Defining and Measuring Synchrophasors Based on Symmetry Principles

Kempei Seki

***Abstract*—This paper defines and measures synchrophasors based on the symmetry principles. The paper first defined the frequency associated with a rotation phase angle, and creates a symmetry group for measuring frequency and amplitude. Then, the paper defines synchrophasors in the complex plane and creates symmetry groups for measuring synchrophasors. Furthermore, the paper defines and measures time synchrophasors and space synchrophasors. The paper also introduces symmetry breaking criteria to raise the precision of the novel method. At last, a numerical example shows the novel method is effective for power systems.**

***Keywords*: frequency measurement, group theory, invariant, smart meters, spiral vector theory, synchrophasors, PMU.**

I. Introduction

Synchrophasors definition is very important for developing PMU(phasor measurement unit). It is popular to define and measure synchrophasors associated with nominal system frequency (50Hz or 60Hz) and DFT [1-2]. On the other hand, the symmetry principles play a very important role in relative theory and quantum mechanics [3]. In the process of applying spiral vector theory to the analysis of AC systems [4-11], we discovered that spiral vectors have symmetry properties. Therefore, we have been applying group theory to the analysis of AC systems [12-13]. This paper introduces development results on synchrophasors. The paper is organized as following: Section II covers measuring frequency and amplitude with symmetry groups; Section III presents defining and measuring synchrophasors with symmetry groups; Section IV provides a numerical example. Section V is conclusions of the paper.

II. Measuring Frequency and Amplitude with Symmetry Groups

In this section we shall propose frequency and amplitude measuring method based on group theory.

A. Defining the frequency associated with a rotation phase angle

Let us recall that in [12], we defined the frequency with a rotation phase angle as

$$\frac{f}{f_S} = \frac{\alpha}{2\pi} \tag{1}$$

where f is real frequency, f_S is the sampling frequency, and α is the rotation phase angle in an interval of time T that is expressed as

$$T = \frac{1}{f_S} \tag{2}$$

In addition, the angular velocity ω can be expressed as

$$\omega = 2\pi f \tag{3}$$

With the relation of (1), we switched the task for measuring the frequency to the task for measuring the rotation phase angle. Fortunately, we discover that the rotation phase angle can be obtained with symmetry groups and next we show the detailed procedure.

B. Creating a symmetry group for obtaining invariants depending on the rotation phase angle

Let's consider four rotating voltage vectors in the complex plane as in Fig.1 as

$$\begin{cases} v_1(t) = Ve^{j(\omega t + \frac{3\alpha}{2})} \\ v_1(t-T) = Ve^{j(\omega t + \frac{\alpha}{2})} \\ v_1(t-2T) = Ve^{j(\omega t - \frac{\alpha}{2})} \\ v_1(t-3T) = Ve^{j(\omega t - \frac{3\alpha}{2})} \end{cases} \tag{4}$$

where V is voltage amplitude, ω is the angular velocity, and α is the rotation phase angle. Then we can build a rotational

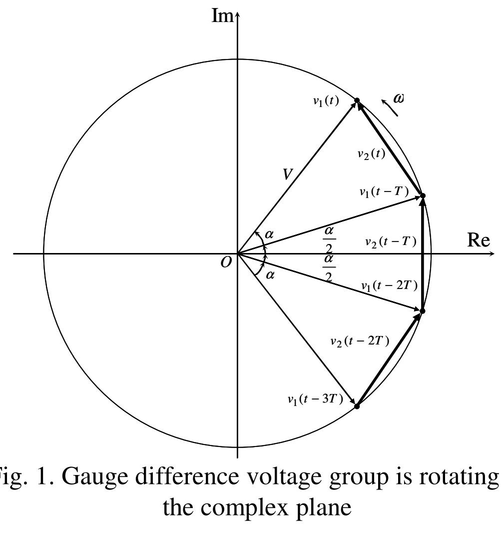

Fig. 1. Gauge difference voltage group is rotating in the complex plane

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group containing three difference voltage vectors as

$$\begin{cases} v_2(t) = v_1(t) - v_1(t-T) \\ v_2(t-T) = v_1(t-T) - v_1(t-2T) \\ v_2(t-2T) = v_1(t-2T) - v_1(t-3T) \end{cases} \quad (5)$$

Substituting (4) into the above expression, we can obtain

$$\begin{cases} v_2(t) = V[e^{j(\omega t + \frac{3\alpha}{2})} - e^{j(\omega t + \frac{\alpha}{2})}] \\ v_2(t-T) = V[e^{j(\omega t + \frac{\alpha}{2})} - e^{j(\omega t - \frac{\alpha}{2})}] \\ v_2(t-2T) = V[e^{j(\omega t - \frac{\alpha}{2})} - e^{j(\omega t - \frac{3\alpha}{2})}] \end{cases} \quad (6)$$

Considering the above group, we can find that three members are symmetrical about the middle one geometrically. Suppose (6) is a symmetry group, we propose an invariant formula as

$$V_{gd} = \sqrt{v_{22}{}^2 - v_{21}v_{23}} \quad (7)$$

where v_{21}, v_{22}, and v_{23} are the real or imaginary parts of three members of (6). For evaluating the above formula, we substitute

$$\begin{cases} v_{21} = \mathrm{Re}[v_2(t)] = V[\cos(\omega t + \frac{3\alpha}{2}) - \cos(\omega t + \frac{\alpha}{2})] \\ v_{22} = \mathrm{Re}[v_2(t-T)] = V[\cos(\omega t + \frac{\alpha}{2}) - \cos(\omega t - \frac{\alpha}{2})] \\ v_{23} = \mathrm{Re}[v_2(t-2T)] = V[\cos(\omega t - \frac{\alpha}{2}) - \cos(\omega t - \frac{3\alpha}{2})] \end{cases} \quad (8)$$

into (7) and obtain the following result. Here Re indicates the real part of the complex number.

$$V_{gd} = 2V \sin\alpha \sin\frac{\alpha}{2} \quad (9)$$

In spite of this, we substitute

$$\begin{cases} v_{21} = \mathrm{Im}[v_2(t)] = V[\sin(\omega t + \frac{3\alpha}{2}) - \sin(\omega t + \frac{\alpha}{2})] \\ v_{22} = \mathrm{Im}[v_2(t-T)] = V[\sin(\omega t + \frac{\alpha}{2}) - \sin(\omega t - \frac{\alpha}{2})] \\ v_{23} = \mathrm{Im}[v_2(t-2T)] = V[\sin(\omega t - \frac{\alpha}{2}) - \sin(\omega t - \frac{3\alpha}{2})] \end{cases} \quad (10)$$

into (7) and obtain (9) again. Here Im indicates the imaginary part of the complex number. Because both the real and imaginary parts of three members of (6) lead the same result (9), it shows that (6) is truly a symmetry group. Therefore we call invariant V_{gd} as gauge difference voltage and call (6) as gauge difference voltage group. Next we shall discover another invariant of this symmetry group.

C. Obtaining frequency coefficient with gauge difference voltage group

Hereafter we propose frequency coefficient formula as

$$f_C = \frac{v_{21} + v_{23}}{2v_{22}} \quad (11)$$

where v_{21}, v_{22}, and v_{23} are the real or imaginary parts of three members of (6). For evaluating the above formula, we substitute (8) into (11) and obtain

$$f_C = \cos\alpha \quad (12)$$

In spite of this, we substitute (10) into (11) and obtain the above result again. Same as (9), this shows that frequency coefficient f_C is an invariant of gauge difference voltage group. Next we shall employ frequency coefficient and gauge difference voltage to obtain the frequency and voltage amplitude.

D. Obtaining the frequency and voltage amplitude with gauge difference voltage group

Thus, (12) shows that we can obtain the rotation phase angle as

$$\alpha = \cos^{-1} f_C \quad (13)$$

As a natural consequence of this, substitute the above equation into (1), we obtain the frequency as

$$f = \frac{f_S}{2\pi} \cos^{-1} f_C \quad (14)$$

Furthermore, (12) shows that we can obtain the sine of the rotation phase angle as

$$\sin\alpha = \sqrt{1 - \cos^2\alpha} = \sqrt{1 - f_C{}^2} \quad (15)$$

and the sine of half rotation phase angle as

$$\sin\frac{\alpha}{2} = \sqrt{\frac{1 - \cos\alpha}{2}} = \sqrt{\frac{1 - f_C}{2}} \quad (16)$$

Fortunately, substituting (15) and (16) into (9), we obtain voltage amplitude as

$$V = \frac{V_{gd}}{2\sin\alpha \sin\frac{\alpha}{2}} = \frac{\sqrt{2}V_{gd}}{2(1 - f_C)\sqrt{1 + f_C}} \quad (17)$$

The above solution is useful for real time control systems because it contains only arithmetic operations. Next we evaluating frequency spectrum of the novel method.

E. Evaluating frequency spectrum of the novel method

Hereafter we act a simulation with the sampling frequency of 600 Hz and assume that test waveforms are pure sinusoids. The simulation results are shown in Fig.2 and Fig.3 respectively. Like (12), Fig. 2 shows that frequency coefficient is the cosine function of the rotation phase angle. Figure 3 shows that the novel method can obtain real frequency if it is lower than the Nyquist frequency (here 300 Hz) and this agrees with the sampling theorem.

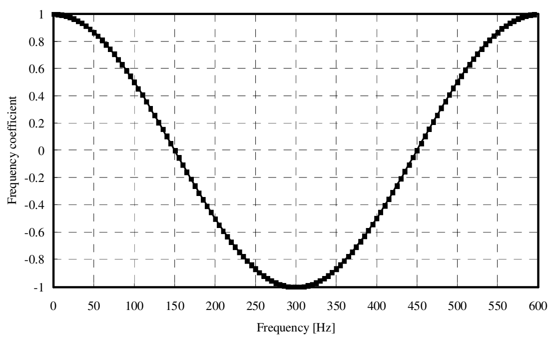

Fig. 2. Simulation results of frequency coefficient for the sampling frequency of 600Hz

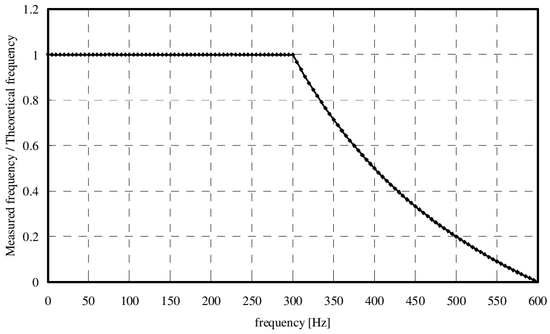

Fig. 3. Simulation results of measurement precision for the sampling frequency of 600Hz

In the following, we shall employ obtained rotation phase angle and voltage amplitude to measure synchrophasors.

III. Defining and Measuring Synchrophasors with Symmetry Groups

In this section we shall define and measure synchrophasors based on group theory.

A. Defining a synchrophasor in the complex plane

At first, we define a synchrophasor as an instantaneous phase angle of a counterclockwise rotating vector with the velocity of real frequency in the complex plane. Thus we aim to obtain instantaneous phase angle φ in Fig.4. It is clearly that we can calculate this phase angle straightforwardly as [9]

$$\varphi = \cos^{-1}\left(\frac{v_{re}}{V}\right) \tag{18}$$

where v_{re} is instantaneous voltage and V is voltage amplitude. Because the arccosine function is always positive, the above rotating phase angle has two rotating directions. Namely, one is a counterclockwise direction; the other is a clockwise direction. Unfortunately, two rotating directions will cause problems in calculating time synchrophasors and space synchrophasors that are the difference phase angle of two synchrophasors. Though in [9], we developed a latch mode for avoiding this difficult, it is still a big problem to fast determine time synchrophasors and space synchrophasors at any time.

Fortunately, in the process to create symmetry groups for measuring active power and reactive power [13], we got an idea that employing gauge power group to measure synchrophasors. The idea is that replacing rotating current vectors in gauge power group with fixed unit vectors and calculating phase angle between rotating voltage vectors and fixed unit vectors. In the next way, we shall show the idea is effective to calculate synchrophasors.

B. Creating symmetry groups for measuring synchrophasors

At first, in Fig.4, we build a rotational group composed of three rotating voltage vectors and two fixed unit vectors as five members as

$$\begin{cases} v_1(t) = Ve^{j\varphi} \\ v_1(t-T) = Ve^{j(\varphi-\alpha)} \\ v_1(t-2T) = Ve^{j(\varphi-2\alpha)} \\ v_{10}(0) = e^{j0} \\ v_{10}(1) = e^{-j\alpha} \end{cases} \tag{19}$$

where α is the rotation phase angle and φ is the synchrophasor.

Hereafter, we choose two rotating voltage vectors and two fixed unit vectors from (19) as four members to build a rotational group as

$$\begin{cases} v_1(t-T) = Ve^{j(\varphi-\alpha)} \\ v_1(t-2T) = Ve^{j(\varphi-2\alpha)} \\ v_{10}(0) = e^{j0} \\ v_{10}(1) = e^{-j\alpha} \end{cases} \tag{20}$$

Suppose the above group is a symmetry group, we propose an invariant formula as

$$SA_P = v_{12}v_{101} - v_{13}v_{100} \tag{21}$$

where v_{12},v_{13},v_{100}, and v_{101} are the real or imaginary parts of four members of (20). For evaluating the above formula, we substitute the real parts of four members

$$\begin{cases} v_{12} = \mathrm{Re}[v_1(t-T)] = V\cos(\varphi-\alpha) \\ v_{13} = \mathrm{Re}[v_1(t-2T)] = V\cos(\varphi-2\alpha) \\ v_{100} = \mathrm{Re}[v_{10}(0)] = 1 \\ v_{101} = \mathrm{Re}[v_{10}(1)] = \cos\alpha \end{cases} \tag{22}$$

into (21) and obtain

$$SA_P = V\sin\alpha\sin(\alpha-\varphi) \tag{23}$$

In spite of this, we substitute the imaginary parts of four members

$$\begin{cases} v_{12} = \mathrm{Im}[v_1(t-T)] = V\sin(\varphi-\alpha) \\ v_{13} = \mathrm{Im}[v_1(t-2T)] = V\sin(\varphi-2\alpha) \\ v_{100} = \mathrm{Im}[v_{10}(0)] = 0 \\ v_{101} = \mathrm{Im}[v_{10}(1)] = -\sin\alpha \end{cases} \tag{24}$$

into (21) and obtain (23) again. This shows that (20) is truly a symmetry group and SA_P is an invariant of (20). However, the expression of SA_P is familiar with the expression of gauge

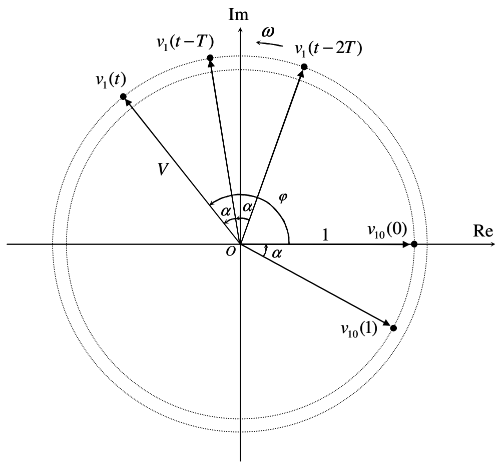

Fig. 4. Gauge synchrophasor group in the complex plane

active power [13], we call SA_P as gauge active synchrophasor and call (20) as gauge active synchrophasor group.

In a similar way, we choose another two rotating voltage vectors and two fixed unit vectors from (19) as four members to build a rotational group as

$$\begin{cases} v_1(t) = Ve^{j\varphi} \\ v_1(t-T) = Ve^{j(\varphi-\alpha)} \\ v_{10}(0) = e^{j0} \\ v_{10}(1) = e^{-j\alpha} \end{cases} \tag{25}$$

Suppose the above group is a symmetry group, we propose an invariant formula as

$$SA_Q = v_{11}v_{101} - v_{12}v_{100} \tag{26}$$

where v_{11},v_{12},v_{100}, and v_{101} are the real or imaginary parts of four members of (25). For evaluating the above formula, we substitute the real parts of four members

$$\begin{cases} v_{11} = \mathrm{Re}[v_1(t)] = V\cos\varphi \\ v_{12} = \mathrm{Re}[v_1(t-T)] = V\cos(\varphi-\alpha) \\ v_{100} = \mathrm{Re}[v_{10}(0)] = 1 \\ v_{101} = \mathrm{Re}[v_{10}(1)] = \cos\alpha \end{cases} \tag{27}$$

into (26) and obtain

$$SA_Q = -V\sin\alpha\sin\varphi \tag{28}$$

In spite of this, we substitute the imaginary parts of four members

$$\begin{cases} v_{11} = \mathrm{Im}[v_1(t)] = V\sin\varphi \\ v_{12} = \mathrm{Im}[v_1(t-T)] = V\sin(\varphi-\alpha) \\ v_{100} = \mathrm{Im}[v_{10}(0)] = 0 \\ v_{101} = \mathrm{Im}[v_{10}(1)] = -\sin\alpha \end{cases} \tag{29}$$

into (26) and obtain (28) again. This shows that (25) is truly a symmetry group and SA_Q is an invariant of (25). However, the expression of SA_Q is familiar with the expression of gauge reactive power [13], we call SA_Q as gauge reactive synchrophasor and call (25) as gauge reactive synchrophasor group.

Recall the group (19), it is called as gauge synchrophasor group due to it contains gauge active synchrophasor group and gauge reactive synchrophasor group.

Therefore, we combine (23), (28) and obtain the cosine of the synchrophasor as

$$\cos\varphi = \frac{SA_P - SA_Q\cos\alpha}{V\sin^2\alpha} \tag{30}$$

Then, substituting the sine and cosine function of the rotation phase angle into the above equation, we determine the synchrophasor as

$$\varphi = \begin{cases} \cos^{-1}\left(\dfrac{SA_P - SA_Q f_C}{V(1-f_C{}^2)}\right), & SA_Q \le 0 \\ -\cos^{-1}\left(\dfrac{SA_P - SA_Q f_C}{V(1-f_C{}^2)}\right), & SA_Q > 0 \end{cases} \tag{31}$$

At last, we truly obtain the synchrophasor that has only one rotating direction. In addition, the range of synchrophasors is from -180 to +180 degrees. Next we shall define and measure time synchrophasors.

C. Defining and measuring time synchrophasors

We define time synchrophasors as the difference phase angle between the present time and an interval of time T_0 ago. In addition, the range of time synchrophasors is from -180 to +180 degrees. Therefore time synchrophasors is calculated as

$$\varphi_{TP} = \begin{cases} \varphi_t - \varphi_{t-T_0} - 2\pi, & \varphi_t - \varphi_{t-T_0} > \pi \\ \varphi_t - \varphi_{t-T_0} + 2\pi, & \varphi_t - \varphi_{t-T_0} < -\pi \\ \varphi_t - \varphi_{t-T_0}, & others \end{cases} \tag{32}$$

where φ_t is the synchrophasor at the present time, and φ_{t-T0} is the synchrophasor at T_0 time ago. Time synchrophasors can be employed in synchronous switching controller (SSC). SSC will forecast rotating time between the present time point and a give point by calculating the difference phase angle between these two points. Usually, a given point indicates that real value of current is zero and the synchrophasor of current is -90 or 90 degrees at this point. Next we shall define and measure space synchrophasors.

D. Defining and measuring space synchrophasors

We define space synchrophasors as the difference phase angle between two synchrophasors at the same time. Then the synchrophasor should synchronized to Universal time coordinated (UTC) for calculating space synchrophasors that is the difference phase angle between two different nodes of power system. In addition, the range of space synchrophasors is from -180 to +180 degrees. Therefore space synchrophasors is calculated as

$$\varphi_{SP} = \begin{cases} \varphi_1 - \varphi_2 - 2\pi, & \varphi_1 - \varphi_2 > \pi \\ \varphi_1 - \varphi_2 + 2\pi, & \varphi_1 - \varphi_2 < -\pi \\ \varphi_1 - \varphi_2, & others \end{cases} \tag{33}$$

where φ_1 is the synchrophasor of node 1, and φ_2 is the synchrophasor of node 2 respectively. Assuming time tag of node1 is lead to node2. We determine the synchrophasor φ_1 as

$$\varphi_1 = \varphi_1(t_1) \tag{34}$$

where t_1 is time tag of UTC in node 1, we determine the synchrophasor φ_2 as

$$\varphi_2 = \begin{cases} \varphi_{2t2} + 2\pi f_2(t_1 - t_2), & \varphi_{2t2} + 2\pi f_2(t_1 - t_2) \le \pi \\ \varphi_{2t2} + 2\pi f_2(t_1 - t_2) - 2\pi, & \varphi_{2t2} + 2\pi f_2(t_1 - t_2) > \pi \end{cases} \tag{35}$$

where t_2 is time tag of UTC in node 2. The above equation corrected slip phase angle between two nodes. Space synchrophasors can be applied to PMU and PDC (Phasor data concentrators) to monitor stability of power systems. Space synchrophasors can also be applied to automatic synchronizers (ASY) for estimating time from now to the point that space synchrophasors of two sides is zero.

Next we introduce symmetry breaking criteria for raising the precision of the novel method.

E. Symmetry breaking criteria

According to group theory, if the invariant of a symmetry group is changed, symmetry is broken. Employing this concept, we can introduce symmetry breaking criteria to distinguish between small disturbance and large disturbance. Hereafter we propose following formula as

$$\left| f_{C(t)} - f_{C(t-T)} \right| > f_{BRK} \tag{36}$$

where $f_{C(t)}$ is frequency coefficient at the present time, $f_{C(t-T)}$ is frequency coefficient at an interval of time T ago, and f_{BRK} is symmetry breaking setting respectively.

Firstly, we deal with the case of large disturbance. Because symmetry is broken in large disturbance condition, we can't calculate the synchrophasor and the others. Thus, we latch the rotation phase angle, the frequency, and voltage amplitude as

$$\begin{cases} \alpha_t = \alpha_{t-T} \\ f_t = f_{t-T} \\ V_t = V_{t-T} \end{cases} \tag{37}$$

where α_t, f_t, V_t are data at the present time and α_{t-T}, f_{t-T}, V_{t-T} are data at an interval T ago respectively. On the other hand, we estimate the synchrophasor at the present time as

$$\varphi_t = \begin{cases} \varphi_{t-T} + 2\pi f_t T, & \varphi_t + 2\pi f_t T \le \pi \\ \varphi_{t-T} + 2\pi f_t T - 2\pi, & \varphi_t + 2\pi f_t T > \pi \end{cases} \tag{38}$$

where φ_{t-T} is the synchrophasor at an interval of time T ago.

Secondly, we deal with the case of small disturbance. For reducing influence of additive noise in small disturbance condition, we can employ several symmetry groups for averaging invariants.

We can use several gauge difference voltage groups for averaging frequency coefficient as

$$f_C = \frac{1}{n-2} \sum_{k=2}^{n-1} \frac{v_{2(k-1)} + v_{2(k+1)}}{2 v_{2k}}, \quad n \ge 3 \tag{39}$$

where v_{2k} are difference voltages.

We can use several gauge difference voltage groups for averaging gauge difference voltage as

$$V_{gd} = \sqrt{\frac{1}{n-2} \left(\sum_{k=2}^{n-1} v_{2k}{}^2 - v_{2(k-1)} v_{2(k+1)} \right)}, \quad n \ge 3 \tag{40}$$

where v_{2k} are difference voltages.

We can use several gauge active synchrophasor groups for averaging gauge active synchrophasor as

$$SA_P = \frac{1}{n-2} \left(\sum_{k=2}^{n-1} (v_{1k} v_{10(k-1)} - v_{1(k+1)} v_{10(k-2)} \right), n \ge 3 \tag{41}$$

where v_{1k} is instantaneous voltages, v_{10k} are fixed unit vectors and calculated as

$$v_{10k} = \cos(k\alpha), \quad k = 0,1,\ldots n-2 \tag{42}$$

We can use several gauge reactive synchrophasor groups for averaging gauge reactive synchrophasor as

$$SA_Q = \frac{1}{n-2} \left(\sum_{k=2}^{n-1} (v_{1(k-1)} v_{10(k-1)} - v_{1k} v_{10(k-2)} \right), n \ge 3 \tag{43}$$

where v_{1k} is instantaneous voltages, v_{10k} are fixed unit vectors.

For the reason smart meters are becoming more and more important for developing smart grid, we shall apply the novel method to smart meters.

F. Optimal sampling frequency for smart meters

Let us recall Fig.2 and Fig.3, we consider the frequency is equal to half of the Nyquist frequency and calculate frequency coefficient, voltage amplitude, and the synchrophasor as following respectively.

$$f_C = 0 \tag{44}$$

$$V = \frac{\sqrt{2}}{2} V_{gd} \tag{45}$$

$$\varphi = \begin{cases} \cos^{-1}\left(\dfrac{SA_P}{V}\right), & SA_Q \le 0 \\ -\cos^{-1}\left(\dfrac{SA_P}{V}\right), & SA_Q > 0 \end{cases} \tag{46}$$

Therefore we can propose optimal sampling for smart meters. One is that we recommend the sampling frequency of 200 Hz for the rated frequency of 50 Hz power systems, the other is that we recommend the sampling frequency of 240 Hz for the rated frequency of 60 Hz power systems.

In summary, we give the procedure for measuring synchrophasors.

G. Procedure for measuring synchrophasors

Figure 5 shows the procedure for measuring synchrophasors. Step 4 promises measured frequency, amplitude, and synchrophasors are results of approximation sinusoids. However, for particular application, it is not necessary to employ all these twelve steps. For example, we can use step 1-6 to build up a low-voltage protective relay that is not influenced by DC offset components.

Next, we give a numerical example to test the novel method.

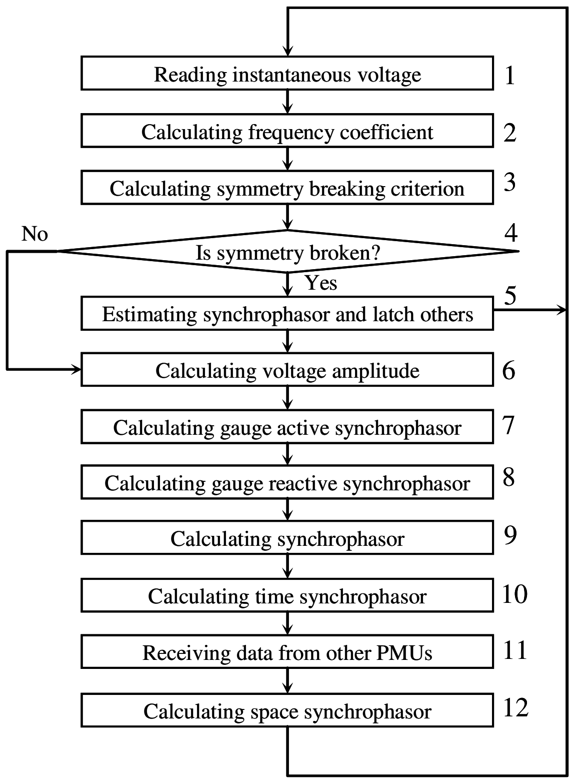

Fig. 5. Procedure for measuring synchrophasors

IV. NUMERICAL EXAMPLE

In this section, we shall give a phase step test numerical example that taken from IEEE Standard [2].

According to Table I, the real-valued function of test waveform can be expressed as

$$v = \begin{cases} \cos(390.19t + 0.4363), & t \le 0.5 \\ \cos(390.19t + \varphi_C + \pi/2), & t > 0.5 \end{cases} \quad (47)$$

where φ_C is the phase angle at the point of variation.

We plot some simulation results from Fig.6 to Fig.10.

As shown in Fig.6, frequency coefficient is calculated as

$$f_C = \frac{v_{21} + v_{23}}{2v_{22}} = 0.6874 \quad (48)$$

According to Table I, we calculate symmetry breaking criterion as

$$\left| f_{C(t)} - f_{C(t-T)} \right| > 0.01 \quad (49)$$

Figure 6 shows that three points after the point of variation changed largely and symmetry is broken. In the time of symmetry breaking duration, we latch the rotation phase angle, the frequency, voltage amplitude and estimate the synchrophasor respectively.

As shown in Fig. 7, we calculate the rotation phase angle as

$$\alpha = \cos^{-1} f_C = 46.575 \text{ (degree)} \quad (50)$$

And, the frequency is obtained as

$$f = \frac{f_S}{2\pi}\alpha = 62.10 \text{ (Hz)} \quad (51)$$

As shown in Fig. 8, we calculate voltage amplitude as

$$V = \frac{\sqrt{2}V_{gd}}{2(1 - f_C)\sqrt{1 + f_C}} = 1.0 \quad \text{(V)} \quad (52)$$

And, gauge difference voltage is obtained as

$$V_{gd} = \sqrt{{v_{22}}^2 - v_{21}v_{23}} = 0.5743 \quad \text{(V)} \quad (53)$$

Figure 8 shows that gauge difference voltage is influenced by phase step that is large disturbance. This shows that same as frequency coefficient, we can use the difference of two gauge difference voltage as symmetry breaking criterion to distinguish between small disturbance and large disturbance.

As shown in Fig. 9, simulation results of synchrophasors that are calculated based on (31) agree with definition proposed in this paper. One is that the range of synchrophasors is -180 to +180 degrees; the other is that the rotation direction of synchrophasor is a counterclockwise direction.

TABLE I
PARAMETERS OF PHASE STEP TEST

Symbol	Item	Parameter
f_S	Sampling frequency	480 Hz
n	Sampling points	4
f_{BRK}	Symmetry breaking setting	0.01
f	Real frequency	62.1 Hz
V	Voltage amplitude	1 V
φ	Initial voltage phase angle	25 Degree
Tc	Phase change time	90 degree rise in 0.05S
$Tend$	Simulation time range	0-0.1 S

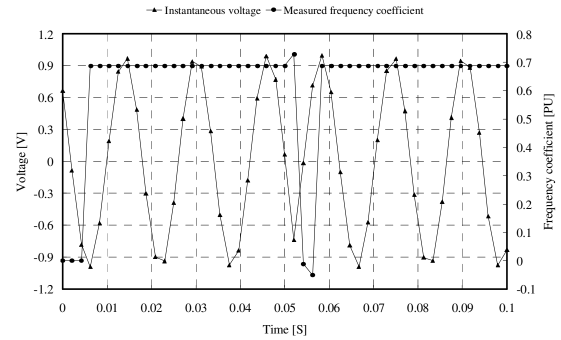

Fig. 6. Simulation results of frequency coefficient

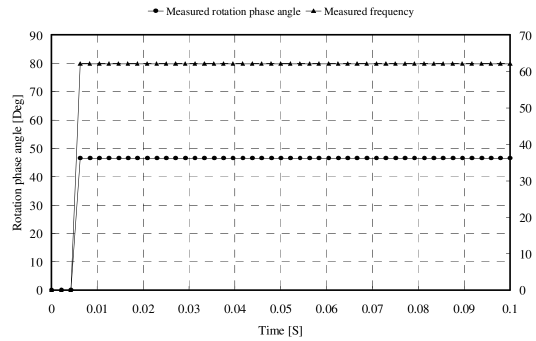

Fig. 7. Simulation results of the rotation phase angle and the frequency

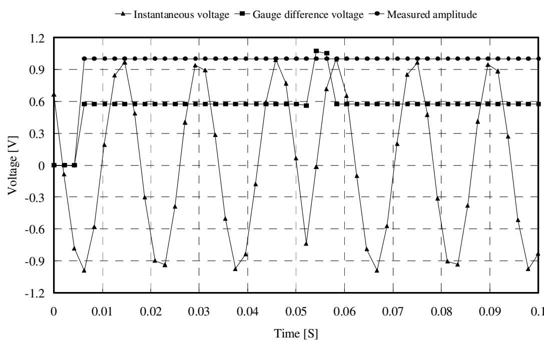

Fig. 8. Simulation results of gauge difference voltage and voltage amplitude

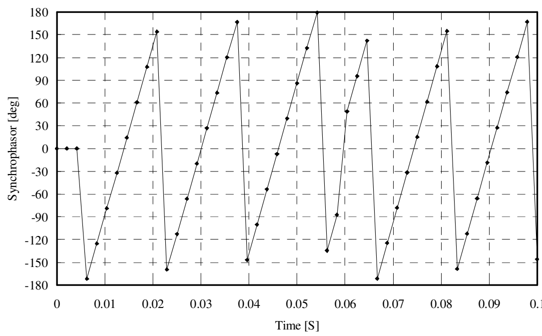

Fig. 9. Simulation results of synchrophasors

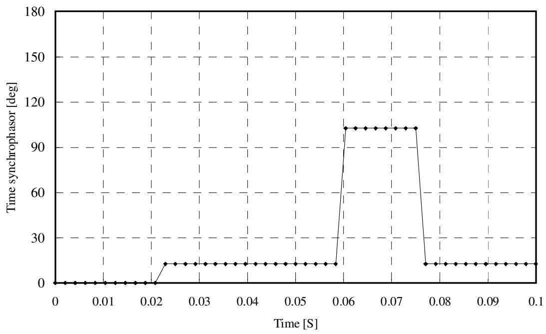

Fig. 10. Simulation results of time synchrophasors

As shown in Fig.10, we can calculate time synchrophasors before the point of variation as

$$\varphi_{TP} = \varphi_t - \varphi_{t-T_0} = \frac{62.1-60}{60} \times 360 = 12.6 \text{ (degree)} \qquad (54)$$

Here the interval time of time synchrophasors is set as one cycle of the rated frequency as

$$T_0 = \frac{1}{60} \text{ (seconds)} \qquad (55)$$

In the transition area, we can calculate time synchrophasors as

$$\varphi_{TP} = \varphi_t - \varphi_{t-T_0} = 12.6 + 90 = 102.6 \text{ (degree)} \qquad (56)$$

Figure 10 shows that time synchrophasors can take snapshot to record flicker of voltage.

At last, we give conclusions.

V. CONCLUSIONS

Guiding by symmetry thinking, we created gauge difference voltage group to calculate frequency coefficient and gauge difference voltage. Employing these invariants of symmetry groups we proposed formulae for calculating the frequency and voltage amplitude. Then, after creating gauge active synchrophasor group to obtain gauge active synchrophasor, and creating gauge reactive synchrophasor group to obtain gauge reactive synchrophasor, we got the synchrophasor. In the next way, we defined and measured time synchrophasors and space synchrophasors. Furthermore, for raising the precision of the novel method, we introduced symmetry breaking criteria. The numerical example illustrated that the novel method is effective.

At last, we show the difference between the novel method and popular methods. Firstly, the former develops synchrophasors based on group theory, the latter develops synchrophasors based on DFT; secondly, the former take synchrophasors associated with real frequency, the latter take synchrophasors associated with nominal system frequency; lastly, the formers deal with waveforms of single phase, the latter deal with waveforms of three phase.

We shall have been applying the symmetry principles to power systems continuously.

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