between modal attenuation and phase constants calculated using the proposed full-wave modal analysis approach and those calculated using formulations commonly-used in power system transient simulation. It is shown how the differences become more significant at higher frequencies and how the proximity effect influences the propagating modes in a system of cables. Frequency-dependent per-unit-length resistance, inductance, and capacitance are also calculated and compared. This approach is not limited to any specific geometry and can be applied to cables with an arbitrary cross section such as sector-shaped cables.

II. Full-wave Modal Analysis

In this approach, it is assumed that electromagnetic fields propagate in the axial direction of the cable, z axis, as given by,

$$\mathbf{E}(x,y,z) = \hat{\mathbf{E}}(x,y)e^{-(\alpha+j\beta)z}, \quad (1a)$$

$$\mathbf{H}(x,y,z) = \hat{\mathbf{H}}(x,y)e^{-(\alpha+j\beta)z}, \quad (1b)$$

where, α and β are the attenuation and phase constants of the electric field, **E**, and the magnetic field, **H**. Solving an eigenvalue problem using the Finite Element Method[1] (FEM) will determine the electromagnetic fields at any point in the 2D space and the corresponding attenuation constant, α, and phase constant, β.

To calculate the voltage of the i^{th} conductor of the cable (V_i), the electric field is integrated along path ℓ_i from the reference point at the boundary of the lossy ground (O) to a point on the conductor (C_i) using (see Fig. 1),

$$V_i = -\int_O^{C_i} \mathbf{E}\cdot d\mathbf{l}. \quad (2)$$

The current of the i^{th} conductor of the cable (I_i) is determined by integrating the current density over the surface of the i^{th} conductor as given by,

$$I_i = \int_{S_i} \mathbf{J}\cdot d\mathbf{s} = \sigma\int_{S_i} E_z\,dx\,dy, \quad (3)$$

where, σ is the conductivity of the conductor. The so-calculated line voltage and current will also propagate with the

---

Support from Natural Sciences and Engineering Research Council (NSERC) of Canada is acknowledged.

S. Habib and B. Kordi are with the Department of Electrical and Computer Engineering, University of Manitoba, Winnipeg, MB, Canada R3T 5V6 (e-mail of corresponding author: Behzad_Kordi@UManitoba.CA, e-mail: umhabib@cc.umanitoba.ca).

Paper submitted to the International Conference on Power Systems Transients (IPST2011) in Delft, the Netherlands June 14-17, 2011

[1] Comsol Multiphysics was used in this work for the FEM analysis.

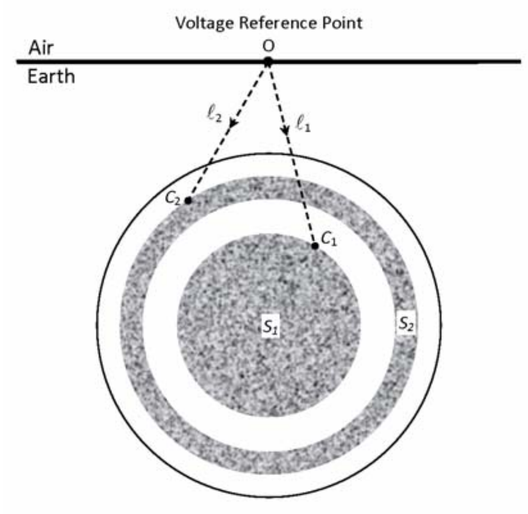

Fig. 1. The geometry of a two-conductor cable buried in lossy ground and the integration paths for the calculation of the conductors voltages.

same propagation constant, $\gamma = \alpha + j\beta$, as the electromagnetic fields.

Now consider an n-conductor cable system whose voltages and currents satisfy the multiconductor transmission line equations given by [4],

$$\frac{d}{dz}\mathbf{V}(z) = -\mathbf{ZI}(z)\,, \tag{4a}$$

$$\frac{d}{dz}\mathbf{I}(z) = -\mathbf{YV}(z)\,. \tag{4b}$$

In (4), $\mathbf{V}$ is the $n\times1$ vector of line voltages, and $\mathbf{I}$ is the $n\times1$ vector of line currents of the cable system. $\mathbf{Z}$ and $\mathbf{Y}$ are $n\times n$ per-unit-length impedance and admittance matrices, respectively. In the modal excitation of the cable system, for each mode, the electromagnetic fields, and consequently, the line voltages and currents propagate with the same propagation constant. In other words, for each propagating mode we can write,

$$\mathbf{V}(z) = \mathbf{V}_m e^{-\gamma z}\,, \tag{5a}$$

$$\mathbf{I}(z) = \mathbf{I}_m e^{-\gamma z}\,. \tag{5b}$$

The $n\times1$ vectors $\mathbf{V}_m$ and $\mathbf{I}_m$ in (5) are the modal line voltages and currents that are calculated by integrating the electromagnetic fields. In the case of 2D simulation of a 3D structure, the third dimension is assumed to be infinitely long. That makes the transmission line reflectionless. Substitution of (5) in (4) yields,

$$\gamma\mathbf{V}_m = \mathbf{ZI}_m\,, \tag{6a}$$

$$\gamma\mathbf{I}_m = \mathbf{YV}_m\,, \tag{6b}$$

where, γ, $\mathbf{V}_m$, and $\mathbf{I}_m$ are known and have been determined using the full-wave modal analysis of the cable system.

Once we write (6) for all n propagating modes, we have a set of linear equations whose solution will provides us with the unknown per-unit-length impedance and admittance matrices. For example, for the case of a two-conductor cable, where we have two propagating modes, (6) can be written as,

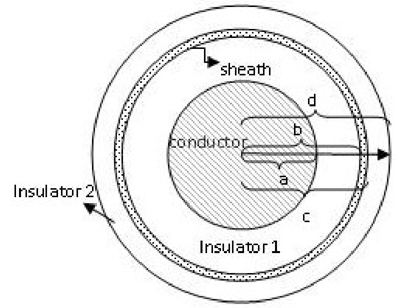

Fig. 2. Dimensions of the single coaxial cable used for the simulation.

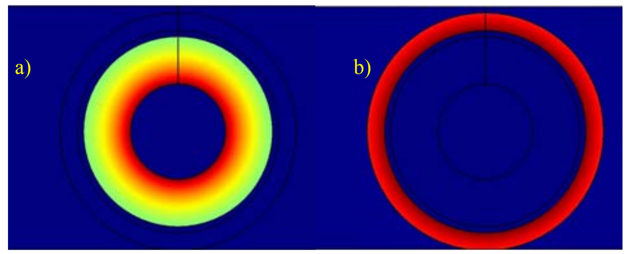

Fig. 3. Electric field distribution for the propagating modes of a two-conductor coaxial cable; a) coaxial mode, b) ground mode.

$$\underbrace{\begin{bmatrix}\gamma^1 V_1^1 & \gamma^2 V_1^2\\ \gamma^1 V_2^1 & \gamma^2 V_2^2\end{bmatrix}}_{\mathbf{V}_\gamma} = \begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix}\begin{bmatrix}I_1^1 & I_1^2\\ I_2^1 & I_2^2\end{bmatrix}, \tag{7a}$$

$$\underbrace{\begin{bmatrix}\gamma^1 I_1^1 & \gamma^2 I_1^2\\ \gamma^1 I_2^1 & \gamma^2 I_2^2\end{bmatrix}}_{\mathbf{I}_\gamma} = \begin{bmatrix}Y_{11} & Y_{12}\\ Y_{21} & Y_{22}\end{bmatrix}\begin{bmatrix}V_1^1 & V_1^2\\ V_2^1 & V_2^2\end{bmatrix}. \tag{7b}$$

In (7), γ^k represents propagation constant of the k^{th} mode, and V_i^k and I_i^k are the voltage and current of the i^{th} conductor for the k^{th} mode. The impedance and admittance matrices can be calculated using,

$$\begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix} = \mathbf{V}_\gamma \begin{bmatrix}I_1^1 & I_1^2\\ I_2^1 & I_2^2\end{bmatrix}^{-1}, \tag{8a}$$

$$\begin{bmatrix}Y_{11} & Y_{12}\\ Y_{21} & Y_{22}\end{bmatrix} = \mathbf{I}_\gamma \begin{bmatrix}V_1^1 & V_1^2\\ V_2^1 & V_2^2\end{bmatrix}^{-1}. \tag{8b}$$

III. Case Studies

A. Single Coaxial Cable

To verify the validity of the proposed method, we will first analyze a single coaxial cable and compare the results with those obtained from analytical formulation used in PSCAD/EMTDC. The cable used for the simulation has dimensions as shown in Fig. 2. The radii a, b, c, and d are 2cm, 4cm, 4.25cm, and 5cm, respectively. The conductivity of core and sheath are 10^8 and 5×10^7 S/m, respectively. The relative permittivity of the ground, insulator 1, and insulator 2 are 1, 3, and 2, respectively. Ground conductivity is 0.01 S/m. We will also present a parametric analysis when the burial

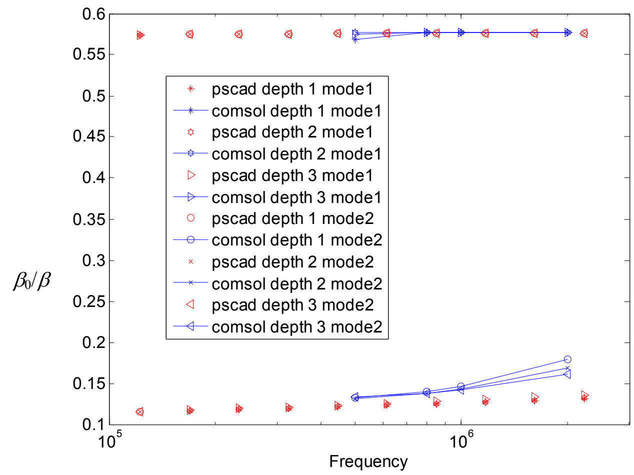

Fig. 4. Relative phase constant (relative to that of free space) for the coaxial mode (mode 1) and the ground mode (mode 2) and for different burial depths. The value of frequency is in [Hz].

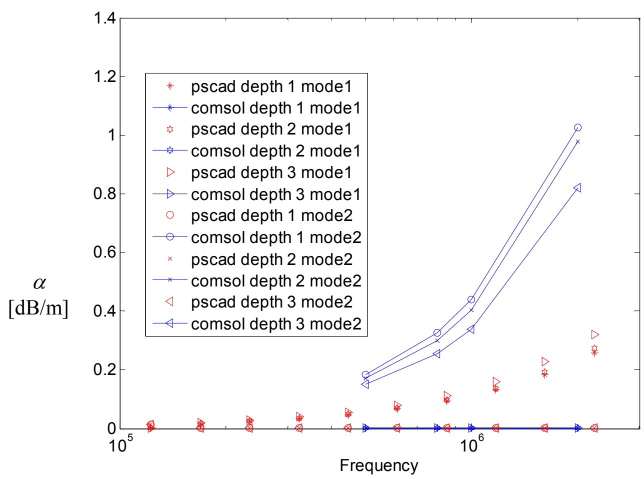

Fig. 5. Attenuation constant for the coaxial mode (mode 1) and the ground mode (mode 2) and for different burial depths. The value of frequency is in [Hz].

depth varies.

The single coaxial cable will have two propagating transmission-line modes. They are coaxial and ground modes. The electric field distributions of the modes are given in Fig. 3. Three different burial depths were chosen. They are 0.055m (depth 1), 0.15m (depth 2) and 1m (depth 3). The depths are measured from ground surface to the center of cable. In Figs. 4 and 5, we have plotted the variation of the attenuation and propagation constants with frequency. By β_0/β we mean the ratio of the phase constant in free space over that in the given cable which is equal to the propagation speed of the mode over the speed of light. The phase constant and the attenuation constant for coaxial modes are the same in PSCAD/EMTDC and full-wave method. But, for the ground mode, both phase and propagation constant are different especially at higher frequencies. The effect of varying the ground depth is opposite in the two methods. When the burial depth of the cable is decreased, the attenuation increases in the full-wave method. This can be justified as when the cable moves closer

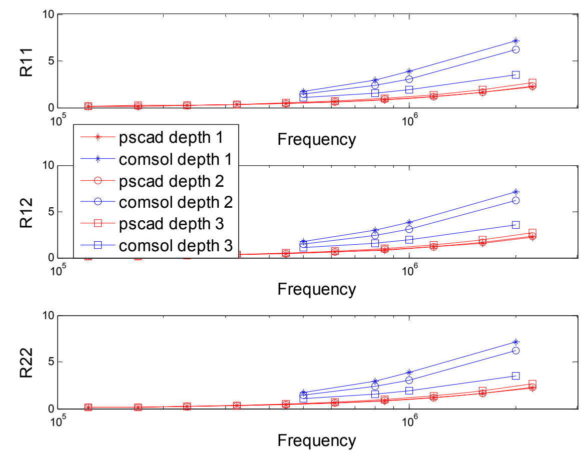

Fig. 6. Elements of the per-unit-length resistance matrix for three different burial depths. The value of resistance is in [Ω/m] and frequency is in [Hz].

to the ground, the ground current has a smaller area to flow, so, conceptually the attenuation should increase. Same effect can be observed in the elements of the resistance matrix as shown in Fig. 6. In the full-wave method, the resistance decreases with an increase in the burial depth. Also, the full-wave simulation shows a more significant change as the burial depths varies, especially for the first two depths.

When the cable is near ground (for depths 1 and 2) even a change in depth of about 10 cm causes a significant change in the attenuation constant and, consequently, the elements of the resistance matrix. The inductance values (see Fig. 7) are also different at higher frequencies. At lower frequencies they seem to be converging to those calculated by current formulations. The values of the elements of capacitance matrix (see Fig. 8), calculated by both PSCAD/EMTDC and our method, are consistent and show almost no significant variation with frequency.

B. *Sector Shaped Cable*

An advantage of the proposed method is that it is not limited to any specific cross sectional geometry of the cables system. To demonstrate this, we choose a sector shaped cable as given in [5]. The dimensions are given in Fig. 9. The cable is buried at a depth of 1m in the ground. Ground conductivity and relative permittivity are 0.01 S/m and 1, respectively. The electric field distribution for this cable is shown in Fig. 10. We can see four propagating modes in this figure. The first two modes use two of the sector shaped conductors, whereas, in mode 3, the current flows in the sector shaped conductors and returns through the sheath. The fourth mode consists of a transmission-line mode between the sheath conductor and the lossy ground.

Both the phase and attenuation constants (see Fig. 11) are not changing much with frequency for the inter-core modes (modes 1 and 2) and the coaxial mode (mode 3). Values for these three modes are very close. However, the ground mode (mode 4) is significantly slower than the other modes and its propagation characteristics are highly dependent on the frequency.

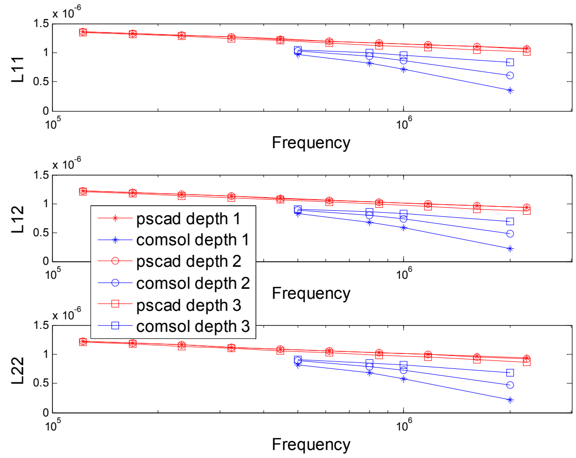

Fig. 7. Elements of the per-unit-length inductance matrix for three different burial depths. The value of inductance is in [H/m] and frequency is in [Hz].

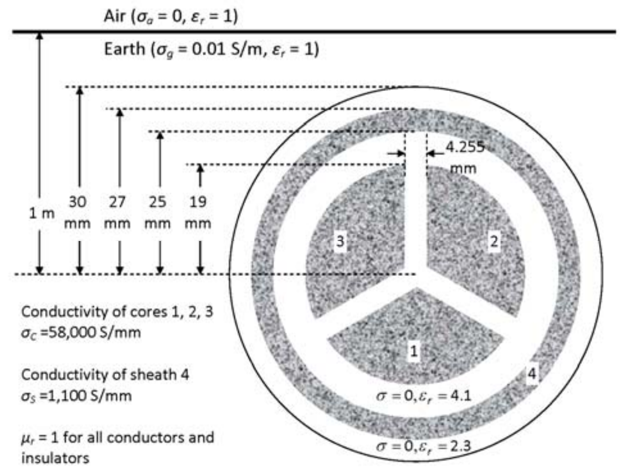

Fig. 9. Dimensions of the sector shaped cable. The cable is buried in a depth of 1m in a lossy ground with a resistivity of 0.01 S/m and a relative permittivity of 1.

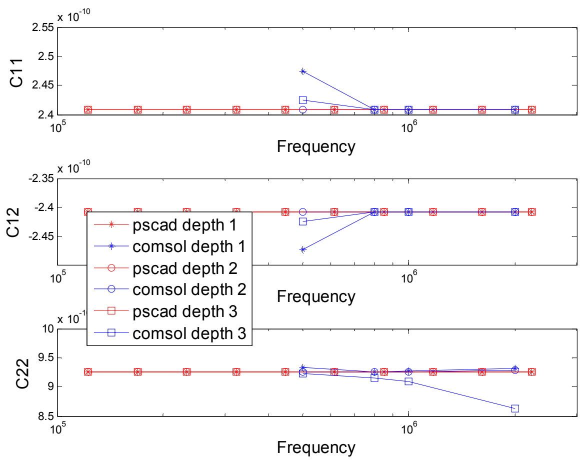

Fig. 8. Elements of the per-unit-length capacitance matrix for three different burial depths The value of capacitance is in [F/m] and frequency is in [Hz].

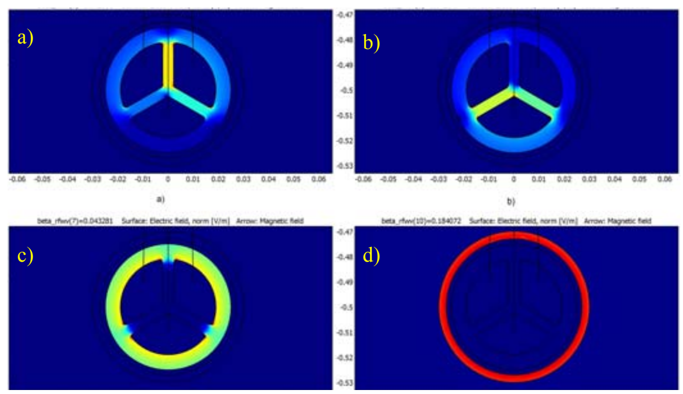

Fig. 10. Electric field distributions of the sector shaped cable for different propagating modes; a) inter-core mode 1 (mode 1), b) inter-core mode 2 (mode 2), c) coaxial mode (mode 3), d) ground mode (mode4).

The resistance, inductance, and capacitance values for different frequencies are plotted in Figs. 12 and 13. All the elements in capacitance matrices except C_{44} are almost constant for all frequencies.

The same sector shaped cable, with no ground, has been simulated in [5] using the partial subconductor equivalent circuit (PSEC) method. The values of the self and mutual resistance and inductance obtained using the PSEC method and the full-wave method (at 500kHz) are compared in Table I. This comparison shows a good agreement between the methods. Our full-wave modal analysis technique is also able to evaluate the capacitance. The self and mutual capacitances for the sector shaped cable of [5] are 614 and 169.66 nF/km, respectively.

TABLE I
COMPARISON BETWEEN PSEC AND FULL-WAVE METHOD AT 500KHZ WHEN THE SECTOR SHAPED CABLE IS SIMULATED WITHOUT GROUND.

	R_{self} (ohm/km)	R_{mutual} (ohm/km)	L_{self} (μH/km)	L_{mutual} (μH/km)
PSEC	14	8	105	40
Full-wave	13.87	8.1	100.2	38.3

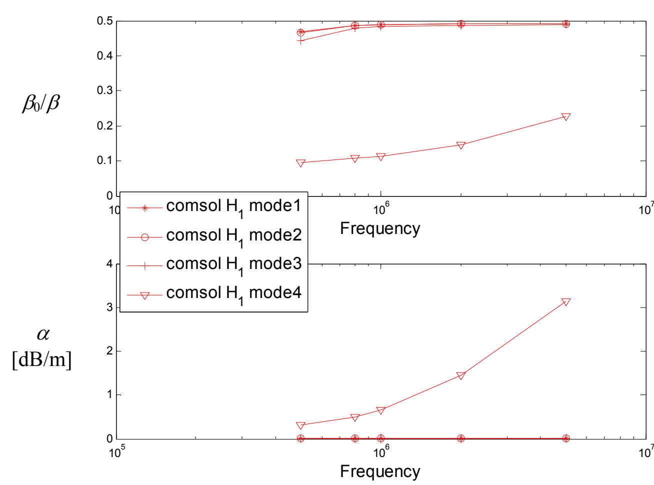

Fig. 11. Attenuation and phase constants for all fours modes of the sector shaped cable shown in Fig. 9. The value of frequency is in [Hz].

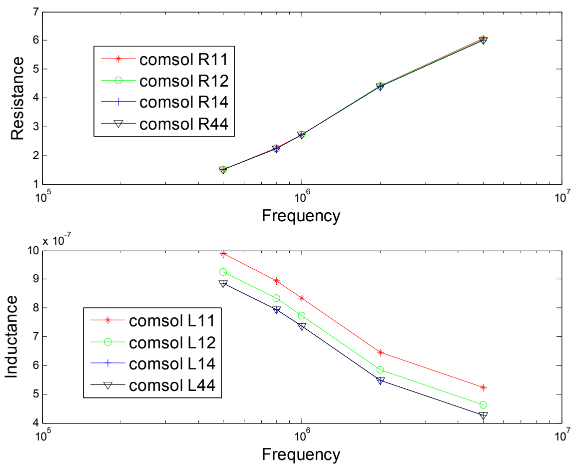

Fig. 12. Elements of the per-unit-length resistance and inductance matrices calculated using the modal analysis method for the sector shaped cable. The value of resistance and inductance are in [Ω/m] and [H/m], respectively, and frequency is in [Hz]. Due to symmetry we have not plotted all the elements.

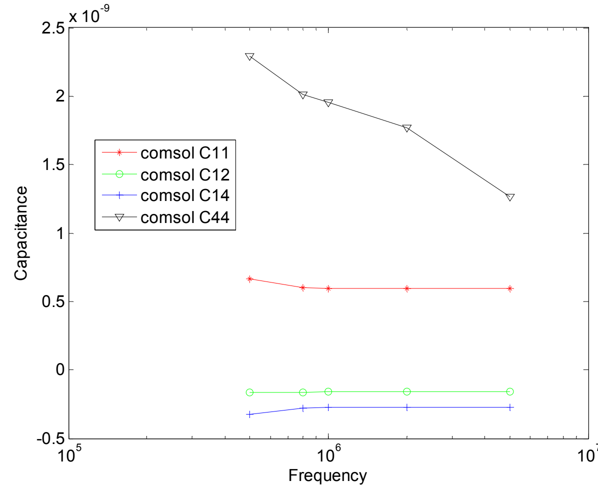

Fig. 13. Elements of the capacitance matrix calculated using the modal analysis method for sector shaped cable. The value of capacitance is in [F/m] and frequency is in [Hz]. Due to symmetry we have not plotted all the elements.

IV. Conclusions

In this paper, a full-wave modal analysis technique was introduced for the calculation of frequency-dependent per-unit-length parameters of underground cables buried in lossy earth. This method makes no approximation for the calculation of ground impedance and is valid at high frequencies as well. The proposed method is capable of simulating cables of any shape. It is also not limited by the number of conductors. Skin effect and proximity effect are considered in the calculation of the cables per-unit-length impedance and admittance matrices. We examined two examples in this paper. A single two-conductor coaxial cable and a four-conductor sector shaped cable were studied and their parameters were compared with those obtained from other techniques and formulations. The full-wave modal analysis is not practical at low frequencies, as at these frequencies the propagation constant of the modes are very close and it is not easy to distinguish them from each other. However, at low frequencies, the quasi-static formulation is very accurate and can be used. Transient simulations in EMTP-type programs will become more accurate if the high frequency per-unit-length parameters calculated in this paper replace those calculated using the available analytical formulations.

V. Acknowledgment

The authors gratefully acknowledge the contributions of Dr. M. Heshmatzadeh and Dr. S. Bonyadi in different parts of the research.

VI. References

[1] L. M. Wedepohl and D. J. Wilcox, "Transient analysis of underground power transmission systems," *Proc. IEE*, vol. 102, pp. 253-260, 1973.

[2] F. Uribe, "Assessing closed-form approximations for underground cable earth impedance," in *Proc. IEEE PES General Meeting*, July 2003.

[3] U. S. Gudmundsdottir, J. De Silva, C. L. Bak, W. Wiechowski, "Double layered sheath in accurate HV XLPE cable modeling" in *Proc. PES General Meeting 2010.*

[4] C. R. Paul, *Analysis of Multi-conductor Transmission Lines.* Wiley Interscience, 1994.

[5] R. A. Rivas and J. R. Marti, "Calculation of frequency-dependent parameters of power cables: matrix partitioning techniques," *IEEE Trans. Power Delivery*, vol. 17, no. 4, pp. 1085-1092, Oct. 2002.