

Fault Location on Transmission Lines Based on Travelling Waves

F. V. Lopes, *Student Member, IEEE*, D. Fernandes Jr., *Member, IEEE*, W. L. A. Neves, *Member, IEEE*

Abstract—Automatic and accurate fault location methods for transmission lines may reduce the search time technicians would take to find out where the fault is, leading to a quick recovery of the system. Usually, transient detection techniques use more than one sample of voltage or current to make possible the required transient analysis. This paper presents a very simple method for fault location on transmission lines based on travelling waves using Park's Transformation to perform the transient detection. An advantage of the proposed method is that only one voltage sample at each phase is used and transients in all three phases are monitored simultaneously by the analysis of only one signal: the direct axis voltage. The method is implemented in the Alternative Transients Program (ATP) using the MODELS language and simulations were carried out for a 230 kV transmission line with a digital fault recorder (DFR) at each terminal to evaluate the applicability of the proposed technique. The results show the effectiveness of the algorithm which can be used as an additional routine for digital protective relays.

Keywords: Fault location, Park's Transformation, transient detection, transmission lines.

I. INTRODUCTION

THE exact fault location on transmission lines is a problem that has been studied for decades. The theory of travelling waves has been recently used for this purpose. Such methods are based on the transient detection at specific points of the electrical system in question and are classified according to the number of monitored points. Several methods for transient detection have been proposed in order to enable an accurate and reliable fault location for transmission lines [1].

Methods based on travelling wave's theory utilize voltage and/or current data captured by DFRs (Digital Fault Recorders) installed in monitored line terminals. According to [2], for double ended fault location methods, it is necessary to use GPS (Global Positioning System) in order to synchronize oscillographic records at both monitored ends. So, digital signal processing techniques are applied to synchronized data from DFRs, making possible the detection of initial transient

instants at both transmission line ends. Then, fault location may be determined using expressions proposed in [3] in which the input data are the propagation velocity of travelling waves, the line length and the initial transient instants in monitored terminals.

In [4], the Discrete Wavelet Transform (DWT) and the Redundant Discrete Wavelet Transform (RDWT) are presented as efficient techniques for transient detection. In [5] is proposed a fault location method based on the analysis of correlation coefficients which are calculated from the voltage oscillographic records in both line ends.

The accuracy of travelling wave methods is a function of the sampling frequency of DFRs, i.e., it depends on the hardware used for data acquisition. In fact, it is rather refer to reliability than accuracy and define a maximum expected error as a function of the sampling frequency.

It is important to know that the transient detection is a critical step for fault location procedure and, thus, the reliability of the fault point estimation by any algorithm depends directly on the reliability of the method used to estimate the initial transient instants. In this paper, a very simple method for transient detection is proposed. Conventional methods usually need a window with more than one sample of current or voltage waveforms to perform the required transient analysis. Here, the window has only one sample, i.e., no previous voltage samples are needed. All three phases are monitored by the direct axis voltage signal V_d obtained through Park's transformation (Tdq0) and the method automatically determines the fault location point immediately after the fault occurrence.

II. DOUBLE ENDED FAULT LOCATION METHODS

The main difference among travelling wave methods is the number of monitored terminals and the technique to detect the transients. In [3] are presented expressions for determination of fault location considering one and two transmission line monitored ends. In [6] is proposed a three-terminal method that avoids the utilization of the propagation velocity of the travelling waves in the fault location procedure. In order to eliminate the propagation velocity variable, three terminals of two neighbor transmission lines are monitored and a unique value of propagation velocity is utilized for both lines. The problem of such method is that the waves present different propagation velocities depending on the ground resistivity and line parameters and, consequently, the utilization of the same propagation velocity for both lines can be considered as a source of error. Other works, as [7] and [8], present single

This work was supported by Brazilian Coordination for the Improvement of Higher Education Personnel (CAPES).

F. V. Lopes is M. Sc. candidate at the Federal University of Campina Grande (UFCG), Av. Aprígio Veloso, 882, Bodocongó, 58.429-140, Campina Grande-PB, Brazil (e-mail: felipe.lopes@ee.ufcg.edu.br).

D. Fernandes Jr. and W. L. A. Neves are with Department of Electrical Engineering of UFCG. (e-mail: damasio@dee.ufcg.edu.br, waneves@dee.ufcg.edu.br).

ended fault location methods. Such method use incident and reflected waves to perform the fault location.

Basically, the key idea of all mentioned methods is the determination of initial transient instants in monitored terminals. Here, the fault location method is based on the analysis of voltage waveforms recorded by DFRs in two transmission line terminals and, so, it is considered to be a double ended method. In this way, only incident travelling waves are used to perform the fault location, avoiding the utilization of reflected waves at the fault point and, consequently, making the method more reliable. A GPS must be used in order to synchronize the DFRs measurements to bring voltage samples to the same timebase and to ensure that the initial transient instants will be correctly obtained. Regarding the GPS usage, it is important to know that, in practice, it can present synchronization errors in the order of $\pm 1\mu\text{s}$. In spite of such limitation, it is possible to use an independent clock on the fault location device that initiates the counter at the moment of the first transient detection and stop the counter at the second transient detection at the opposite bus. In this way, synchronization errors are minimized. To understand the principles of travelling wave methods, consider Fig. 1 in which is shown DFRs placed at Buses 1 and 2. A time space diagram (Lattice diagram) is shown in the middle of the figure. A scheme of the DFRs and GPS application in the proposed double ended method is also presented in Fig. 1.

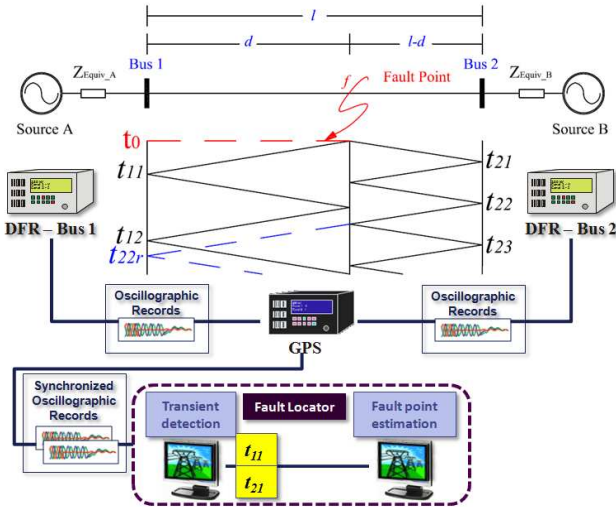


Fig. 1. Double ended fault location method scheme.

Where:

- t_{11} is the initial transient instant at Bus 1;
- t_{21} is the initial transient instant at Bus 2;
- t_{12} is the arrival instant of the reflected wave at Bus 1;
- t_{22}, t_{23} are the arrival instants of reflected waves at Bus 2;
- t_{22r} is the arrival instant of the refracted wave at Bus 1;
- l is the line length;
- d is the distance to the fault point.

The t_{21r} instant was left off in Fig. 1 to simplify the diagram. It is important to know that fault location algorithms consist on two main steps – the transient detection and the fault point location. For the first step, considering double ended methods,

it is necessary to detect the first incident waves at buses 1 and 2 to compute the time instants t_{11} and t_{21} . Secondly, the fault point location is obtained from an expression proposed in [3]:

$$d = \frac{l + (t_{11} - t_{21}) \cdot v}{2}, \quad (1)$$

where v is the propagation velocity of travelling waves and l is the line length. The procedure to perform the transient detection is explained next.

III. TRANSIENT DETECTION USING PARK'S TRANSFORMATION

The proposed method for transient detection is based on the work presented by R. H. Park in the United States in 1929 [9]. Park's transformation (Tdq0) has been used in the whole area of electrical engineering, especially on researches regarding to salient pole synchronous machines where variable inductances in a static reference frame are seen as constant inductances when the reference frame rotates at synchronous speed. Here, the rotating reference system is used to remove power frequency signals from voltage waveforms allowing transient time detection. An analogy between the Tdq0's application in electrical machines research and in transient detection is shown in Fig. 2.

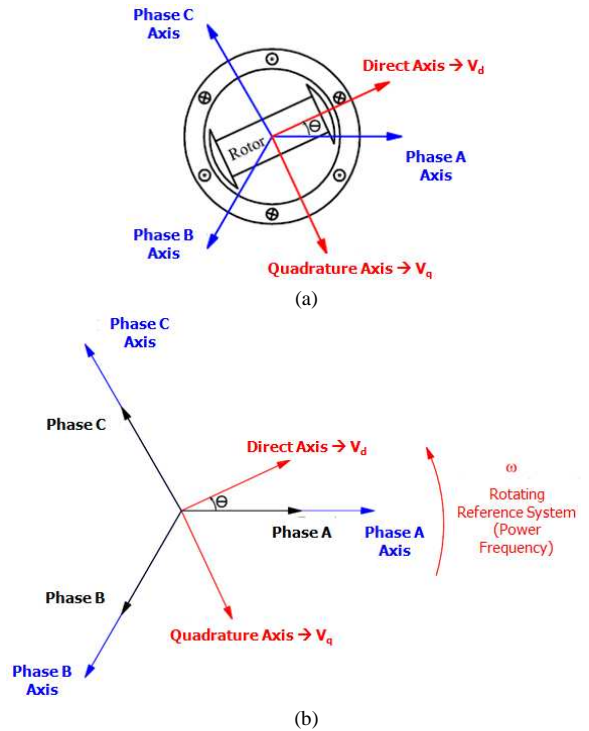


Fig. 2. Park's Transformation (Tdq0) application: (a) Electrical machine researches; (b) Transient detection and fault location.

For power system operating at normal conditions, zero frequency signals are calculated and, if transients occur, oscillatory signals are obtained. Such signals are called direct and quadrature axes components which will be represented from now on as V_d and V_q . Both components may be used for transient detection but, here, only V_d will be considered.

For high impedance fault cases, V_d coefficients presents high attenuation. Thus, to increase the sensitivity of the

proposed algorithm, difference coefficients (c_{dif}) are calculated using Taylor's approximation:

$$c_{dif}(i) = \frac{V_d(i) - V_d(i-1)}{\Delta t}. \quad (2)$$

Where:

V_d is the direct axis component;

i is the sample number;

Δt is the time step.

In the literature, difference quantities are widely used in protective relaying algorithms [10]. In this work, $[c_{dif}]^2$ will be used to detect de initial transient instants in monitored terminals. $[c_{dif}]^2$ makes the transient detection more robust because coefficient related to transient signals are amplified and coefficients related to normal conditions of the system are kept with low magnitude. An example of transient detection using Tdq0 is shown in Fig. 3.

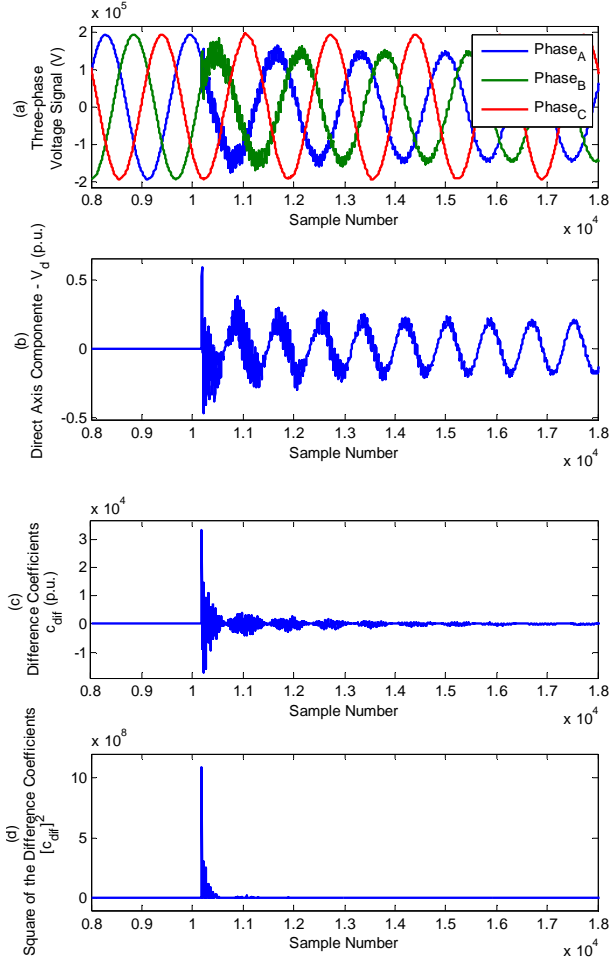


Fig. 3. Transient detection from a double phase to ground fault using Tdq0: (a) Three-phase voltage signal; (b) Direct axis component V_d ; (c) Difference coefficients c_{dif} ; (d) Square of the difference coefficients $[c_{dif}]^2$.

IV. METHOD IMPLEMENTATION

The proposed method implementation is divided into five steps: (a) voltage data acquisition using DFRs in both line ends, (b) calculation of an orthogonal voltage phasor in a static

reference frame, (c) calculation of an orthogonal voltage phasor in a rotating reference frame, (d) determination of the initial transient instants and (e) fault location.

A. Data Acquisition Using DFRs in Both Line Ends

Here, only voltage signals are used. An important characteristic of data acquisition is related to the sampling frequency utilized by DFRs. The low sampling rates of commercially available DFRs (usually in between 15 kHz and 20 kHz) is a limitation for fault location methods based on travelling waves. A 50 μ s time step was used in ATP simulations to simulate DFRs with 20 kHz sampling frequency (333 samples/cycle). Keeping the number of samples/cycle the same, smaller time steps would be even more appropriate to reduce errors produced by numerical integration methods.

Anyway, even with this limitation, algorithms based on travelling waves are more reliable than those based on fundamental frequency components.

B. Orthogonal Voltage Phasors (Static Reference Frame)

The Tdq0's input variables are orthogonal phasors obtained through Clarke's transformation which calculates the aerial modes (V_α , V_β) and the ground mode (V_0):

$$\begin{bmatrix} V_0 \\ V_\alpha \\ V_\beta \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}. \quad (3)$$

Here, only aerial modes are used because their propagation velocity is higher than those of ground mode waves. V_α and V_β are used as Tdq0's inputs variables in order to obtain the orthogonal voltage phasors in a rotating reference frame. Note that Clarke's transformation is used here only as an operator to obtain the orthogonal voltage phasors in a static reference system, differently from other applications where it is used to decouple the three-phase voltage system [11]. Then, the application of the method does not depend on the transmission line transposition scheme, i.e., it may be applicable to untransposed lines as well.

C. Orthogonal Voltage Phasors (Rotating Reference Frame)

This is the most important step in the proposed algorithm because it makes possible the analysis of the aerial modes (V_α , V_β) using a rotating reference frame, allowing the correct transient detection. Park's transformation is applied to the output voltage signals of Clarke's transformation to obtain the voltage V_d , used to determine the fault location:

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \cos(\omega \cdot t + \theta) & -\sin(\omega \cdot t + \theta) \\ \sin(\omega \cdot t + \theta) & \cos(\omega \cdot t + \theta) \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix}. \quad (4)$$

Where:

ω is the angular power frequency;

t is the DFRs clock synchronized by GPS;

θ is the angle between V_d and the voltage phasor at phase A (see Fig. 2b).

Apparently, the non alignment of V_d with the voltage phasor of the phase A may pose a problem. As illustrated in Fig. 2, if θ is greater than zero, a constant non-zero V_d signal is obtained when the system is operating at normal conditions and, so, transients may be wrongly detected. Here, the c_{dif} coefficients overcome that problem because, even getting constant non-zero V_d signals, each c_{dif} coefficient would be nearly zero. Therefore, the proposed method does not require alignment between V_d component and the voltage phasor of the phase A to detect transients correctly. Another good characteristic of the method is that only current voltage samples are used in the calculation of V_d coefficients, simplifying the implementation and making possible the fault location almost immediately after its occurrence. Other techniques as DWT and RDWT, utilize more than one voltage or current sample of each phase to calculate coefficients for the transient analysis of the system, making fault location a more complex procedure.

D. Detection of the Initial Transient Instants

In a two-terminals fault location method, $[c_{dif}]^2$ from both line ends must be calculated. The analysis of the $[c_{dif}]^2$ coefficients allows the identification of the sample number in which occur initial transients. So, to use (1) to estimate de fault point, it is necessary to obtain the initial transient instants in seconds. $[c_{dif}]^2$ with magnitude below a given preset threshold are eliminated and, so, coefficients related to noise and low frequency oscillation are ignored. Consequently, only $[c_{dif}]^2$ coefficients related to transients by the fault occurrence are considered.

As shown in Fig. 4, first non-zero $[c_{dif}]^2$ occurs one sample after the transient beginning. Finally, considering the sampling frequency F_s used by DFRs, the initial transient instants $t_{transient}$ (in seconds) may be determined according to (5a) and (5b).

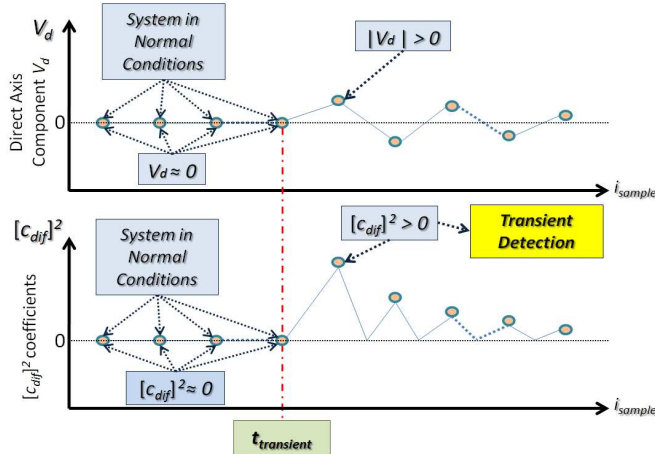


Fig. 4. Detection of the initial transient instants through the $[c_{dif}]^2$ analysis.

$$\text{If } [c_{dif}(i)]^2 = 0 \Rightarrow t_{transient} = \{ \}; \quad (5a)$$

$$\text{If } [c_{dif}(i)]^2 > 0 \Rightarrow t_{transient} = (i - 1)/F_s; \quad (5b)$$

Where:

$t_{transient}$ is the initial transient instant in seconds;

c_{dif} is the difference coefficients calculated from V_d ;

i is the sample number.

E. Fault Location Estimation

Voltage signals from both transmission line ends are used to detect the first transient instants t_{11} and t_{21} at buses 1 and 2 (as shown in Fig. 1). The identification of t_{11} and t_{21} is shown in Fig. 5.

Once t_{11} and t_{21} (in seconds), the line length and the propagation velocity of aerial modes are known, (1) is used to calculate the fault point location.

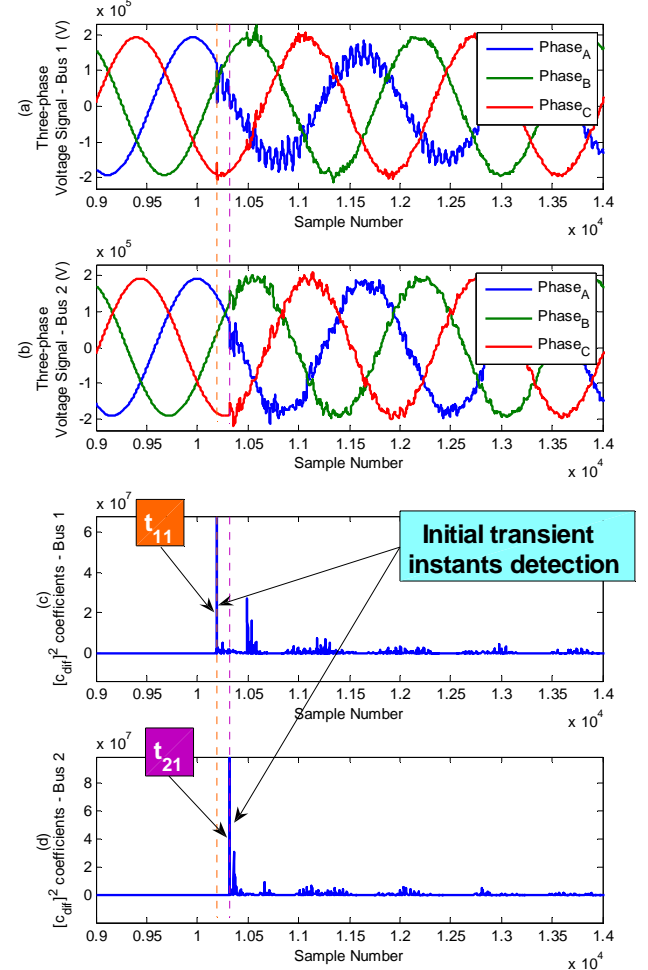


Fig. 5. Example of the initial transient instants detection in both terminals of the transmission line through the $[c_{dif}]^2$ analysis: (a) Three-phase voltage signals at Bus 1; (b) Three-phase voltage signals at Bus 2; (c) $[c_{dif}]^2$ coefficients related to Bus 1; (d) $[c_{dif}]^2$ coefficients related to Bus 2.

V. SIMULATION STUDIES

A. Simulation Model

ATP [12] simulations of the 230 kV system presented in Fig. 6 were performed to evaluate the proposed method. The system parameters are shown in Table I. A constant distributed-parameter transmission line model with a 50 μ s time step was used. The line length is 500 km and the distance d from bus 1 to the fault point is to be calculated. All algorithm steps were implemented using the MODELS language [13] and performed directly in the ATP one sample at a time similar to what occurs in real time protection system, with no need to analyze off line oscillographic records.

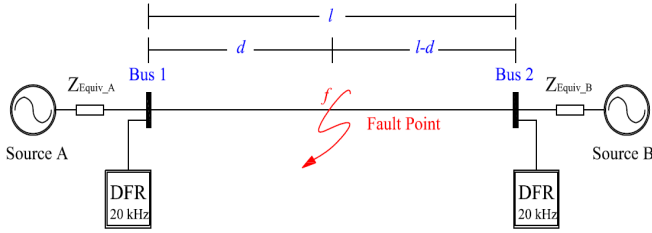


Fig. 6. Three-phase system considered in ATP simulations.

TABLE I
SYSTEM PARAMETERS

SOURCE A	SOURCE B	TRANSMISSION LINE
$V_{G1} = 1,014 \angle 10^\circ pu$	$V_{G2} = 1,000 \angle 0^\circ pu$	$l = 500 km$
$R_0^{G1} = 1,445 \Omega$	$R_0^{G2} = 1,445 \Omega$	$Z_0 = 0,236 + j1,035 \Omega/km$
$L_0^{G1} = 13,996 mH$	$L_0^{G2} = 13,996 mH$	$Y_0^{shunt} = 2,49 \mu mho / km$
$R_+^{G1} = 1,963 \Omega$	$R_+^{G2} = 1,963 \Omega$	$Z_+ = 0,054 + j0,527 \Omega/km$
$L_+^{G1} = 14,982 mH$	$L_+^{G2} = 14,982 mH$	$Y_+^{shunt} = 3,144 \mu mho / km$

B. Applied Disturbances

For validation purposes, situations where the transmission line is transposed and untransposed were analyzed. For each type of transposition scheme, 9600 different cases were run, totaling 19200 different cases in which fault parameters were changed: the fault resistance (5Ω to 95Ω with steps of 30Ω), the fault inception angle (0° to 180° with steps of 20°) and the distance from fault point to Bus 1 ($20 km$ to $480 km$ with steps of $20 km$). Digital simulations were performed for phase to ground, double phase, double phase to ground and three-phase faults. The most adverse cases were considered, as high fault resistance cases, fault inception angle near to zero (or at zero crossing) and faults very close to the substations.

To perform such a high quantity of simulations, batch files were created with Matlab[®] to allow the automatic run of the ATP files.

C. Simulation Results and Analysis

As shown in Table II and Table III, all results show the reliability of the fault location method. It is important to point out that the maximum expected error is a function of the sampling rates, i.e., it is a hardware limitation. The travel time of waves is not interpolated and, thus, the initial transient instants are approximated to the nearly multiple of the time step Δt used by DFRs. In this way, errors in the order of a half time step can be introduced in the calculated initial transient instants and are considered to be admissible errors. So, the module of such admissible errors is calculated using (6).

$$|e| \approx (\Delta t \cdot c) / 2 \quad (6)$$

Where:

c is the speed of light ($\approx 3.10^5 km/s$).

For the used $20 kHz$ sampling frequency, the admissible error is approximately $7.5 km$ in absolute value. This value is used as the error threshold throughout the analysis of the results. So, cases in which the fault location estimation presents error above such threshold are classified as unreliable cases.

TABLE II
FAULT LOCATION SIMULATIONS FOR TRANSPosed TRANSMISSION LINE

Fault type	N° of simulated cases	N° of reliable simulations	Reliable simulations (%)
AG	960	949	98.85%
BG	960	949	98.85%
CG	960	949	98.85%
AB	960	960	100.00%
BC	960	960	100.00%
CA	960	960	100.00%
ABG	960	960	100.00%
BCG	960	960	100.00%
CAG	960	960	100.00%
ABC	960	960	100.00%
Total:	9600	9567	99.66%

TABLE III
FAULT LOCATION SIMULATIONS FOR UNTRANSPosed TRANSMISSION LINE

Fault type	N° of simulated cases	N° of reliable simulations	Reliable simulations (%)
AG	960	944	98.33%
BG	960	947	98.65%
CG	960	950	98.96%
AB	960	954	99.38%
BC	960	960	100.00%
CA	960	954	99.38%
ABG	960	955	99.48%
BCG	960	960	100.00%
CAG	960	959	99.90%
ABC	960	960	100.00%
Total:	9600	9543	99.41%

In Table IV is shown a general analysis of the simulations in which transposed line and untransposed line cases are considered.

In Table V, unreliable simulations cases are analyzed. As expected, in most of unreliable simulations the errors are related to high impedance fault cases and situations in which fault inception angle is nearly to zero.

According to the results, less than 0.5% of 19200 performed cases diverged. Approximately 70% of unreliable results are related to high impedance fault cases and only about 6% are related to fault inception angle nearly to zero crossing cases. So, the proposed method for fault location presents low sensitivity in front of adverse cases related to fault resistance and fault inception angle.

TABLE IV
GENERAL RESULTS OF FAULT LOCATION SIMULATIONS

Considered system	N° of simulated cases	N° of unreliable simulations	N° of reliable simulations	Reliable simulations (%)
Transposed	9600	33	9567	99.66%
Untransposed	9600	57	9543	99.41%
Total:	19200	90	19110	99.53%

TABLE V
ANALYSIS OF UNRELIABLE SIMULATION RESULTS

Considered system	Transp.	Untransp.	Total:
Case analysis			
N ^o of unreliable simulations	33 (100.00%)	57 (100.00%)	90 (100.00%)
N ^o of cases related to high impedance fault (>60 Ω)	22 (66.67%)	41 (71.93%)	63 (70.00%)
N ^o of cases related to fault inception angle nearly to zero crossing (< 10°)	0 (0.00%)	6 (10.53%)	6 (6.67%)
N ^o of cases related to unknown reasons	11 (33.33%)	10 (17.54%)	21 (23.33%)

The sampling frequency used by DFRs is a limitation of fault location methods based on the theory of travelling waves. In fact, increasing the sampling frequency F_s , the maximum expected error decreases and a greater accuracy is obtained. In order to address such considerations, some faults in the system shown in Fig. 6 were performed using a transposed transmission line model and different time steps to simulate DFRs with sampling frequencies of 20 kHz, 100 kHz and 500 kHz. Simulation results for each sampling frequency are shown in Table VI. The error ϵ can be compared to the admissible error calculated from (6).

One can see that all estimated fault locations are within the admissible errors and, consequently, the good accuracy, the great simplicity and the reliability of the proposed technique are attested.

TABLE VI
EFFECT OF THE SAMPLING FREQUENCY ON THE METHOD'S ACCURACY

FT	FP (km)	$F_s=20$ kHz		$F_s=100$ kHz		$F_s=500$ kHz	
		FL (km)	ϵ (km)	FL (km)	ϵ (km)	FL (km)	ϵ (km)
AG	28	23.30	-4.70	27.69	-0.31	28.27	0.27
BG	78	81.80	3.80	77.41	-0.59	78.29	0.29
CG	129	125.68	-3.32	130.07	1.07	129.19	0.19
AB	166	169.56	3.56	165.17	-0.83	166.05	0.05
BC	205	198.81	-6.19	204.66	-0.34	204.95	-0.05
CA	244	242.69	-1.31	244.15	0.15	243.86	-0.14
ABG	311	315.82	4.82	311.43	0.43	310.84	-0.16
BCG	393	388.95	-4.05	393.33	0.33	392.75	-0.25
CAG	432	432.82	0.82	431.36	-0.64	431.65	-0.35
ABC	472	476.70	4.70	472.31	0.31	471.73	-0.27

Where: FT = Fault Type; FP = Fault Point; FL = Fault Location; ϵ = Error; F_s = Sampling Frequency.

VI. CONCLUSIONS

In this paper an approach for fault location on transmission lines monitored in two terminals was presented. Only the current voltage samples are used to detect transients and all three phases are monitored by the direct axis voltage signal V_d obtained through Park's transformation (Tdq0). The method automatically determines the fault location point immediately after the fault occurrence.

The proposed method was applied to transmission lines taking into account the influence of fault resistance, voltage inception angle and the transmission line transposition scheme. An analysis of the effect of the sampling frequency on the method's accuracy was performed. The algorithm was implemented in ATP using the MODELS language. Batch files were created with Matlab[®] to allow the automatic run of the ATP files. A total of 19200 EMTP simulations for a 230 kV transmission line were performed. In more than 99% of the simulated cases, the proposed algorithm presented errors below the admissible value.

Finally, the proposed method is very simple, fairly accurate, do not require phasor estimation and may be used as an additional routine in transmission lines protective relays.

VII. ACKNOWLEDGMENT

The authors would like to thank the reviewers for invaluable suggestions.

VIII. REFERENCES

- [1] H. W. Dommel, J. Michels, "High-speed Relaying Using Travelling Wave Transient Analysis". *IEEE Conference*, Paper No. A78, pp. 214-219, January/February 1978.
- [2] M. Gilany, DK. Ibrahim, E. S. T. Eldin, "Traveling-Wave-Based Fault-Location Scheme for Multiend-Aged Underground Cable System," *IEEE Transactions on Power Delivery*, vol. 22, pp. 82-89, January 2007.
- [3] P. F. Gale, P. A. Crossley, Xu Bingyin, Ge Yaozhong, B. J. Cory, J. R. J. Barker, "Fault Location Based on Travelling Waves," *Fifth International Conference on Developments in Power System Protection*, pp. 54-59, 1993.
- [4] F. B. Costa, B. A. Souza, N. S. D. Brito, K. M. Silva, "Discrete Wavelet Transform applied to the disturbance diagnosis," (in Portuguese), *Brazilian Symposium of Electrical Systems*, Belém, Brazil, 2010.
- [5] D. Lin, P. Jun, S. Wenxia, T. Jun, Z. Jun, "Fault Location for Transmission Line Based on Traveling Waves using Correlation Analysis Method," *International Conference on High Voltage Engineering and Application*, Chongqing, China, November 9-13, 2008.
- [6] Z. Feng, L. Jun, Z. Li, Y. Zhihao, "A New Fault Location Method Avoiding Wave Speed and Based on Traveling Waves for EHV Transmission Line," *Electric Utility Deregulation and Restructuring and Power Technologies-DRPT2008*, Nanjing, China, April 6-9, 2008.
- [7] X. Zheng, X. Li, J. Ding, Z. Duan, "Study on Impedance-traveling Wave Assembled Algorithm in One-terminal Fault Location System for Transmission Lines," *Electric Utility Deregulation and Restructuring and Power Technologies-DRPT2008*, Nanjing, China, April 6-9, 2008.
- [8] M. Aurangzeb, P. A. Crossley, P. F. Gale, "Fault Location on a Transmission Line Using High Frequency Travelling Waves Measured at a Single Line End," *IEEE Power Engineering Society Winter Meeting*, vol. 4, pp. 2437 - 2442, August 06, 2002.
- [9] R. H. Park, "Two-Reaction Theory of Synchronous Machines, Generalized Method of Analysis - Part I," *Winter Convention of the A. I. E. E.*, pp. 716-730, New York, January 28-February 1, 1929.
- [10] G. Benmouyal, J. Mahseredjian, "A Combined Directional and Faulted Phase Selector Element Based on Incremental Quantities," *IEEE Transactions on Power Delivery*, vol. 16, issue: 4, pp. 478-484, October 2001.
- [11] L. M. Wedepohl, "Application of Matrix Methods to the Solution of Travelling-Wave Phenomena in Polyphase Systems," *Proceedings of the IEE*, vol. 110, no. 12, pp. 2200-2212, December 1963.
- [12] Leuven EMTP CENTER, *ATP - Alternative Transients Program: Rule Book*. Heverlee, Belgium, July 1987.
- [13] L. Dube, "Models in ATP: Language Manual," [S.l.], 1996.