A Steady-State and Time-Domain Model of the Coupling between Electrically Conductive Structures

J.-G. Roumy, O. Saad, J. Michaud

Abstract-- **This paper presents the theory and methods used to create a tool for the steady-state and time-domain modeling of the inductive and capacitive coupling between multiple neighbouring electrically conductive structures. Several different types of overhead or underground structures can be modeled such as lines, cables, telecommunication lines, pipelines, metallic fences or other generic structures. First each structure's impedance and admittance matrices are computed using their geometrical and electrical characteristics. Then system matrices are assembled which include the mutual influence coefficients between each conductor of each structure. A robust modal decomposition algorithm is implemented that will handle all cases even when repeated eigenvalues are found, so that a pi-exact representation of the coupled system can be created. The model is compiled as a COM library so that it can be used with a commercial simulation software.**

Keywords : **EMTP, Coupling, Electromagnetic Compatibility, Induction, Transformation Matrix, Modal Decoupling.**

I. INTRODUCTION

Nowadays power structures are omnipresent in the landscape, snaking their way across long distances. Because of the strong AC currents going through the conductors, these structures can generate electro-magnetic fields that have the potential to influence neighbouring parallel running conductive structures. Modeling and analyzing this coupling is very important. It can pose serious security risks, for example due to the voltage induced on a metallic fence. Induced noise on a telecommunications line can also cause serious reliability issues.

While there exist a few specialized software that model the coupling between power structures and neighbouring installations, most are either dating or are specific to a certain type of installation. The problem with older software is they often lack flexibility and are not supported anymore, or lack precision due to the limited computing power available when first developed. More specialized software will be great at doing one thing, such as computing the noise level induced on a telecommunications line by a power line, but do not offer any other modeling capability.

We therefore created a tool to model the coupling between several different types of electrically conductive structures such as overhead power lines, underground cables, telecommunication cables, pipelines or other generic metallic structures. Furthermore, this model can also be used to accurately model the electrical characteristics of a lone structure if so desired. First we compute each structure's impedance and admittance matrices using their geometrical and electrical characteristics. Then we assemble system matrices which include the mutual influence coefficients between each conductor of each structure. The model can output the R, L, G and C system matrices for a steady-state analysis, or create an exact-pi representation of the coupled system for a time-domain simulation.

II. IMPLEMENTATION

We elected to implement the model in the Matlab environment, allowing us to make full usage of this powerful tool's built-in toolboxes and matrix manipulation functions. Furthermore, since this model is to be integrated in a simulation software such as EMTPWorks, we compiled the model as a Microsoft Component Object Model, essentially a registered DLL object which can be accessed from any executable program. We can therefore use the Javascript engine provided in EMTPWorks to call the object with our data, perform the model computation and load the result into circuit elements.

III. INPUT DATA

The modeled structures can be of different nature, such as overhead transmission lines, aboveground or underground cables, pipelines, telecommunication lines or cables, metallic fences or of some other generic nature, and each can include one or more conductors. Power structures can be labeled either as disruptive or disrupted, depending on whether they influence, or are influenced by, other structures. Other types such as telecommunication lines, pipelines, fences, etc… can only be labeled as disrupted. We created input forms for each of these structures in the EMTPWorks environment, through which the user specifies its electrical and geometrical parameters. For a generic structure, the user must specify directly the per-unit-length R', L', G' and C' matrices as well as its geographical location.

When computing the coupling between them, the model assumes that all structures are parallel and of the same length in order to apply the transmission lines theory. To ensure this, we use a so-called 'decoupage' algorithm that takes as input the geographical coordinates of all structures involved and creates groups of smaller, parallel subsegments. All subsegments from disrupted structures within the influence

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area of a disruptive structure are converted to subsegments parallel to this structure to form groups, which may contain one or more disrupted or disruptive subsegments.

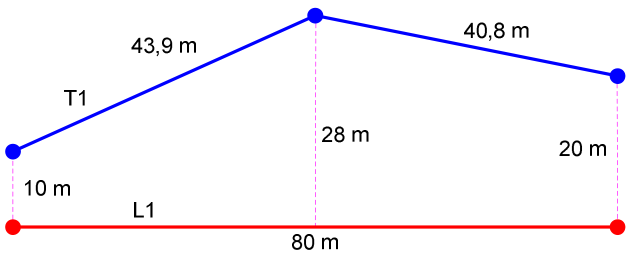

Fig. 1. Coupled structures geographical layout (viewed from above)

Fig. 1 shows a power line L1 running close to a telecommunications cable T1. Since the two structures are not exactly parallel to each other, they must be divided into groups with parallel subsegments as show in Fig. 2. The mean distance between the two subsegments is kept.

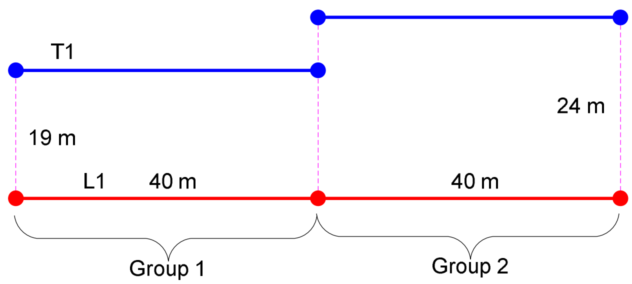

Fig. 2. Coupled structures parallel equivalents, assembled into groups

IV. Structure Modeling

For each type of structure, we must determine its per unit length impedance matrix

$$Z' = R' + j\omega L' \tag{1}$$

and per unit length admittance matrix

$$Y' = G' + j\omega C' \tag{2}$$

A. Overhead Lines

The per unit length series impedance matrix Z' is formed of the diagonal self impedance terms and the mutual coefficients between conductors. The self coefficient are given by

$$Z'_{ii} = Z'_{internal} + Z'_{earth} \tag{3}$$

where $Z'_{internal}$ is the surface impedance of the conductor obtained from the skin effect formula and given with modified Bessel functions [1]. The earth return impedance and the ground return mutual impedance are found using the Deri-Dubanton [2][3] formulas :

$$Z'_{earth} = j\omega\frac{\mu_0}{2\pi}\ln\frac{2(h+p)}{r} \tag{4}$$

$$Z'_{ij} = j\omega\frac{\mu_0}{2\pi}\ln\frac{\sqrt{(h_i+h_j+2p)^2 + x_{ij}^2}}{d_{ij}} \tag{5}$$

Where

$$p = \sqrt{\frac{\rho}{j\omega\mu}} \tag{6}$$

and h_i and h_j are the height of conductors i and j, r the conductor radius, x_{ij} the horizontal separation between the conductors and d_{ij} the total distance between them. μ is the insulation permeability and ρ the conductor resistivity.

The per unit length admittance matrix for overhead lines is assembled from the conductance G' and capacitance C' matrices. We can consider that the influence of the shunt conductance G' is negligible on lines, except at low frequencies approaching DC where it is estimated at $0.2\text{x}10^{-12}$ S/m. The capacitance matrix is obtained from the inverse of the potential matrix P', with the self and mutual coefficients of P' computed using

$$P'_{ii} = \frac{1}{2\pi\varepsilon_0}\ln\frac{2h_i}{r_i} \tag{7}$$

$$P'_{ij} = \frac{1}{2\pi\varepsilon_0}\ln\frac{D_{ij}}{d_{ij}} \tag{8}$$

Where ε_0 is the permittivity of free space, d_{ij} the distance between two conductors and D_{ij} the distance between one conductor and the image of the other in the ground.

B. Overhead / underground cables

The cables parameters of a coaxial arrangement are derived as equations for coaxial loops, each loop being formed with an inner conductor and outer conductor as return. The outermost conductor uses the earth for return. Each loop equation therefore has the form

$$Z'_{loop} = Z'_{inner} + Z'_{insulation} + Z'_{outer} \tag{9}$$

Where the insulation impedance is given by

$$Z'_{insuation} = \frac{j\omega\mu}{2\pi}\ln\frac{r}{q} \tag{10}$$

With r and q respectively the outer and inner insulation radii.

The internal impedance and the mutual impedance of a tubular conductor are function of frequency due to the skin effect, and are found with modified Bessel functions [1].

For above ground cables, the earth return impedance is computed the same way as for lines. When cables are underground, the mutual and self earth return impedance is computed using the notion of complex depth [4] with :

$$Z'_{mutual} = \frac{\rho m^2}{2\pi}\left\{K_0(md) + \frac{2}{4+m^2x^2}e^{-\ell m}\right\} \tag{11}$$

$$Z'_{earth} = \frac{\rho m^2}{2\pi}\left\{K_0(mR) + \frac{2}{4+m^2R^2}e^{-2hm}\right\} \tag{12}$$

where ℓ is the vertical distance between conductor i and the image of conductor j in the air, and x the horizontal distance between the conductors. K_0 is a modified Bessel function to compute the skin effect, and m = p^{-1} as defined previously

To use the equations in a form suitable for EMTP models, we must transform the loop voltages and current to phase quantities knowing that $V_{phase} = V_{inner} - V_{outer}$ and the phase current I_{phase} is the sum of all the conductor currents of the phase.

C. Pipelines

The per-unit-length impedance of a pipeline can be modeled as the sum of the ground resistance R'_{gnd} and the tube internal impedance Z'_{int} [5]. The self coefficients are given by :

$$R'_{gnd} = \frac{\mu_0 \omega}{8} \quad (13)$$

$$Z'_{int} = \frac{\rho_c}{S_{ca}} \quad (14)$$

Where S_{ca} is the surface of the tube section where current is flowing. This section is computed using the tube diameter and the complex depth of penetration of the current in the tube. ρ_c is the resistivity of the tube material.

The mutual resistance is simply the ground resistance as computed in (13). The mutual inductance between two tubes is similar to the self inductance (imaginary part of (14)), replacing the tube radius with the distance between the two tube axes.

The per unit length admittance matrix for pipelines is assembled from the conductance G' and capacitance C' matrices. Only for underground conductors, the self conductance is computed from the tube insulation resistance per unit length

$$G'_{ii} = \frac{\pi d}{R_{isol}} \quad (15)$$

where d is the tube diameter and R_{isol} the insulation resistance per unit surface. There is no conductance for overhead conductors.

In the case of an underground pipe, the imaginary part of the total admittance is computed using the tube self capacitance, which is a cylindrical capacitor. Since there is no mutual capacitance, therefore no off-diagonal element, the capacitance matrix can be computed directly using

$$C'_{ii} = \frac{\pi \varepsilon_0 \varepsilon_r d}{\delta_i} \quad (16)$$

Where δ_i is the insulation thickness and ε_0 and ε_r are the air and insulation permittivity.

For overhead pipes. since there will be mutual capacitance between the conductors, it is necessary to compute the potential coefficients. The self and mutual coefficients are computed using

$$P'_{ii} = \frac{\ln\left(\frac{h_i + \sqrt{h_i^2 + a^2}}{a}\right)}{2\pi\varepsilon_0} \quad (17)$$

$$P'_{ij} = \frac{\ln\left(\frac{D_{ij}}{d_{ij}}\right)}{2\pi\varepsilon_0\varepsilon_r} \quad (18)$$

Where a is the tube radius.

Finally the capacitance matrix is computed as the inverse of the potential matrix.

D. Fence structures

The per unit length impedance matrix of a metallic fence is computed from its linear resistance and inductance matrices [6]. The linear resistance R' is computed using

$$R' = \frac{\rho_{wire}}{S_{wire} N_{wire}} \quad (19)$$

Where ρ_{wire} is the wire resistivity, S_{wire} the wire section and N_{wire} the number of wires constituting the fence.

The linear inductance is computed using the complex images theory. Considering $\overline{p}$ is the depth of penetration of current in the ground :

$$\overline{\rho} = \sqrt{\frac{\rho_{gnd}}{j\omega\mu_0}} \qquad k = \frac{|\overline{p}|}{h_{eq}\sqrt{2}} \quad (20)$$

The inductance is given as :

$$L'\omega = \frac{\mu_0}{2\pi}\left[\ln\left(\frac{2h_{eq}}{r_{eq}}\right) + \ln\sqrt{(1+k)^2 + k^2} + j \cdot \arctan\left(\frac{k}{1+k}\right)\right] \quad (21)$$

Where h_{eq} and r_{eq} are respectively the average conductor height above ground and the mean radius of the beam comprising all conductors.

The fence conductance is computed from the post resistance R_{piq} and the ground resistance R_{gnd} due to their burying.

$$G' = \frac{1}{d_{piq}\left(R_{piq} + R_{gnd}\right)} \quad (22)$$

where d_{piq} is the distance between posts, and

$$R_{piq} = \frac{\rho_{piq}}{S_{piq}} h_{piq} \quad (23)$$

$$R_{gnd} = \frac{\rho_{gnd}}{2\pi D_{piq}}\left[\ln\left(\frac{4D_{piq}}{\sqrt{\frac{S_{piq}}{\pi}}}\right) - 1\right] \quad (24)$$

ρ_{piq} is the posts resistivity, S_{piq} their section, h_{piq} their height above ground. D_{piq} represents the depth at which the posts are buried.

The fence linear capacitance is given by :

$$C' = \frac{2\pi\varepsilon_0}{\ln\left(\frac{2h_{eq}}{\sqrt{\frac{S_{piq}}{\pi}}}\right)} \quad (25)$$

V. SYSTEM MATRICES

Using modeling techniques consequent with its nature, we compute each structure's per-unit-length phase impedance and admittance matrices independently of other structures. We then assemble coupled system matrices and compute the structures' mutual influence coefficients to obtain M-phase admittance and impedance matrices, where M represents the sum of the conductor count from all structures.

An n-structure system per-unit-length admittance matrix has the form

$$Y' = \begin{bmatrix} Y'_1 & \cdots & Y'_{n1} \\ \vdots & \ddots & \vdots \\ Y'_{1n} & \cdots & Y'_n \end{bmatrix} \quad (26)$$

With Y'_i representing the ith structure self admittance matrix, and Y'_{ij} representing the mutual impedance coefficients matrix between the ith and jth structure conductors.

There is no mutual admittance between underground structures, or between an underground and overhead structure. In the case of overhead structures, there will be mutual capacitance between the conductors

$$Y'_{mut} = j\omega C'_{mut} \quad (27)$$

An n-structure system per-unit-length impedance matrix has the form

$$Z' = \begin{bmatrix} Z'_1 & \cdots & Z'_{n1} \\ \vdots & \ddots & \vdots \\ Z'_{1n} & \cdots & Z'_n \end{bmatrix} \tag{28}$$

With Z'_i representing the i^{th} structure self impedance matrix, and Z'_{ij} representing the mutual impedance coefficients matrix between the i^{th} and j^{th} structure conductors.

In the case of underground structures, there is mutual inductance between the conductors. The mutual inductance coefficients between the conductors are computed using the same equations and methods as in the case of parallel underground cables, as shown in (11).

If shielded structures are involved, the value of the mutual inductance applies to the structure outer conductor.

For overhead structures, there is mutual inductance between the conductors. The mutual inductance coefficients between the conductors are computed using the same equations and methods as in the case of parallel overhead lines, where the value of the mutual impedance between two conductors is computed as shown in (5)

In the case where overhead shielded structures are involved (such as telecommunication cables), the value of the mutual inductance applies to the structure outer conductor.

In the case of mixed overhead / underground structures, there is mutual inductance between the conductors. Assuming the j^{th} conductor is in the air and the i^{th} one in the earth, consider the action of the j^{th} circuit on the i^{th} one to compute the mutual impedance. The mutual impedance coefficients are computed using the Lucca [7] approximation of the exact integral :

$$Z'_{ij} = j\omega\frac{\mu_0}{2\pi}\left(\ln\frac{\bar{d}}{d} - \frac{2\bar{y}}{3y^3}\left[\frac{\bar{y}^2 - 3a^2}{\bar{d}^6}\right]\right) \tag{29}$$

with the earth conductivity $\sigma_e = \frac{1}{\rho}$ and the horizontal distance between the conductors denoted as a.

$k_e = -j\omega\mu_0\sigma_e$ and $\gamma = jk_e$

$\bar{y} = h_i - h_j + \frac{2}{\gamma}$

$\bar{d} = \bar{y}^2 + a^2$

VI. Transformation Matrix

To facilitate the solution of the M-phase system coupled equations, they can be transformed into M decoupled equations, which can then be solved as single-phase equations.

Modal decomposition is the adopted theory to decouple a system. Starting from the voltage current relation :

$$\begin{aligned} \frac{dV_{phase}}{dx} &= -Z'_{phase} \cdot I_{phase} \\ \frac{dI_{phase}}{dx} &= -Y'_{phase} \cdot V_{phase} \end{aligned} \tag{30}$$

By differentiating the first equation with respect to x and replacing the current derivative with the second equation, a second-order equation for voltage only is obtained :

$$\frac{d^2V_{phase}}{dx^2} = Z'_{phase} \cdot Y'_{phase} \cdot V_{phase} \tag{31}$$

Similarly a second-order equation can be developed for the current :

$$\frac{d^2I_{phase}}{dx^2} = Y'_{phase} \cdot Z'_{phase} \cdot I_{phase} \tag{32}$$

We can find voltage and current transformation matrices such that

$$\begin{aligned} V_{mode} &= T_v^{-1} \cdot V_{phase} \\ I_{mode} &= T_i^{-1} \cdot I_{phase} \end{aligned} \tag{33}$$

The frequency-dependent modal voltage (T_v) and current (T_I) transformation matrices are respectively the column eigenvectors of the matrix products $Z'_{phase} \cdot Y'_{phase}$ and $Y'_{phase} \cdot Z'_{phase}$.

The current transformation matrix is related to the voltage transformation matrix through :

$$T_i = \left[T_v^t\right]^{-1} \tag{34}$$

It is therefore sufficient to calculate only one of the transformation matrices. However, this bi-orthonormality cannot be satisfied [8] unless the voltage and current modal transformation matrices themselves form an orthonormal basis. This is true for the vast majority of cases encountered in the power system studies where the per unit phase admittance and impedance matrices are either Hermitian or normal [9][10]. This type of matrix ensures that the eigenvalues are distinct and their associated eigenvectors are unique and linearly independent.

Problems arise when the matrix product $Z'_{phase} \cdot Y'_{phase}$ has repeated eigenvalues [11]. There are cases when some eigenvalues are repeated yet a linearly independent set of *n* eigenvectors can be found. There are however other cases with repeated eigenvalues where the eigenvectors (columns of T_v and T_i) corresponding to those repeated eigenvalues are not unique. The eigenvectors associated with a repeated set of eigenvalues can be transformed with a nonsingular transformation to another set which retains the ability to decouple the second-order equations. Structures that exhibit certain types of symmetry (such as pipe type cables) can result in repeated eigenvalues and hence give rise to this nonunique assignment of the columns of T_v or T_i associated with those repeated eigenvalues. The nonunique assignment of these eigenvectors will not affect the diagonalization of the second-order equations but will affect the diagonalization of the first order equations for Z'_{phase} and Y'_{phase}. For this type of structure the eigenvector matrices T_v and T_i need to be evaluated separately.

The eigenvalues and eigenvectors are evaluated using the Matlab command *eig*. This function uses the well known Lapack [12] routines to evaluate the eigenvalues and eigenvectors. The *eig* function is very robust and provides the correct solution [13] when a matrix has no repeated eigenvalues. The eigenvectors themselves are always independent and the eigenvector matrix V diagonalizes the original matrix A if applied as a similarity transformation. However, if a matrix has repeated eigenvalues, it is necessary to find a full (independent) set of eigenvectors. If the eigenvectors are not independent then the original matrix is said to be defective. Even if a matrix is defective, the solution from *eig* satisfies A*X = X*D. Imposing this condition in the *eig* function results in incorrect selection of the eigenvectors associated with the repeated roots. Thus a more rigorous approach has to be investigated for the evaluation of T_v and T_i.

The number of repeated eigenvalues (geometrical multiplicity) is determined and the largest possible set of linearly independent eigenvectors is evaluated using the *rref* Matlab function. This function produces the reduced row echelon form of ZY (for T_v) and YZ (for T_i) using Gauss Jordan elimination with partial pivoting [13]. A tolerance variable is included in the reduced row echelon form computation that allows the exact number of eigenvectors to be found for each eigenvalue depending on how many times it is repeated, smoothing out oscillations when too many or two few vectors are found for a particular eigenvalue.

This algorithm works in all practical cases. The only draw back is the eigenvector matrices T_v and T_i must be evaluated separately and the bi-orthonormality property cannot be used to avoid the matrix inversion of T_i and T_v.

The modal impedances and admittances are specified in conjunction with the eigenvectors used in their calculation. To obtain them, transform equation (30) to modal quantities :

$$-\frac{dV_{mode}}{dx} = T_v^{-1} \cdot Z'_{phase} \cdot T_i \cdot I_{mode} \tag{35}$$

The triple matrix product in (35) is diagonal [14] representing the elements of the modal series impedance. This modal impedance becomes :

$$Z'_{mode} = T_v^{-1} \cdot Z'_{phase} \cdot T_i \tag{36}$$

Similarly, the modal shunt admittance matrix is given by :

$$Y'_{mode} = T_i^{-1} \cdot Y'_{phase} \cdot T_v \tag{37}$$

VII. Pi-Exact Model

To use the model in an EMTP-type software for time-model simulations, it is implemented in terms of Y-matrix representations of the series and shunt branches of a multiphase pi-circuit.

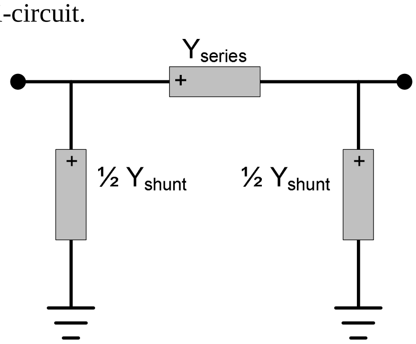

Fig. 3. Coupled Circuit representation of a pi-exact model

For each mode *j* of a multiphase line, the equivalent-pi representation as shown in fig is :

$$Y_{series} = \frac{1}{Z_{series}} \text{ with } Z_{series} = \frac{ZY_{mode}}{Y_{mode}} \frac{\sinh(\gamma\ell)}{\gamma} \tag{38}$$

and

$$\frac{1}{2} Y_{shunt} = Y_{mode} \frac{\tanh\left(\frac{\gamma\ell}{2}\right)}{\gamma} \tag{39}$$

where γ is the propagation constant.

$$\gamma = \sqrt{ZY_{mode}} \tag{40}$$

To obtain an equivalent M-phase pi-circuit, the phase quantities are first transformed to modal quantities as shown previously. For each mode an equivalent single-phase pi-circuit is then found in the same way as for single-phase lines. These single-phase modal pi-circuits each have a series admittance $Y_{series-mode}$ and two equal shunt admittance $\frac{1}{2} Y_{shunt-mode}$. By assembling these admittances as diagonal matrices, the admittance matrices of the M-phase PI-circuit in phase quantities are obtained from

$$\begin{aligned} Y_{series} &= T_i \cdot Y_{series-mode} \cdot T_v^{-1} \\ \frac{1}{2} Y_{shunt} &= T_i \cdot Y_{shunt-mode} \cdot T_v^{-1} \end{aligned} \tag{41}$$

This representation can be loaded directly into EMTPWorks devices, allowing us to perform time-domain simulations to study the resulting waveforms on all modeled structure conductors.

VIII. Validation

Since this new tool can be used to perform inductive and capacitive coupling computations for different types of structures, we used a variety of established softwares (such as EMTP software, electromagnetic compatibility tools and induced noise on telecommunication lines calculators) to compare and validate the results. The validation process showed very good concordance with the references for all tested configurations. We also tested several cases where the matrix product $Z'_{phase} \cdot Y'_{phase}$ has repeated eigenvalues in order to validate the robustness of the algorithm, which in all cases was able to find a linearly independent set of eigenvectors.

The next step in the validation process is a comparison of the computed results with measurements taken in the field. Induced current and voltage measurements will be taken on a number of structures adjacent to aerial or buried power lines, which will then be compared to the computed values.

IX. Conclusion

We demonstrated a tool used to accurately model structures of different nature and the coupling between them. This tool can output matrices representing the complete coupled system to perform steady-state studies in a commercial simulation software, or create a pi-exact representation for time-domain simulations. To obtain the necessary decoupled equations, we implemented an algorithm to compute the voltage and current transformation matrices that will work in all cases, even with configurations that exhibit a certain type of symmetry, resulting in repeated eigenvalues for the ZY matrix. By using the Matlab framework, we created a high-level code that is easily expendable and modifiable, and when compiled as a COM object can be called by any executable program.

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