Investigation on the Occurrence of Delayed Current Zeros Phenomena in Power Stations and Related Stress Imposed on Generator Circuit-Breakers

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Abstract—The requirements imposed on generator circuit-breakers greatly differ from the requirements imposed on general purpose transmission and distribution circuit-breakers. Due to the location of installation between the generator and the associated step-up transformer a generator circuit-breaker must meet high technical requirements with respect to the interruption of fault currents. In addition to their generally high magnitude, those currents might exhibit delayed current zeros. The present paper specifically investigates the possible occurrence of fault currents which show delayed current zeros. The capability of the generator circuit-breaker to interrupt those currents is ascertained by means of calculations which take into account the effect of the arc-voltage of the circuit-breaker on the prospective fault current.

In order to carry out a more thorough investigation on the interrupting capability of generator circuit-breakers a comparison between different extinguishing technologies is also provided.

Keywords: Generator circuit-breaker, delayed current zeros, arc-voltage.

I. INTRODUCTION

The requirements imposed on generator circuit-breakers greatly differ from the requirements imposed on general purpose transmission and distribution circuit-breakers. Due to the location of installation between the generator and the associated step-up transformer a generator circuit-breaker (hereinafter GenCB) must meet high technical requirements with respect to the interruption of fault currents. In addition to their generally high magnitude, these currents can be characterized by delayed current zeros (hereinafter DCZ).

The present paper specifically investigates the possible occurrence of fault currents which show such a waveshape. In addition to the well known DCZ phenomena associated with generator terminal faults and synchronizing under out-of-phase conditions, a third case is analyzed. The electrical layout of the power station considered for this purpose is depicted in Fig. 1.

It consists of two generators connected to a three-winding step-up transformer via GenCBs. Generator #1 is a gas-turbine generator and generator #2 is the associated steam-turbine generator.

The capability of the GenCB to interrupt fault currents which show DCZ is ascertained by calculations that take into account the effect of the arc-voltage of the circuit-breaker on the prospective fault current. The arc-voltage versus current characteristic of the GenCB is transferred into a mathematical model and implemented in the Electromagnetic Transients Program (EMTP) [1] as a non-linear time-varying resistance. It is then inserted into the simulation at the time of the separation of the contacts of the circuit-breaker.

In order to carry out a thorough investigation on the interrupting capability of GenCBs, a comparison between SF₆ and vacuum extinguishing technologies is also provided.

II. GENERATOR CIRCUIT-BREAKER MODEL ADOPTED FOR THE SIMULATIONS

According to [2], [3], demonstrating the capability of a GenCB to interrupt short-circuit currents with DCZ may be difficult and limited in high power testing stations.

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Fig. 1. Electrical layout of a combined cycle power station with two generators connected to the HV-system by means of a three-winding step-up transformer
Considering that various designs of generators behave differently and that the pre-load of the generator influences the course of the fault current, it can be impossible to simulate the required current shape in the testing station [4]. Therefore the capability of a circuit-breaker to interrupt a short-circuit current with DCZ has to be ascertained by calculations that take into account the effect of the arc-voltage of the GenCB on the prospective short-circuit current. The arc-voltage model to be used for this purpose has to be derived from tests [2]. The GenCB’s arc-resistance is an additional resistance which forces the d.c. component of the short-circuit current to decay faster. It is of utmost importance that the magnitude of the arc-voltage is high enough to force a fast decay of the d.c. component of the short-circuit current, so that current zeros are produced within the maximum permissible arcing time of the GenCB. The arc-voltage of a GenCB depends on different physical quantities, e.g. the instantaneous value of the current and the type of the extinguishing medium, its pressure, the intensity of its flow and the length of the arc. In order to investigate the behaviour of the GenCB during the interruption of short-circuit currents with DCZ, the arc-voltage versus current characteristic has to be transferred into a mathematical model. From the arc-voltage and the current the arc-resistance is obtained. A non-linear time-varying resistance is inserted into the simulation at the time of the separation of the contacts of the circuit-breaker to model the behaviour of the GenCB.

According to [2] the following two cases shall be investigated:

- fault initiation at voltage zero in one phase which implies that the current in the corresponding phase exhibits the maximum degree of asymmetry;
- fault initiation at voltage maximum in one phase which implies that the current in the corresponding phase is symmetrical.

The capability of a GenCB to interrupt a given short-circuit current can be considered as being demonstrated when the following conditions are met:

- the maximum operating voltage is less or equal to the power frequency recovery voltage during the short-circuit test with the corresponding (symmetrical) short-circuit current;
- the symmetrical short-circuit current is less or equal to the symmetrical short-circuit breaking current demonstrated by a short-circuit test;
- the asymmetrical short-circuit current is less or equal to the asymmetrical short-circuit breaking current demonstrated by a short-circuit test;
- the rate-of-rise and the peak value of the transient recovery voltage are less or equal to the rate-of-rise and the peak value of the transient recovery voltage during the short-circuit test with the corresponding (symmetrical) short-circuit current;
- in case of a short-circuit current with DCZ the GenCB is capable of forcing the current to zero within the time interval in which it is able to interrupt a current (i.e. within the maximum permissible arcing time).

In order to correctly simulate the behaviour of the GenCB, the arc-voltage model used for this investigation has been derived from tests (see Fig. 2 and Fig. 3).

### III. GENERATOR TERMINAL FAULTS

The current to be interrupted by the GenCB in case of faults between the terminals of the GenCB and the LV-windings of the step-up transformer is called generator-source short-circuit current. If the fault initiation takes place when the voltage in one phase passes through zero the resulting fault current in that phase exhibits the maximum degree of asymmetry. The symmetrical component decays with the subtransient and transient time constants of the generator; the d.c. component decays with the armature time constant. If the symmetrical component of the fault current decays faster than the d.c. component, it can happen that, for a certain period of time following the initiation of the fault, the magnitude of the d.c. component of the fault current is bigger than the peak value of its symmetrical component. In such a case the degree of asymmetry of the fault current is higher than 100%, thus leading to DCZ. In addition the magnitude of the a.c. component of the generator-source short-circuit current and its degree of asymmetry can vary if the generator is unloaded or delivering power with lagging power factor (i.e. working in the under-excited mode) or leading power factor (i.e. working in the over-excited mode) prior to the fault. The degree of
asymmetry of the generator-fed fault current is typically about 130%. Special attention should be paid if the generator is loaded with leading power factor before fault initiation. In such a case the degree of asymmetry of the fault current can reach very high values. In order to simulate accurately the behaviour of the generator in case it is loaded prior to the fault computer simulations are necessary.

The course of the generator-source short-circuit current in case of a three-phase fault between generator circuit-breaker #2 and the LV-terminals of the step-up transformer (refer to fault location F2 in Fig. 1) is depicted in Fig. 4. Fault initiation takes place at 100 ms and a bolted fault has been assumed (i.e. that there is no arc-voltage at the fault location). The fault initiation occurs at voltage zero in phase A.

Fig. 5 and Fig. 6 show the corresponding calculation results with the GenCB closing into a three-phase fault. In the computation the arc-voltage of a SF$_6$ GenCB starting at contact separation is taken into account. Fig. 5 represents the case with fault initiation at voltage zero and Fig. 6 shows the case with fault initiation at voltage maximum in one phase. In all the calculations it has been assumed that the contacts of the GenCB part 39 ms after fault initiation.

In order to carry out a deeper investigation the interrupting capability of a GenCB employing vacuum extinguishing technology is also analysed.

Following the same procedure as the one applied to SF$_6$ GenCBs, the arc-voltage versus current characteristic has been transferred into a mathematical model and a non-linear time-varying resistance has been inserted into the simulation at the time of the separation of contacts.

Fig. 7 and Fig. 8 show the corresponding calculation results. Fig. 7 represents the case with fault initiation at voltage zero and Fig. 8 shows the case with fault initiation at voltage maximum in one phase.

The interrupting capability of the GenCB in case the generator is delivering power either with lagging or with leading power factor prior to fault has also been assessed. The results of the simulations are summarized in Table I.

It is evident that the fault current in case the generator is delivering power with leading power factor prior to fault cannot be handled by the GenCB based on vacuum technology as the calculated arcing time exceeds the maximum permissible value.
A method sometimes adopted to reduce the arcing time of the circuit-breaker is to introduce an intentional tripping delay. A value in the range of 100 ms – 200 ms is usually sufficient to limit the degree of asymmetry of the fault current at contact separation to values the GenCB can cope with. It has to be noted that this solution would lead to longer fault duration and consequently to severe damages to power station equipment with consequent long downtime for repair. Fault durations exceeding 100 ms are usually sufficient to let the step-up transformer explode in case of internal failures. For this reason many power station operators consider the solution of intentionally delaying the tripping as not recommendable. Therefore the preferred method to handle the delayed current zeros phenomena is to choose a GenCB having an arc-voltage magnitude sufficiently high to force current to zero without the aid of any intentional tripping delay.

**TABLE I**

<table>
<thead>
<tr>
<th>Generator condition prior to fault</th>
<th>Applied voltage at fault initiation</th>
<th>Arcing time (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unloaded</td>
<td>$U_A = 0$</td>
<td>$SF_E$</td>
</tr>
<tr>
<td>Unloaded</td>
<td>$U_A = max$</td>
<td>$Vacuum$</td>
</tr>
<tr>
<td>Delivering power with lagging power factor</td>
<td>$U_A = 0$</td>
<td>16.6</td>
</tr>
<tr>
<td>Delivering power with lagging power factor</td>
<td>$U_A = max$</td>
<td>19.4</td>
</tr>
<tr>
<td>Delivering power with leading power factor</td>
<td>$U_A = 0$</td>
<td>33.6</td>
</tr>
<tr>
<td>Delivering power with leading power factor</td>
<td>$U_A = max$</td>
<td>37.1</td>
</tr>
</tbody>
</table>

**IV. OUT OF PHASE SYNCHRONIZING**

Out-of-phase synchronising occasionally occurs in power plants [5]. The main reasons for out-of-phase synchronising are wiring errors made during commissioning or during maintenance when connecting voltage transformers and synchronising equipment. These wiring errors lead to particular out-of-phase angles, i.e. multiples of 60° el.. Polarity errors at a voltage transformer cause synchronising at 180° el. out-of-phase angle; phase connection errors lead to 60° el. and 120° el. out-of-phase angles. Besides these particular out-of-phase angles any value may be caused by inadequate settings of the synchronising equipment, e.g. due to an incorrect value of the closing time of the circuit-breaker or due to manual synchronisation. The current resulting from out-of-phase synchronising may show DCZ; their causes are totally different compared to generator terminal faults. The rapid movement of the rotor from initial out-of-phase angle $\delta_0$ to $\delta = 0$ results in a very small symmetrical component of the fault current and a dominant d.c. component when the condition of $\delta = 0$ is reached. As the instant when the $\delta = 0$ condition is reached is determined by the movement of the rotor, the inertia constants of turbine, rotor and excitation equipment of the generator are of special importance. Because the fault current to be interrupted by the GenCB is characterized by DCZ it is extremely important to prove that the circuit-breaker by means of its arc-voltage is capable of forcing current to zero within its maximum permissible arcing time. The most important parameters which influence the waveshape of the fault current resulting from out-of-phase synchronizing and the occurrence of DCZ are power plant equipment parameters, out-of-phase angle $\delta_0$, power frequency of the system and instant when the synchronisation is initiated. Even though it is recognized that synchronising with out-of-phase angle up to 180° might occur, [2], [3] cover only requirements for a maximum of 90°. Therefore for the present work simulations referring to such a fault conditions have been performed. The waveshape of the out-of-phase current in case of $\delta_0 = 90°$ is depicted in Fig. 9.

From Fig. 9 it is evident that at the time when $\delta = 0$ (about 170 ms after fault initiation) the fault current is dominated by a d.c. component.

The simulation results are depicted in Fig. 10, Fig. 11, Fig. 12 and Fig. 13. Fig. 10 and Fig. 11 show the course of the fault current in case a SF$_E$ GenCB is employed. Fig. 12 and Fig. 13 show the corresponding results in case of use of a vacuum interrupter. Fig. 10 and Fig. 12 represent the case of synchronization occurring when the voltage across the open contacts of pole A ($U_A$) of the GenCB is zero, while Fig. 11 and Fig. 13 show the case when $U_A$ is at its maximum value. In all the simulations the contacts of the GenCB part 39 ms after fault initiation.
The results summarized in Table II show that the fault current resulting from out-of-phase synchronizing can impose extremely severe interrupting conditions if the GenCB closes when the voltage across its contacts in one pole is at its maximum value and the arc-voltage of the circuit-breaker is not high enough to force current to zero before the condition of $\delta = 0$ is reached.

In modern power stations the protection systems sends the tripping signal to the GenCB before the $\delta = 0$ condition is reached, thus leading to a less severe tripping operation. If the tripping is delayed this might however lead to extremely severe interrupting conditions and even unsuccessful interruption.

It is also shown in published literature that circuit-breakers installed at the HV-side of the step-up transformer may under certain circumstances not be suitable for interrupting fault currents resulting from out-of-phase synchronizing [6].

### V. Fault Occurring at the LV-Terminals of a Three-Winding Step-Up Transformer

In addition to the well known DCZ phenomena associated with generator terminal faults and synchronizing under out-of-phase conditions, a third case is analyzed. The fault current fed by generator #2 in case of a three-phase fault at the LV-terminals of the three-winding step-up transformer (see fault location F1 in Fig. 1) is usually characterized by a very high degree of asymmetry. The waveshape of the fault current is obtained as the superposition of two contributions, i.e. a steady state oscillating at power frequency and a transient one. The course of the latter is dictated by the time constants of the circuit. These delayed current zeros have again totally different causes and are extremely dissimilar in comparison with the fault currents associated with generator terminal faults and out-of-phase synchronizing. The mathematical background of this fault current is thoroughly analyzed and the conditions which make possible the occurrence of such a phenomenon are investigated. The details of this analysis are shown in the Appendix. For simplicity in the simulations only the case of

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<tr>
<td>Unloaded</td>
<td>$U_A = 0$</td>
<td>16.5</td>
</tr>
<tr>
<td>Unloaded</td>
<td>$U_A = \text{max}$</td>
<td>18.2</td>
</tr>
<tr>
<td></td>
<td>$SF_6$</td>
<td>18.9</td>
</tr>
<tr>
<td></td>
<td>Vacuum</td>
<td>206.8</td>
</tr>
</tbody>
</table>
generator unloaded prior to fault has been considered. This approach is justified as the current magnitude is relatively small compared to the generator terminal fault current while the degree of asymmetry can be several times higher. The course of the fault current flowing through generator circuit-breaker #2 in case of a three-phase fault at the LV-terminals of the step-up transformer (refer to fault location F1 in Fig. 1) is depicted in Fig. 14 and in Fig. 15. Fig. 14 represents the case of fault occurring at voltage zero and Fig. 15 shows the case of fault initiation at voltage maximum.

![Fig. 14](image1.png)

**Fig. 14.** Prospective fault current to be interrupted by the generator circuit-breaker #2 in case of three-phase fault located in F1 (refer to Fig. 1) (generator unloaded prior to fault initiation, fault initiation at $U_A = 0$)

The simulation results taking into account the GenCB are depicted in Fig. 16, Fig. 17, Fig. 18 and Fig. 19. Fig. 16 and Fig. 17 show the course of the fault current in case a SF$_6$ GenCB is employed. Fig. 18 and Fig. 19 show the corresponding results in case of use of a vacuum interrupter. Fig. 16 and Fig. 18 represent the case of fault initiation at voltage zero, while Fig. 17 and Fig. 19 show the case when the voltage is at its maximum value.

![Fig. 16](image2.png)

**Fig. 16.** Interruption of fault current occurring at the LV-terminals of the three-winding step-up transformer with a SF$_6$ GenCB (generator unloaded prior to fault initiation, fault initiation at $U_A = 0$, arcing time = 13.5 ms)

![Fig. 17](image3.png)

**Fig. 17.** Interruption of fault current occurring at the LV-terminals of the three-winding step-up transformer with a SF$_6$ GenCB (generator unloaded prior to fault initiation, fault initiation at $U_A = \text{max}$, arcing time = 14.9 ms)

![Fig. 18](image4.png)

**Fig. 18.** Interruption of fault current occurring at the LV-terminals of the three-winding step-up transformer with a vacuum GenCB (generator unloaded prior to fault initiation, fault initiation at $U_A = 0$, arcing time = 138.4 ms)

A possible mitigation method consists in adjusting the

In this specific fault case a GenCB having an arc-voltage of small magnitude is not suitable for interrupting the resulting currents.

The results summarized in Table III show that the arcing time is highly depending upon the magnitude of the arc-voltage and therefore upon the interrupting technology of the GenCB.
settings of the protection system so that the circuit-breaker installed at the HV-side of the step-up transformer clears the fault current before the intervention of the GenCB. This condition has been simulated by assuming that the HV circuit-breaker clears the fault current 40 ms after fault initiation. Fig. 20 and Fig. 21 depict the course of the current flowing through the GenCB influenced by the tripping operation performed by the HV circuit-breaker. Fig. 20 represents the case of fault occurring at voltage zero and Fig. 21 shows the case of fault initiation at voltage max in phase A.

The intervention of the HV circuit-breaker is reflected in a reduced degree of asymmetry and at the same time in a higher magnitude of the fault current flowing through the generator circuit-breaker #2. This solution relies on a correct operation of the HV circuit-breaker.

![Graph](image)

**Fig. 20.** Prospective fault current to be interrupted by the generator circuit-breaker #2 in case of three-phase fault located in F1 (refer to Fig. 1) (generator unloaded prior to fault initiation, fault initiation at \( U_A = 0 \), interruption of the current flowing through the HV circuit-breaker 40 ms after fault initiation)

Therefore also in this case the preferred method to cope with DCZ is to choose a GenCB having an arc-voltage magnitude sufficiently high to force current to zero within a relatively short time.

### VI. Conclusions

The possible occurrence of fault currents in power stations which show DCZ has been investigated. In addition to the well known DCZ phenomena associated with generator terminal faults and synchronizing under out-of-phase conditions the case of faults occurring at the LV-terminals of a three-winding step-up transformer has also been analyzed. The capability of the GenCB to interrupt fault currents which show DCZ has been investigated by calculations that take into account the effect of the arc-voltage of the GenCB on the prospective fault current. In order to carry out a more thorough investigation on the interrupting capability of GenCBs a comparison between SF\(_6\) and vacuum extinguishing technologies has been made.

The results of the simulation are summarized in Fig. 22. In all the cases analyzed the application of a vacuum circuit-breaker results in longer arcing times compared to the SF\(_6\) device. The GenCB employing SF\(_6\) as extinguishing medium is suitable for the application as the calculated arcing time lies well below the maximum permissible value. In case of the generator terminal fault with the generator delivering power with leading power factor prior to fault, the 90° out-of-phase synchronization and the three-winding step-up transformer LV-terminal fault the vacuum circuit-breaker is not suitable as it is not capable of forcing the current to zero within the permissible arcing time.

In all these cases the fault occurring at voltage maximum in

### Table III

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<tbody>
<tr>
<td>Unloaded</td>
<td>( U_A = 0 )</td>
<td>13.5</td>
</tr>
<tr>
<td>Unloaded</td>
<td>( U_A = \text{max} )</td>
<td>14.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>199.4</td>
</tr>
</tbody>
</table>

![Graph](image)

**Fig. 21.** Prospective fault current to be interrupted by the generator circuit-breaker #2 in case of three-phase fault located in F1 (refer to Fig. 1) (generator unloaded prior to fault initiation, fault initiation at \( U_A = \text{max} \), interruption of the current flowing through the HV circuit-breaker 40 ms after fault initiation)
one phase leads to a longer arcing time compared to the case of a fault occurring at voltage zero.

The preferred method in all three cases is to choose a GenCB having an arc-voltage magnitude sufficiently high to force the current to zero within the maximum permissible arcing time.

### VII. APPENDIX

The circuit used to analyze the course of the current in case of a three-phase fault occurring at the LV-terminals of a three-winding step-up transformer is depicted in Fig. 23. $V$ is the rms value of the voltage source, $L_1$, $L_2$ and $L_3$ are the equivalent inductances and $R_1$, $R_2$ and $R_3$ are the equivalent resistances of the circuit. In case of a fault at the terminals of the inductance $L_3$ the current $i_f(t)$ is obtained as the superposition of two contributions, i.e. a steady state oscillating at power frequency and a transient one whose course is dictated by the time constants of the circuit.

The component of $i(t)$ oscillating at power frequency can be expressed by the following formula:

$$i_1(t) = \sqrt{2}V_1 \cos(\alpha t + \phi)$$  \hspace{1cm} (1)

where

- $i_1(t)$ is the component of $i(t)$ oscillating at power frequency;
- $V_1$ is the phasor representing the voltage source;
- $Z_1$, $Z_2$, and $Z_3$ are the impedances of the three branches of the circuit;

The circuit used to analyze the course of the current in case of a three-phase fault occurring at the LV-terminals of a three-winding step-up transformer is shown in Fig. 23.

![Fig. 23. Circuit used to analyze the course of the current in case of a three-phase fault occurring at the LV-terminals of a three-winding step-up transformer](image)

The preferred method in all three cases is to choose a GenCB having an arc-voltage magnitude sufficiently high to force the current to zero within the maximum permissible arcing time.

The preferred method in all three cases is to choose a GenCB having an arc-voltage magnitude sufficiently high to force the current to zero within the maximum permissible arcing time.
From (2) it is evident that the magnitude of \( i_1'(t) \) depends on the values of \( R_2 \) and \( L_2 \). In case branch no. 2 is an open circuit (i.e. \( Z_2 \) tends to infinity) \( I_1 \) can be expressed by the following expression:

\[
I_1 = \frac{V}{Z_1 + Z_3}
\]  

(3)

This condition is depicted in Fig. 20 and Fig. 21 where the magnitude of the current flowing through the generator circuit-breaker #2 increases after the tripping operation performed by the HV circuit-breaker.

In all practical applications the transient component of \( i(t) \) can be expressed by the following equation:

\[
i_1''(t) = \frac{L_2}{A}(L_1 I_{01} + L_3 I_{03}) \left[ \left( \frac{R_2}{L_2} + s_a \right) e^{s_a t} - \left( \frac{R_2}{L_2} + s_b \right) e^{s_b t} \right]
+ \frac{L_3}{A}(L_1 I_{01} - L_2 I_{02}) \left[ \left( \frac{R_3}{L_3} + s_a \right) e^{s_a t} - \left( \frac{R_3}{L_3} + s_b \right) e^{s_b t} \right]
\]

(4)

where

\[
i_1''(t) \text{ is the transient component of } i(t);
\]

\[
s_a = -\frac{B}{2A} + \frac{\sqrt{B^2 - 4AC}}{2A};
\]

\[
s_b = -\frac{B}{2A} - \frac{\sqrt{B^2 - 4AC}}{2A};
\]

\[
A = L_2(L_1 + L_3) + L_1 L_3;
\]

\[
B = L_1 R_3 + L_3 R_1 + L_2(R_1 + R_3) + R_2(L_1 + L_3);
\]

\[
C = R_1 R_3 + R_2 R_1 + R_2 R_3;
\]

\[
B^2 - 4AC > 0;
\]

(10)

\( I_{01}, I_{02} \) and \( I_{03} \) represent the initial conditions of the transient component of the fault current in the three branches of the circuit.

In real applications the terms \( A, B, C \) are positive and assume the same values for the branches of the circuit. Therefore also \( s_a \) and \( s_b \) assume the same values for the three branches of the circuit. As the circuit parameters cannot be negative, the quantities \(-B + \sqrt{(B^2 - 4AC)}\) and \(-B - \sqrt{(B^2 - 4AC)}\) are negative. The resulting waveshape is obtained as the composition of two decaying exponential functions. This waveshape is overdamped as the damping contribution prevails over the oscillating one.

The parameter \( L_3 \) has a decisive influence on the values of \( s_a \) and \( s_b \) and therefore on the course of \( i_1''(t) \). Above certain values (e.g. in case of three-winding step-up transformers) it can lead to the waveshapes depicted in Fig. 14 and Fig. 15.

Assuming \( R_3 \) and \( L_3 \) negligible, \( i_1''(t) \) is reduced to the well-known formula

\[
i_1''(t) = I_{01} e^{-\frac{t}{\tau}}
\]

(11)

where \( \tau = L_1/R_1 \).

VIII. REFERENCES