

A Wideband Strictly Passive Circuit Model of Transformer Windings for the Very Fast Transient Simulation

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Abstract--Power system electromagnetic transient simulation, as well as electromagnetic compatibility analysis needs to build high frequency circuit model for elements and equipments in power system. In this paper, a systemic methodology was proposed to setup the wideband strictly passive circuit model for transformer windings for the very fast transient simulation. To guarantee the simulation stability of the circuit model, the modified vector fitting method was used to approximate the Y -parameters by rational functions which were transformed from S -parameters firstly. And then, the rational functions were optimized by active set method to meet the strictly passive conditions. The π -equivalent circuit model based on Y -parameters was built by means of circuit synthesis method. Moreover, pattern search algorithm was applied to optimize the values of the lumped components of the π -equivalent circuit. The strictly passive circuit model composed of R , L , C , G with only positive values was achieved. The simulation and measured results were presented, confirming the validity of the proposed method.

Keywords: macromodel; strictly passive; active set method; pattern search algorithm; circuit synthesizer; electromagnetic transient simulation.

I. INTRODUCTION

POWER system electromagnetic transient simulation, as well as electromagnetic compatibility analysis needs to build electromagnetic transient model for elements and equipments in power system. Because the transformer type device (including power transformer, potential transformer and current transformer etc.) presents nonlinear behavior as well as frequency-dependent effects under transients, it may be one of the most difficult equipment to model in power system for a transient simulation [1]. Standard EMTP transformer models,

such as BACTRAN, TRELEG, GMTRAN and SEATTLE XFORMER, etc. can accurately be used for transient calculation at power frequency and low frequency (below 10kHz) [2]. However, the main frequency component of lightning over-voltage and very fast transient over-voltage (VFTO) generated in gas insulated substation (GIS) can reach up to a few or tens of Mega Hertz [3]. The given models is far from meeting the requirements of lightning and VFTO transient simulation, therefore, it is necessary to establish the high frequency circuit model of transformer type device.

Macromodeling is an important modeling method mainly used in the Very Large Scale Integrated circuits (VLSI), which adopts black-box techniques to building up port circuit model for elements or apparatus. The general procedure is as follows. The scattering (S) or admittance (Y) parameters are measured or calculated firstly. Then, the frequency domain function model is setup by using rational function approximation [4]. In order to make the circuit model conveniently applied to EMTP, MATLAB or SPICE, etc., a circuit model is further established by circuit synthesis method based on the rational function approximation [1, 5-6].

Establishing macromodel needs to solve an important problem, that is, to guarantee the passivity of the synthesized circuit. The sufficient and necessary condition to realize passive circuit is that the admittance or impedance parameters of the network are positive real rational function matrix [7]. In [8], mandatory correction method of passive fitting for Y -parameter matrix is put forward. This passive criterion is that the real part of the admittance matrix need to be positive definite matrix. The model built based on the passive criterion can ensure macro-passivity of the circuits, which is the port passivity of the circuit. However, it can not guarantee the passivity of each circuit element, meaning that the circuit model is stable, but not necessarily passive. If the model is used as a subsystem in a whole system simulation, the simulation may generate unstable results. In [9], a method based on sequential quadratic programming to ensure the circuit strictly passivity was presented, which optimized the Y -parameters of the printed circuit boards (PCBs). This method is very effective when the order of the rational function approximation of the PCBs's Y -parameters is relatively low. However, when using this method to model the transformer type device for high frequency simulation, due to their higher order rational function approximation of the Y -parameters, the strictly passive condition is difficult to meet.

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In this paper, a systematic methodology for establishing a strictly passive circuit model for transformer windings based on active set method and pattern search algorithm is presented. The approximation of Y -parameters is explained in Part II. In Part III, the passive optimization process is discussed. In Part IV, circuit synthesis method is applied to build the circuit model, and its experimental verification is provided in Part V.

II. APPROXIMATION OF Y -PARAMETERS

The two-port S -parameters of transformer windings was measured by means of spectrum network analyzer, and then transformed into Y -parameters by equation (1).

$$Y = [R(E + S)]^{-1}(E - S) \quad (1)$$

where R is the matched-impedance matrix associated to the S -parameter measurements, E is the identity matrix. The two-port π -equivalent circuit is setup based on the admittance matrix, as shown in Fig.1.

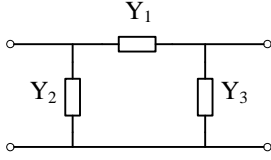


Fig. 1. π -equivalent circuit

According to two-port circuit theory, the relationship between the branch admittance of the π -equivalent circuit and the elements of Y is shown in equation (2).

$$\begin{cases} Y_1 = -Y_{12} = -Y_{21} \\ Y_2 = Y_{11} + Y_{12} \\ Y_3 = Y_{22} + Y_{12} \end{cases} \quad (2)$$

Modified vector fitting method (MVF) is adopted to approximate the circuit Y -parameters [10].

$$Y_{ij}(s) = \sum_{n=1}^{N_r} \frac{r_{rn}}{s - p_{rn}} + \sum_{n=1}^{N_c} \left(\frac{r_{cn}}{s - p_{cn}} + \frac{r_{cn}^*}{s - p_{cn}^*} \right) + h \quad (3)$$

where N_r and N_c are the number of the real poles and complex poles, p_{rn} and r_{rn} are the real poles and residues, p_{cn} , p_{cn}^* , r_{cn} , r_{cn}^* are pairs of complex and conjugate poles and residues, respectively.

III. PASSIVE OPTIMIZATION OF Y -PARAMETERS

A. Strictly passive condition

Strict passivity, also known as the classic passivity, is that each component values composed of circuit are positive, and the circuit does not contain controlled sources. Strictly passivity is an important property of the circuit to guarantee stable simulation under any conditions, because that stable, but not passive macromodels can produce unstable simulation when connected to other stable, even passive loads [11].

To implement strictly passivity, that is the ultimately achieved circuit is constituted by only passive components, expression (3) must fulfill the following conditions [12].

a) Let complex pole $p=a+bi$ and complex residue $r=c+di$, they satisfy the condition $ac \pm bd \leq 0$.

b) The real part of complex residue must be positive.

c) The constant h must be positive.

The relationship between the poles and residues is shown in Fig.2, the shaded figure is the range of residues.

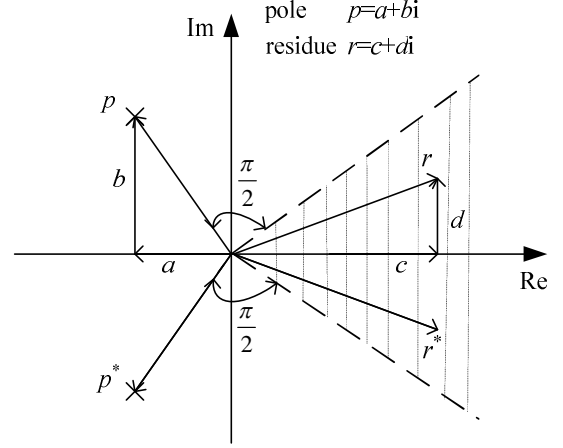


Fig. 2. Relationship between the complex poles and residues

B. Quadratic programming problem

This optimization problem is equivalent to minimizing the error between the optimal network function and the measured results to meet the constraint conditions.

Real poles and residues will be treated as complex and conjugate poles and residues. The order of fitting expression is $2N$. Let the admittance calculated by (1) be $Y=[y_1, y_2, \dots, y_M]$, where M is the number of frequency sampling points. Let k -th pair of pole-residue pair be $p_{2k-1}=a_k \pm b_k i$, $r_{2k-1}=c_k \pm d_k i$, respectively. The variable X to be optimized is $X=[c_1, d_1, \dots, c_N, d_N, h]^T$, and the coefficient matrix A is of dimensions $(2N+1) \times (2N+1)$.

$$A = \begin{bmatrix} a_1 & b_1 & 0 & \cdots & 0 & 0 \\ a_1 & -b_1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & \cdots & a_N & b_N & 0 \\ 0 & 0 & \cdots & a_N & -b_N & 0 \\ 0 & 0 & \cdots & 0 & 0 & -1 \end{bmatrix}$$

$$B = [0, 0, \dots, 0]^T$$

$$E = [0, -\infty, \dots, 0, -\infty, 0]^T$$

$$F = [\infty, \infty, \dots, \infty, \infty, 0]^T$$

where X, B, E, F are column vector of dimensions $(2N+1) \times 1$. These forms the constraints $AX \leq B, E \leq X \leq F$, which are the strictly passive conditions mentioned above. They can be combined as $A_j X \leq B_j$, where A_j is a matrix of dimensions $(6N+3) \times (2N+1)$, B_j is a column vector of dimensions $(6N+3) \times 1$.

Then build the objective function. C is a matrix of dimensions $2M \times (2N+1)$. If $1 \leq i \leq M, 1 \leq j \leq N$,

$$C_{i,2j-1} = \frac{-a_j}{a_j^2 + (\omega_i - b_j)^2} + \frac{-a_j}{a_j^2 + (\omega_i + b_j)^2},$$

$$C_{i,2j} = \frac{(\omega_i - b_j)}{a_j^2 + (\omega_i - b_j)^2} + \frac{-(\omega_i + b_j)}{a_j^2 + (\omega_i + b_j)^2} \cdot$$

$$C_{i,2N+1} = 1.$$

If $M+1 \leq i \leq 2M$, $1 \leq j \leq N$,

$$C_{i,2j-1} = \frac{(b_j - \omega_{i-M})}{a_j^2 + (\omega_{i-M} - b_j)^2} + \frac{-(b_j + \omega_{i-M})}{a_j^2 + (\omega_{i-M} + b_j)^2},$$

$$C_{i,2j} = \frac{-a_j}{a_j^2 + (\omega_{i-M} - b_j)^2} + \frac{a_j}{a_j^2 + (\omega_{i-M} + b_j)^2} \cdot$$

$$C_{i,2N+1} = 0.$$

$$D = [\text{Re}(y_1), \text{Im}(y_1), \dots, \text{Re}(y_M), \text{Im}(y_M)]^T$$

where D is a column vector of dimensions $2M \times 1$, $\text{Re}(\ast)$, $\text{Im}(\ast)$ is to obtain the real and imaginary part of complex \ast , respectively.

According to principle of least squares method, minimize $CX-D$, namely minimize $(CX-D)^T(CX-D) = X^T C^T CX - D^T CX - X^T C^T D = X^T GX + HX$, where $G = C^T C$, $H = -2D^T C$.

Therefore, the optimization problem is equivalent to minimizing $f(x) = X^T GX + HX$ with the inequality constraint $A_j X \leq B_j$, which is quadratic programming problem with inequality constraints.

C. Active set method

The quadratic programming problem with inequality constraints can be solved by the active set method. The basic principle of this method is the setting of a feasible point known as the starting point, the active constraints at the feasible point as the equality constraints, and removing the inactive constraints in each iteration. Under the new constraints, minimize the objective function and search new feasible point repeatedly until the convergence.

Let x_k be the feasible point known after k iterations, I_k be the active constraints at the feasible point. It needs to solve the problem with equality constraints,

$$\begin{cases} \min & f(x) \\ \text{s.t.} & A_i x = b_i, i \in I_k \end{cases}$$

where A_i is the i th row vector in matrix A , b_i is the i th element in vector b . Let the search direction be $d_k = x - x_k$. The problem is equivalent to

$$\begin{cases} \min & \frac{1}{2} d_k^T G d_k + \nabla f(X)^T d_k \\ \text{s.t.} & A_i d_k = 0, i \in I_k \end{cases}$$

This is a quadratic programming problem with equality constraints, which can be solved by direct elimination method or Lagrange multiplier method.

After obtained the optimal solution, process d_k in three cases. (1) If $x_k + d_k$ is the feasible point and $d_k \neq 0$, then $x_{k+1} = x_k + d_k$ after $k+1$ iterations. (2) If $x_k + d_k$ is not the feasible point, then $x_{k+1} = x_k + \lambda_k d_k$, where λ_k is the step and $a_i(x_k + \lambda_k d_k) \geq b_i$ for all $i \notin I_k$. If it exists $a_p(x_k + \lambda_k d_k) = b_p$, then $I_k = I_k \cup \{p\}$. (3) If $d_k = 0$, then x_k is the optimal solution.

Anyway, when dealing with large numbers of poles over a

wideband frequency range, condition a) has proven to be very difficult to be enforced. The nonlinear optimization ends with a Foster expansion that is passive in the whole, but some partial fraction terms are active. In order to overcome this problem, a last optimization is done by means of pattern search algorithm (PSA).

D. PSA optimization

The PSA is a direct search method, which does not need to solve the objective function. Besides, it is easy to achieve directly by calling the PATTERNSEARCH optimization toolbox in MATLAB. The basic idea of PSA is to find the direction and the points which are conducive to the optimal values of the objective function. Here the optimization problem is to minimize the function with n variables $x = \{x_1, x_2, \dots, x_n\}$. The general process is as follows.

Step 1. Let initial point $x^{(k)} \in \mathbb{R}^n$, e_1, e_2, \dots, e_n be the axial direction, δ be the step, accelerated factor $a \geq 1$, reduced rate $\beta \in (0, 1)$, tolerance $\varepsilon > 0$, $y^{(1)} = x^{(1)}$, $j=1$, $k=1$, respectively.

Step 2. If $f(y^{(j)} + \delta e_j) < f(y^{(j)})$, $y^{(j+1)} = y^{(j)} + \delta e_j$. Then, turn to step 4. Otherwise, turn to step 3.

Step 3. If $f(y^{(j)} - \delta e_j) < f(y^{(j)})$, $y^{(j+1)} = y^{(j)} - \delta e_j$. Then turn to step 4. Otherwise, $y^{(j+1)} = y^{(j)}$, and turn to step 4.

Step 4. If $j < n$, $j=j+1$, then turn to step 2. Otherwise, turn to step 5.

Step 5. If $f(y^{(n+1)}) < f(x^{(k)})$, turn to step 6. Otherwise, turn to step 7.

Step 6. Let $x^{(k)} = y^{(n+1)}$, $y^{(1)} = x^{(k+1)} + a(x^{(k+1)} - x^{(k)})$, $k=k+1$, $j=1$, then turn to step 2.

Step 7. If $\delta \leq \varepsilon$, the iteration will be stopped and $x^{(k)}$ will be obtained. Otherwise, $\delta = \beta \delta$, $y^{(1)} = x^{(k)}$, $x^{(k)} = x^{(k+1)}$. Let $k=k+1$, $j=1$, then turn to step 2.

In this paper, let all the component values of the synthesized circuit be the initial point, $x_i > 0$ be the constraints, $i=1, 2, \dots, n$. The objective function of the optimization is

$$\min \left\{ \sum_{k=1}^M \left| Y(\omega_k) - Y^{meas}(\omega_k) \right|^2 \right\} \quad (4)$$

where M is the number of frequency sampling points, $Y(\omega_k)$ is the specified Y -parameters calculated from the equivalent circuit selected by PSA, $Y^{meas}(\omega_k)$ is the Y -parameters derived from the S -parameters.

The PSA is terminated after a prescribed number of iterations (here is 50) and the best points is chosen.

IV. CIRCUIT SYNTHESIS

The admittance expression in (3) can be synthesized in circuit shown in Fig.3.

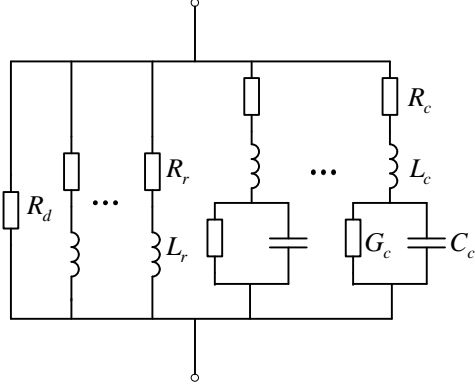


Fig. 3. RLCG equivalent circuit

The component values in Fig.3 are the following:

$$R_r = -\frac{p_r}{r_r} \quad (5)$$

$$L_r = \frac{1}{r_r} \quad (6)$$

$$L_c = \frac{1}{2c_c} \quad (7)$$

$$R_c = \frac{-a_c c_c + b_c d_c}{2c_c^2} \quad (8)$$

$$C_c = \frac{2c_c^3}{b_c^2(c_c^2 + d_c^2)} \quad (9)$$

$$G_c = -\frac{2c_c^2(a_c c_c + b_c d_c)}{b_c^2(c_c^2 + d_c^2)} \quad (10)$$

$$R_d = 1/h \quad (11)$$

Let p_r and r_r be a pair of real pole-residue pair, and $p_c = a_c + b_c i$ and $r_c = c_c + d_c i$ a complex conjugate pole-residue pair, respectively. The two-port equivalent circuit of transformer windings can be obtained by connecting the Y_1 , Y_2 , Y_3 branch circuit in π type.



Fig. 4. Transformer windings

V. EXPERIMENTAL VERIFICATION

Fig.4 shows a typical structure of continuous type transformer windings. The accurate dimension of the windings is as follows: there are 20 sections of coils, each section includes 9 turns; and the height of winding is 315 mm. In addition, the inner diameter and outer diameter are 789 mm

and 935 mm, respectively. Given the symmetry of the winding's distribution, here it was dealt with ten sections of the top part. The two-port S-parameters of transformer windings were measured by means of a spectrum network analyzer (Agilent 4395A). The range of measuring frequency is from 100kHz to 50MHz, and the measurement diagram is illustrated in Fig.5.

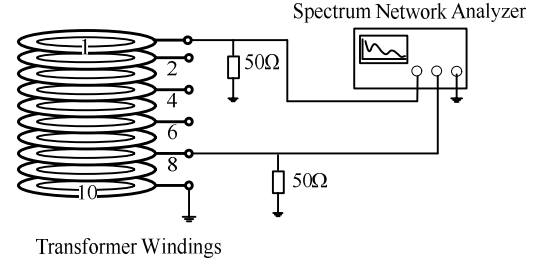
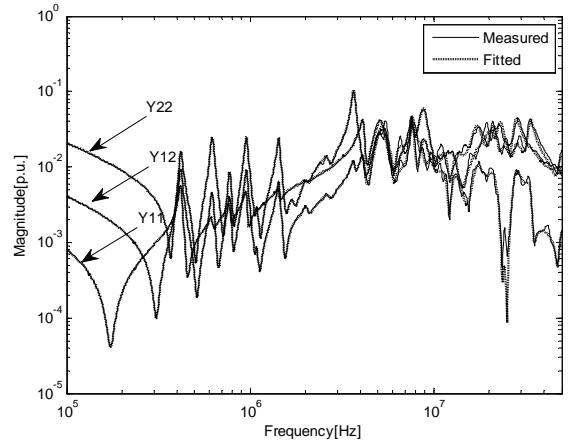
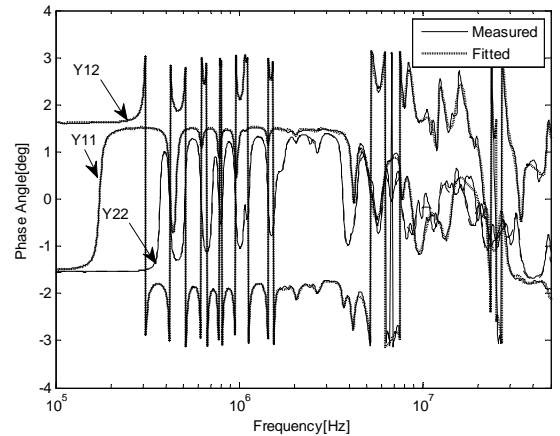


Fig.5. The measurement diagram of scattering parameter

As Y is symmetric matrix ($Y_{12}=Y_{21}$), Y_{11} , Y_{12} , Y_{22} are fitted together by MVF, as shown in Fig.6. Solid and dotted lines represent the measured and fitted curve, respectively. It can be seen from the figure, the fitted rational function has a high accuracy.



(a) Magnitude characteristic



(b) Phase characteristic

Fig. 6. Frequency characteristic of admittance

A π -equivalent circuit model could be built by means of Foster synthesis method based on the data fitted by MVF, which composed of 183 components (R, L, C, G) with 61

negative components. This method can ensure macro-passivity of the circuit model; however, it can not guarantee its strict passivity. The active set method was here applied to optimize the parameters of the rational functions, after building the π -equivalent circuit; the PSA was applied to further optimize the values of the lumped components of circuit. The results obtained from the equivalent circuit showed that the circuit model is synthesized by 170 components with positive values, realizing the strictly passivity of the circuit model. Take Y_1 circuit for example, synthesized element values before and after passive optimization are listed in Table I.

TABLE I
ELEMENT VALUES OF Y_1 CIRCUIT BEFORE AND AFTER PASSIVE OPTIMIZATION

Before optimization	R(Ω)	L(H)	C(F)	G(S)
1	6.00	3.52E-04		
2	-1.35E+03	-2.85E-04		
3	-1.91E+02	1.07E-03	1.33E-10	3.80E-05
4	-2.07E+02	1.67E-03	3.91E-11	1.13E-05
5	-4.78E+02	1.73E-03	2.41E-11	1.15E-05
6	6.80E+01	5.29E-04	5.27E-11	6.26E-06
7	3.87E+02	6.29E-04	1.96E-11	-5.86E-06
8	2.13E+04	1.76E-03	1.51E-12	-1.75E-05
9	9.88E+02	9.75E-05	1.56E-11	-1.37E-04
10	1.85E+01	9.33E-06	9.58E-11	2.06E-04
11	-6.06	5.77E-06	6.73E-11	6.69E-04
12	1.86E+03	-3.83E-05	-3.87E-12	-2.48E-04
13	1.02E+05	2.64E-04	2.50E-14	-9.28E-06
14	-3.24E+02	-8.59E-06	-6.64E-12	9.20E-05
15	-5.25E+04	5.61E-05	1.90E-14	1.83E-05
16	7.22E+04	2.19E-04	2.90E-14	-9.39E-06
Rd	-2.43E+04			
After optimization	R(Ω)	L(H)	C(F)	G(S)
1	6.00	3.52E-04		
2	1.35E+03	1.76E+13		
3	5.62E-06	1.07E-03	1.34E-10	1.44E-05
4	1.23E-05	1.67E-03	3.92E-11	6.53E-06
5	9.01E-06	1.73E-03	2.42E-11	4.85E-06
6	6.80E+01	5.29E-04	5.27E-11	5.18E-06
7	1.99E+02	6.28E-04	1.96E-11	6.25E-21
8	9.82E+02	5.63E+14	2.45E-12	0
9	2.39E+02	9.65E-05	1.82E-11	2.51E-20
10	1.85E+01	9.33E-06	9.58E-11	2.06E-04
11	4.00E+00	5.76E-06	6.77E-11	6.02E-04
12	8.02E+03	3.21E-04	8.58E-13	6.37E-15
13	2.90E+03	2.05E-04	6.40E-13	4.26E-15
14	1.46E-11	2.15E+09	1.24E-13	2.96E-06
15	0	5.37E+08	5.95E-13	6.06E-06
16	2.01E+03	7.04E+13	9.35E-14	0
Rd	1.60E+15			

In order to simulate VFTO, impulse voltage wave generated by nanosecond impulse voltage generator (GMY-1) is exerted on the first section of the transformer windings. Time domain signal of first and eighth section can be gathered by digital oscilloscope (Agilent DSO5034A), and the input waveform is shown in Fig.7. Based on the SIMULINK of MATLAB, the

simulation of π -equivalent circuit model composed by R, L, C and G with positive values can be obtained. The simulated result is compared with the measured data of the output port, namely the eighth section, which is shown in Fig.8. From the figure, simulated 1 and 2 represent simulation waveforms before and after passive optimization, respectively. Both of the effects of the first two oscillation simulations are good, however, the 3rd and 4th oscillation simulations have differences that are probably resulted from measurement or simulation error. In addition, simulated 2 is not good as simulated 1 during the other oscillations, which maybe due to the changes occurs in the residue optimization process. Above all, the overall simulation result is fairly good.

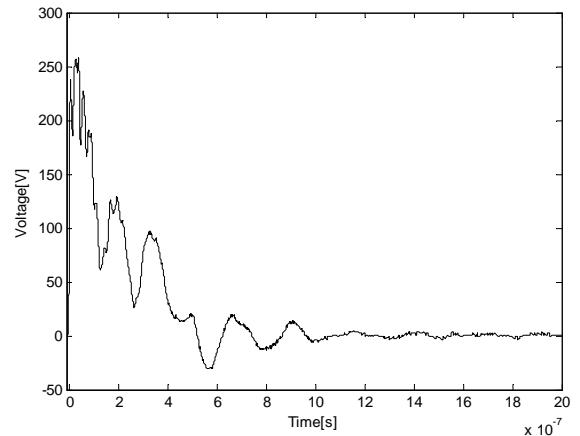


Fig.7. Input waveform

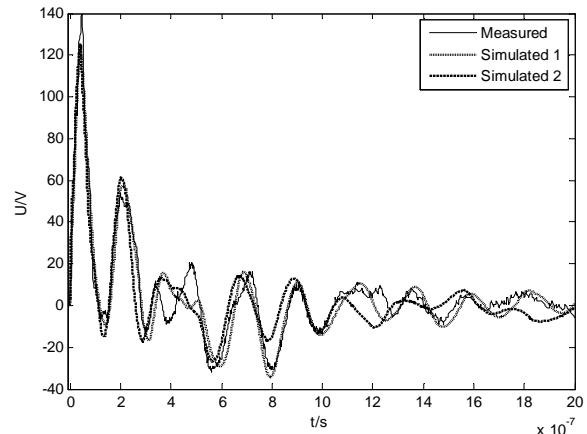


Fig.8. The measurement and simulation waveform of the transformer windings

VI. CONCLUSIONS

This paper has presented a systematic method for establishing wideband strictly passive circuit model of transformer type device for the very fast transient simulation. By now, for building transient circuit model by means of black-box technique, it is difficult to achieve the conditions of strictly passive circuit by the existing rational function approximation method directly. In order to solve this difficulty, active set method was here applied to optimize the parameters of the rational function, PSA was used to further optimize the

values of the lumped components of the equivalent circuit, and strictly passive circuit model was achieved. The simulation and measured results of a practical transformer windings were presented, confirming the validity of the proposed method. This method is suitable for any transformer type device, as long as their parameters of the multi-port network are obtained.

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