# Interharmonic Analysis via Thévenin Iterations

Mauricio Caixba and Abner Ramirez

*Abstract*— **This article proposes a hybrid technique to calculate the steady state solution of a network that includes linear/nonlinear elements and electronic devices considering interharmonics. The discrete Fourier transform (DFT) is chosen as a natural domain for the representation of interharmonics. Basically, the solution is obtained through partitioning of the network into linear and nonlinear parts, where the former is represented by its Thévenin equivalent. Initially, a voltage on the interface of the linear and nonlinear parts is proposed. By using the proposed voltage, the current entering the nonlinear part is internally found in the time domain and converted back into the frequency domain via DFT operations. Then, the resultant current is injected into the Thévenin equivalent. This results in an updated voltage at the interface. This iterative solution scheme permits to obtain interharmonic voltages and currents at the buses where nonlinear loads and electronic devices are connected. A Newton-type solution scheme is used to compare the Thévenin scheme proposed here, presenting the corresponding numerical issues and differences between both methods.**

*Index Terms*—**Fourier transforms, frequency domain analysis, interharmonics.**

## I. Introduction

ONE of the traditional techniques for harmonic analysis in the power systems area is the Harmonic Domain (HD) [1]. The HD technique arranges the Fourier coefficients in a matrix/vector form to represent linear time-periodic systems. However, the HD is limited only to the analysis of integer multiples of the fundamental frequency of the electrical system.

Due to the increasing inclusion of nonlinear loads and electronic devices in the power systems, it is also necessary to consider interharmonics in the analysis of voltages and currents of the network.

This work uses a technique named modified harmonic domain (MHD) [2], which is based on the discrete Fourier transform (DFT) formulated in a matrix/vector form similar to the HD. In the MHD is possible to include non-integer multiples of the fundamental frequency, i.e., interharmonics [3]. Additionally, the MHD permits to analyze electrical systems in the presence of nonlinear loads and electronic devices by readily interfacing frequency and time domain. Those time-varying elements are solved in the latter.

To determine the steady state of a purely linear network in the presence of interharmonics using the MHD, the procedure is relatively simple. On the other hand, when the network contains time-varying elements as well as interharmonic sources, it is no longer a trivial task. The complexity is enhanced for large networks with many time-varying elements due to the need of Jacobians and matrix inversions in traditional techniques, such as Newton-type ones.

This paper proposes the use of a Thévenin equivalent in the MHD for the linear part of the network. Time-varying elements are solved in the time domain in which either numerical integration or specialized software can be used. Interfacing the Thévenin equivalent and the resultant variables from the time-varying elements is made efficiently via DFT operations. Using an illustrative example, the method proposed here is compared with a Newton-type solution scheme [2].

The proposed method is based on compensation techniques where nonlinear elements are simulated as current injections. For instance, a two-iterative-loops method for calculating harmonics from transformer saturation is described in [4]. In [4], one iteration procedure is used to obtain an approximate solution at fundamental frequency and a second one is applied to incorporate higher frequencies (harmonics). In [5], a step-by-step solution method to incorporate nonlinear elements into a general network is described. In this widely utilized method, local iterations are used to solve the nonlinear element at each time step; alternatively, the intersection of the linear network and the piece-wise approximation of the nonlinear element has to be found. Roughly speaking, there are three major differences between the proposed method and the ones from [4], [5]. Firstly, in the former no local iterative loops are needed. Secondly, the algebraic-based arrangement that takes all interharmonics at once makes the proposed method attractive for fast computations. Thirdly, interharmonics have been added in the proposed method.

The inclusion of power flow constraints has been proposed in a harmonic power flow technique described in [6] and it represents a subsequent step in this research work, considering interharmonics.

## II. MHD Basic Theory

The MHD methodology is briefly described in this Section, for further details the reader can refer to [2].

### A. DFT Basic Relations

The Fourier transforms are given by [7]:

$$f(t)=\frac{2}{\pi}\int_{o}^{\infty} F(j\omega)e^{j\omega t}\,d\omega, \tag{1a}$$

Financial support from Conacyt-Mexico is gratefully acknowledged.
M. Caixba and A. Ramirez are with CINVESTAV-Guadalajara, Mexico (e-mails: mcaixba@gdl.cinvestav.mx, abner.ramirez@cts-design.com )

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$$F(j\omega) = \int_{o}^{\infty} f(t) e^{-j\omega t} dt, \tag{1b}$$

where the pair ($F$, $f$) corresponds to the frequency domain signal and its time domain counterpart, respectively (assumed here as causal). Supplementary, in (1) $\omega$ and $t$ represent the angular frequency (in rad/s) and time (in s), respectively. Also, consider the discretization of ($F$, $f$) by using the definitions:

$$\omega = k\Delta\omega, \qquad t = n\Delta t. \tag{2}$$

Then, (1) gives the corresponding inverse discrete Fourier transform (IDFT) and (DFT) expressed here as:

$$f_n = \frac{1}{\Delta t}\left[\frac{1}{N}\sum_{k=0}^{N-1} F_k \sigma_k e^{jk\Delta\omega n\Delta t}\right], \qquad n = 0,1,\cdots,N-1 \tag{3a}$$

$$F_k = \Delta t\left[\sum_{n=0}^{N-1} f_n e^{-jk\Delta\omega n\Delta t}\right], \qquad k = 0,1,\cdots,N-1 \tag{3b}$$

with:

$$\Delta\omega = \frac{2\pi}{T}, \tag{4}$$

where $T$ corresponds to the observation period of time, $N$ is the number of samples of the signal, and $\sigma$ is a data window used for decreasing Gibbs phenomenon due to truncation.

It can be noticed that the terms between square parentheses in (3a) and (3b) can be calculated efficiently through inverse fast Fourier transform (IFFT) and fast Fourier transform (FFT) algorithms, respectively [8].

The discretization given by (2) and (3) permits to handle frequencies that are non-integer multiple of the fundamental power frequency, i.e., interharmonics [3]. The periodic steady state can be determined in a direct form by considering the DFT coefficients as constants. In a similar way to the HD, the transient behavior of interharmonics can be achieved from the consideration of the coefficients being slowly time-varying [9].

### *B. Matrix-Vector Expressions*

In a similar way to the traditional harmonic domain, it can be demonstrated that the scalar ordinary differential equation given by:

$$\dot{x} = ax + bv, \tag{5}$$

can be converted into the algebraic system of equations:

$$\mathbf{D}X = \mathbf{A}X + \mathbf{B}V. \tag{6}$$

Application of (3a) to each term in (5), matching the exponentials for each frequency, and expressing the resultant equalities in matrix-vector form leads to (6). In (6), the new variables are defined as ($T_r$ denotes transpose):

$$X = \begin{bmatrix} X_o & X_1 & \cdots & X_{N-1} \end{bmatrix}^{T_r}, \tag{7a}$$

$$\mathbf{D} = diag\begin{bmatrix} 0 & j\Delta\omega & j2\Delta\omega & \cdots & j(N-1)\Delta\omega \end{bmatrix}. \tag{7b}$$

Also, in (6) **A** (and **B**) corresponds to a Toeplitz-type matrix with the frequency content (not necessarily harmonics) of $a$ as elements [10].

It is important to mention that the coefficient arrangement in (7a) corresponds to the IDFT formula as in (3); hence the subscript $o$ corresponds to the DC component, subscript 1 to the $\Delta\omega$ component, and so forth, until the $N$-th sample is represented.

For the case in which the coefficients $a$ and $b$ are constant, (6) becomes:

$$\mathbf{D}X = aX + bV. \tag{8}$$

Thus, the steady state can be calculated directly either from (6) or (8).

It should be clarified at this point that from the total number of samples, $N$, only the frequencies under interest, $N_p$, are taken into account for frequency domain calculations. Thus the MHD variables in (7) are handled as truncated variables, for more details please see [2].

The network elements, which usually are represented by differential equations as in (5), can be then represented as admittances (or impedances) in the MHD as in (6).

## III. Thévenin Equivalent in the MHD

### *A. Solution Scheme*

The total network is represented by a Thévenin equivalent, which consists of the linear part of the network in the MHD, and elements with non-linear (or switching) features, as shown in Fig. 1.

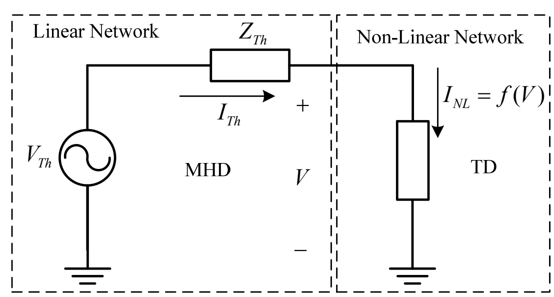


**Fig. 1.** Network representation.

In general, nonlinear elements or switching devices can be represented by a relation between voltage and current as:

$$I_{NL} = f(V), \tag{9}$$

or, in linearized form:

$$\Delta I_{NL} = \mathbf{J}\Delta V, \tag{10}$$

which will be used for the convergence analysis in a subsequent Section.

The proposed technique is described by the following steps. It can be applied to single and multi-phase networks with one or more time-varying elements.

**Step 1.** Start with an initial guess voltage, $V_0$, at the interface of linear and nonlinear networks. With this value calculate the entering current into the nonlinear element or electronic device, $I_{NL}$. This calculation is efficiently done in the time domain.

**Step 2.** Application of Kirchhoff's voltage law to the Thévenin equivalent gives a new voltage, $V_n$, at the interface.

**Step 3**. The new voltage $V_n$ is replaced in step 1 as an improved guess voltage. The process is stopped until the following convergence criterion is satisfied:

$$\left\| V_{k+1} - V_k \right\| \le tolerance, \tag{11}$$

or, until a pre-defined stopping iteration number.

The flow-chart of the procedure outlined above is depicted in Fig. 2.

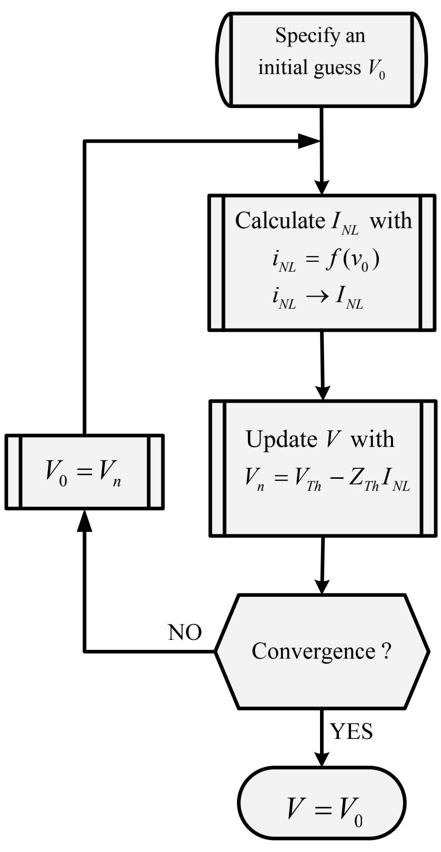


**Fig. 2.** Iterative solution scheme.

*B. Convergence Analysis*

The following optional analysis may be performed to assure convergence before applying the method outlined in the preceding sub-section.

In the proposed hybrid method the *k*-th iteration can be written as:

$$V_{k+1} = V_{Th} - \mathbf{Z_{Th}} I_{NL,k}. \tag{12}$$

If $V_{k+1}$ converges to $V_e$ and $I_{NL,k}$ to $I_{NLe}$, then (12) takes the form:

$$V_e = V_{Th} - \mathbf{Z_{Th}} I_{NLe}. \tag{13}$$

Subtracting (13) from (12) gives:

$$\Delta V_{k+1} = -\mathbf{Z_{Th}} \Delta I_{NL,k}. \tag{14}$$

Using the linearized relation for the time-varying element (10), (14) becomes:

$$\Delta V_{k+1} = -\mathbf{Z_{Th}} \mathbf{J} \Delta V_k. \tag{15}$$

Thus the Thévenin process is characterized by the iteration matrix:

$$\mathbf{T_h} = -\mathbf{Z_{Th}} \mathbf{J}. \tag{16}$$

All eigenvalues of $\mathbf{T_h}$ must be < 1 (in absolute value) to assure convergence and the largest eigenvalue gives the final (slowest) convergence rate, when all the faster modes have already converged. It can be noticed that the iteration matrix depends on parameters of the linear network and on the Jacobian of the nonlinear element.

## IV. APPLICATION EXAMPLE

*A. Network Description*

Consider the 10-bus network shown in Fig. 3 with the corresponding data described in the Appendix. The network is formed by ten distributed-parameters frequency dependent transmission lines with parameters calculated by using the complex ground concept [11]. Bus 7 has a nonlinear load [2], [12] and a TCSC [13] has been connected between buses 4 and 5 with the aim of improving the power factor, based on the corresponding *RL* load. In addition to the fundamental power frequency, in the sources $u_1$ and $u_2$ it has been arbitrarily considered the injection of several interharmonics, as seen in (17) and (18).

The MHD parameters are: $N$ = 1024 samples, $\Delta\omega$ =377/4, and $N_p$ = 320 samples. Thus, interharmonics spaced at ¼ of the fundamental power frequency are represented.

$$u_1(t) = \sin(\omega_0 t) + 0.1\sin(2.75\omega_0 t) + \\ +0.4\sin(3.25\omega_0 t) + 0.05\sin(4\omega_0 t), \tag{17}$$

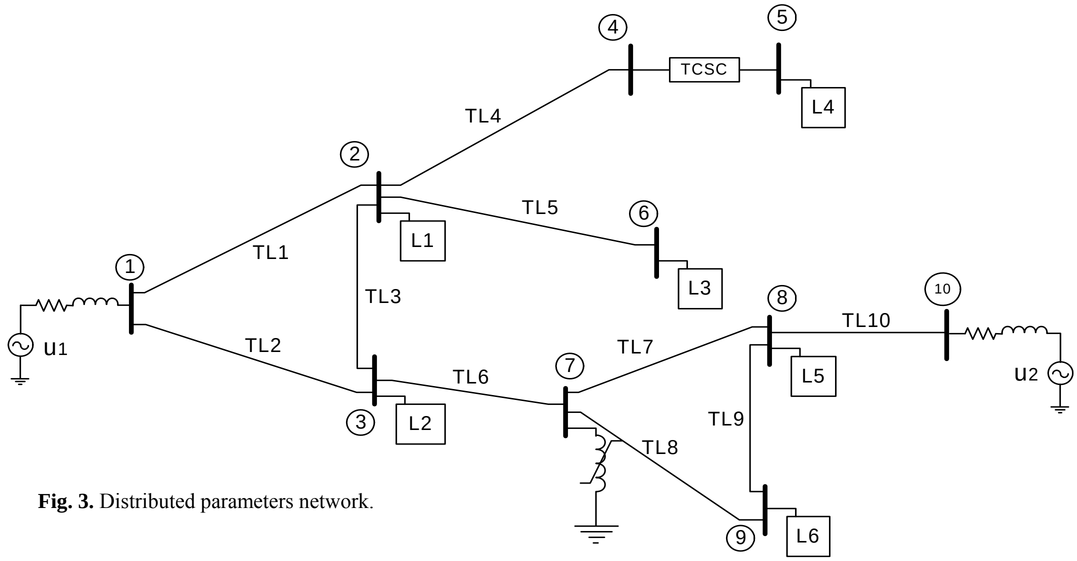


**Fig. 3.** Distributed parameters network.

$$u_2(t) = \sin(\omega_0 t) + 0.3\sin(2.5\omega_0 t) + \\ +0.1\sin(3.5\omega_0 t). \qquad (18)$$

For bus 7, a nonlinear reactor has been considered with a current/flux relation given by

$$i = \alpha\phi + \beta\phi^3. \qquad (19)$$

*B. Network Solution*

The network is represented by a Thévenin equivalent in its multiport version, which is shown in Fig. 4.

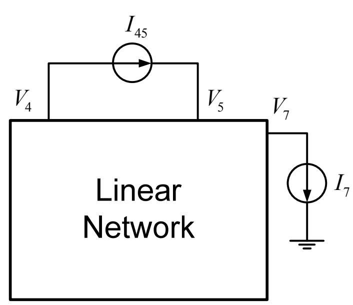


**Fig. 4.** Thévenin equivalent – multiport version.

The total network can be represented by its nodal form as:

$$\begin{bmatrix} -I \\ I_S \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V \\ V_S \end{bmatrix}, \qquad (20)$$

where $I$ and $V$ are vectors with the currents and voltages of the time-varying elements, respectively. Solving (20) for $V$, results in:

$$V = \underbrace{-\left(Y_{11} - Y_{12}Y_{22}^{-1}Y_{21}\right)^{-1}}_{\mathbf{Z_{Th}}} I \underbrace{-\left(Y_{11} - Y_{12}Y_{22}^{-1}Y_{21}\right)^{-1} Y_{12}Y_{22}^{-1}I_S}_{\mathbf{V_{Th}}}. \qquad (21)$$

In this case the matrix dimensions in (21) are: $Y_{11}$ (960x960), $Y_{12}$ (960x2240), $Y_{21}$ (2240x960), and $Y_{22}$ (2240x2240).

In the proposed methodology, it is suggested that, to obtain a better initial guess, the linear network be first solved. This can be obtained by taking an equivalent reactance of the electronic device and making zero the coefficient of the nonlinear term in the polynomial flux/current relation. According to the Authors' experience, the linear network solution is usually achieved in two Thévenin iterations.

*C. Numerical Results*

In Figs. 5 and 6 the voltage and the current between buses 4 and 5 are shown. Their corresponding frequency content is depicted in Figs. 7. With reference to bus 7, the voltage and current in this bus are presented in Fig. 8. The corresponding frequency content is depicted in Fig. 9. From Fig. 9 it can be noticed the coupling of the sources frequencies and the generation of new frequencies as well.

For comparison purposes, the results presented in Figs. 5 to 9 were also obtained by using a Newton-type solution scheme.

The computational time by using the proposed method is equal to 0.44 s while using Newton has resulted in 5.74 s; that is, about thirteen times faster with the former. Both methods were programmed under a Matlab® environment, the convergence tolerance has been set equal to $10^{-10}$ (reached in 15 iterations for both of them). The computer used to obtain the results is a 2GB RAM, 3 GHz processor.

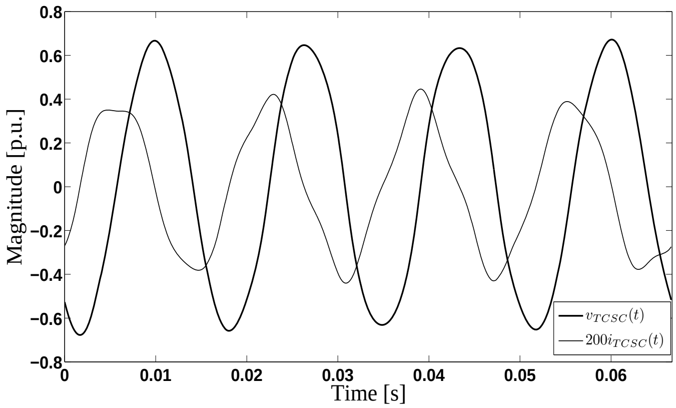


**Fig. 5.** Voltage at TCSC's terminals and current into the TCSC.

Finally, Fig. 10 shows the convergence pattern of both the Newton-type scheme and the Thévenin iteration technique proposed in this paper. It should be mentioned here that the convergence pattern from the latter in this particular example is not characteristic of the method; however, for several distinct parameters (results not shown here) the convergence pattern did not change substantially.

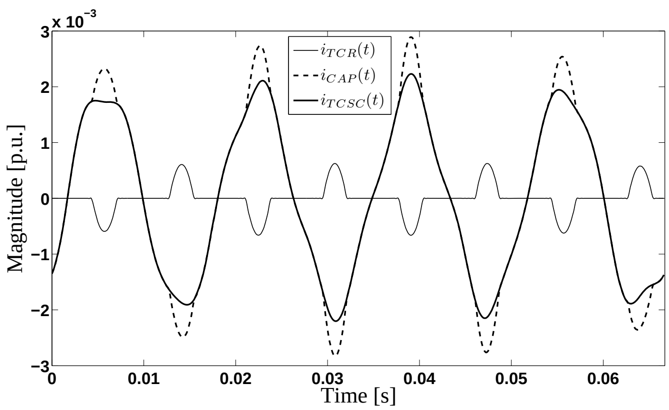


**Fig. 6.** TCSC entering currents.

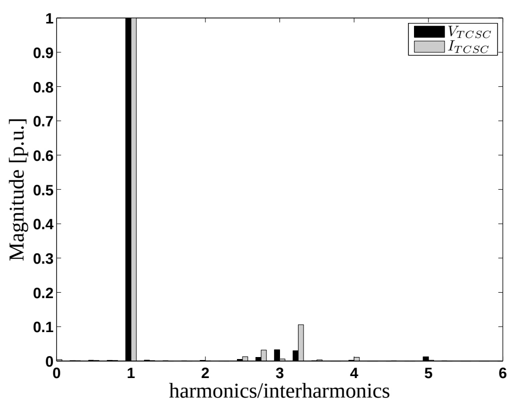


**Fig. 7.** Frequency content of the voltage at TCSC's terminals and of the current entering the TCSC.

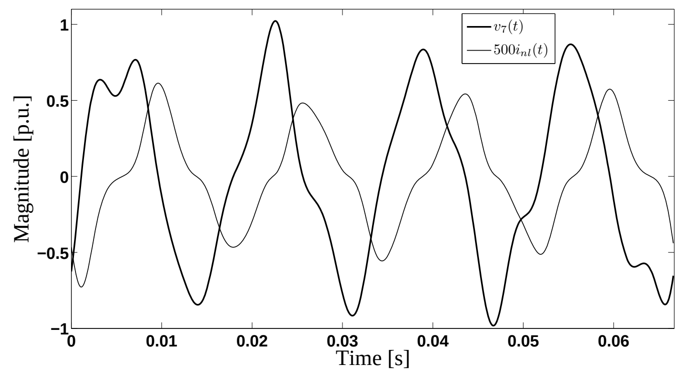


**Fig. 8.** Waveforms of voltage and current at bus 7.

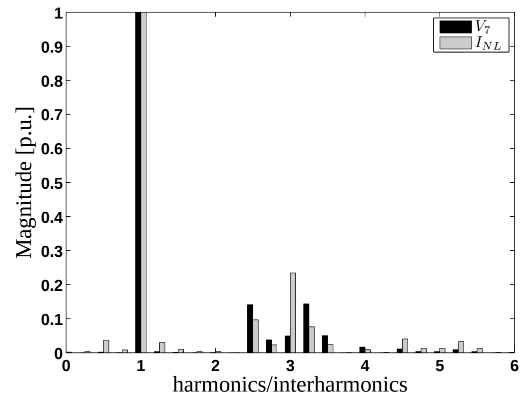


**Fig. 9.** Frequency content of the voltage and current entering the nonlinear load at bus 7.

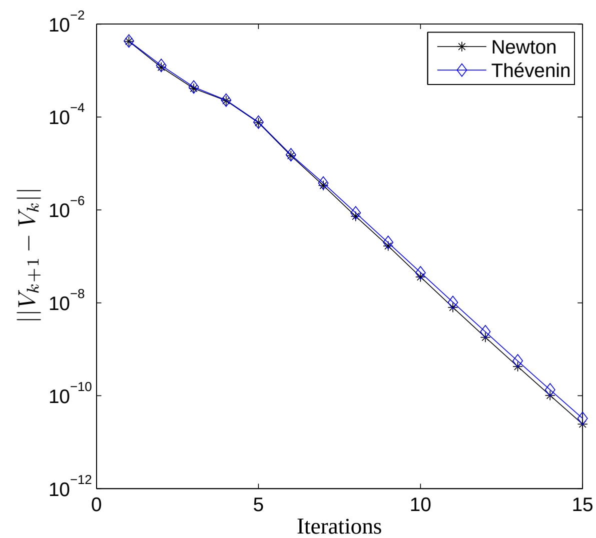


**Fig. 10.** Convergence pattern.

## V. CONCLUSIONS

The periodic steady state computation of a network that includes time-varying elements has been proposed in the modified harmonic domain, aimed to account for interharmonics. The backbone of the proposed hybrid method is that it is based on a Thévenin equivalent of the linear part of the network (solved in the MHD) whilst the time-varying elements are resolved in the time domain.

The Thévenin-based method has shown advantages over a Newton-type method in computational time while keeping similar number of iterations for both. This can be attributed to the fact that the former does not perform inversion of large matrices in the iterative process. In addition, Thévenin does not require calculation of Jacobians as in traditional Newton technique.

The proposed method is intended for large networks by using computational resources such as parallel processing and graphic processor units (GPUs). This may overcome the issue of dimensionality, though it is left as a future research topic.

## APPENDIX

### DISTRIBUTED-PARAMETERS NETWORK DATA

The lengths of the transmission lines forming the network in Fig. 3 are listed in Table I. For all of them we have used the conductor data: 0.02 m radius and 18 m height. The source parameters are: $R_o$ = 1 mΩ and $L_o$ = 50 mH, and the load data are listed also in Table I. The firing angle of the TCSC is set equal to 152º, with $L_{TCSC}$ = 0.4 H and $C_{TCSC}$ = 8.8 μF. For the nonlinear load connected at bus 7, and referring to (19), $\alpha$ is equal to 0.4 and $\beta$ is equal to $10^5$.

TABLE I
PARAMETERS OF NETWORK

| Transmission Lines | | | |
|---|---|---|---|
| Label | Length (Km) | Label | Length (Km) |
| TL1 | 80 | TL6 | 120 |
| TL2 | 80 | TL7 | 80 |
| TL3 | 100 | TL8 | 80 |
| TL4 | 120 | TL9 | 100 |
| TL5 | 80 | TL10 | 150 |

| Loads | | | | | |
|---|---|---|---|---|---|
| Label | *R* (Ω) | *L* (mH) | Label | *R* (Ω) | *L* (mH) |
| L1 | 300 | 10 | L4 | 400 | 0.6 |
| L2 | 250 | 10 | L5 | 300 | 20 |
| L3 | 300 | 10 | L6 | 250 | 20 |

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## BIOGRAPHIES

**Mauricio Caixba** received his B.Sc. and M.A.Sc. from San Andres Tuxtla Institute of Technology in 2004, and from the Center for Research and Advanced Studies of Mexico (CINVESTAV) Campus Guadalajara, in 2010, respectively. He is currently pursuing the Ph.D. degree at CINVESTAV. His interests are power system harmonic analysis and power electronic devices.

**Abner Ramirez** received his B.Sc., M.A.Sc. and Ph.D. from University of Guanajuato, Mexico, in 1996, University of Guadalajara, Mexico, in 1998 and from the Center for Research and Advanced Studies of Mexico (CINVESTAV) Campus Guadalajara, in 2001, respectively. He was a postdoctoral fellow in the Department of Electrical and Computer Engineering of the University of Toronto, Canada, from November 2001 to January 2005. Currently, he is a Professor at CINVESTAV-Guadalajara. He is a member of the Mexican Association of Professionals and Students A.C. (PLAPTSAC). His interests are electromagnetic transient analysis in power systems and numerical analysis of electromagnetic fields.