Inaccuracies in Network Realization of Rational Models Due to Finite Precision of RLC Branches

Antonio C. S. Lima, Bjørn Gustavsen, and Alécio B. Fernandes

Abstract-- One of the most challenging points in the simulation of power system transients is the modeling of frequency dependent impedances or admittances and their inclusion in the simulation environment. Highly accurate models can be identified via rational function approximations in the frequency domain, for instance by Vector Fitting. One way of including the obtained model in a time domain simulation is via an equivalent electrical circuit. This paper shows a case where the modeling via an equivalent circuit in ATP gives an inaccurate representation of the rational model. It is shown that the finite precision of the branch cards causes a catastrophic cancellation error in the representation of an RLCG branch. The sensitivity to the output format is investigated.

Keywords: System identification, Vector fitting, Frequency dependence, Network synthesization, Rational approximation.

I. Introduction

THE accurate modeling of frequency dependent networks in time-domain electromagnetic programs such as ATP, EMTP-RV or PSCAD/EMTDC, can be quite challenging. The main approach is to approximate the frequency response with rational functions on the form of state space models or pole-residue terms. This procedure allows to efficiently implement the frequency response in time-domain programs using recursive convolutions [1]. Network equivalencing [2],[3],[12] and wide band transformer modeling [4],[5] are examples where such an application is needed.

For the synthesis of the electric network one may use linear or nonlinear techniques. The main drawback of nonlinear techniques is the dependence on the starting point (initial guess). For linear techniques, the challenge lies with high order fitting over wide frequency bands. One procedure that is becoming increasingly more popular is the pole relocating algorithm known as Vector Fitting (VF). It is essentially a robust reformulation of the Sanathanan–Koerner iteration [6] that uses rational basis functions (partial fractions) instead of polynomials, and pole relocation instead of weighting [7]. The approach allows for high order functions and a wide frequency band.

One way of including the rational model in an EMTP programs is via an equivalent electrical network given in a computer generated file of branch cards. In this paper we demonstrate that usage of finite precision branch cards can severely reduce the accuracy of the model.

II. Rational Approximation by Vector Fitting

We briefly review the basic structure of the VF approach with relaxation (RVF). Consider first the original formulation of VF [8] where a frequency response *f(s)* (generally a vector, hence the designation VF) to be approximated as

$$f(s) = \sum_{m=1}^{N} \frac{r_m}{s - a_m} + d + se \qquad (1)$$

where the terms d and e are optional. VF first identifies the poles by solving in the least squares sense, the linear problem

$$\sigma(s) f(s) = p(s) \qquad (2)$$

where

$$\sigma(s) = 1 + \sum_{m=1}^{N} \frac{\tilde{r}_m}{s - q_m} \qquad (3)$$

$$p(s) = d + se + \sum_{m=1}^{N} \frac{r_m}{s - q_m} \qquad (4)$$

$\sigma(s)$ is a scalar while *p(s)* is generally a vector and the set of initial poles is $\{q_m\}$. VF is adequate for physical systems as all poles and residues come in complex conjugate pairs or are real, and d and e are real. It can be shown [8] that the poles of *f(s)* must be equal to the zeros of $\sigma(s)$ which are calculated as the eigenvalues of a matrix

$$\{a_m\} = eig\left(\mathbf{A} - \mathbf{b}\mathbf{c}^{\mathrm{T}}\right) \qquad (5)$$

In (5), $\mathbf{A}$ is a diagonal matrix with the initial poles $\{q_m\}$, $\mathbf{b}$ is a column vector of ones and $\mathbf{c}^{\mathrm{T}}$ is a row vector holding the residues $\{\tilde{r}_m\}$. This procedure can be applied in an iterative manner where (2)–(5) are solved repeatedly with the new poles $\{a_m\}$ replacing the previous poles $\{q_m\}$. This pole relocation procedure usually converges in 3–5 iterations. After the poles have been identified, the residues of (1) are finally calculated by solving the corresponding least squares problem

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with known poles.

An improved convergence can be achieved by removing the asymptotic requirement of the least squares problem (2) [9]. This *relaxed* VF (RVF) is obtained by replacing (3) with the constraint

$$\sigma(s) = \left(\tilde{d} + \sum_{m=1}^{N} \frac{\tilde{r}_m}{s_k - q_m} \right) \tag{6}$$

where $\sigma(s)$ is real. To avoid the trivial (null) solution, one equation is added to the resulting least squares problem,

$$\operatorname{Re}\left\{ \sum_{k=1}^{N_s} \left(\tilde{d} + \sum_{m=1}^{N} \frac{\tilde{r}_m}{s_k - a_m} \right) \right\} = N_s \tag{7}$$

where N_s is the number of samples and s_k is the sampled frequency.

Equation (7) enforces that the sum of the real part of $\sigma(s)$ over the given frequency samples equals some nonzero value without fixing any of the free variables. As the convergence is approached $\sigma(s)$ tends to unity at all frequencies. From this, it follows that the criterion (7) does not impose any constraint on the LS problem (2) other than preventing $\sigma(s)$ from becoming zero.

Finally, the rational model is perturbed in order to give a passive model [10],[11], thereby ensuring a stable time domain simulation.

Although VF (and RVF) can be applied to wide range of applications, as the focus here is electric circuit application, the main goal is to find an RLC admittance matrix to represent accurately the fitted function. For an efficient synthesis of the RLC network, all the elements of the fitted admittance matrix must share the same set of poles.

III. Synthesis of The Equivalent Network

The procedure to obtain the network equivalent used is based on [3],[4] and summarized here. Consider an admittance matrix Y fitted with VF, creating a matrix $Y_{fit}(s)$ in which the elements are

$$Y_{fit}(s)_{ij} = \sum_{m=1}^{N} \frac{r_{m_{ij}}}{s_k - a_m} + d_{ij} + se_{ij} \tag{8}$$

The network has branches between all nodes and ground and between all nodes. Branch between node and ground are given by:

$$y_{ii} = \sum_{j=1}^{K} Y_{fit}(s)_{ij} \tag{9}$$

while branches between node i and node j are

$$y_{ij} = -Y_{fit}(s)_{ij} \tag{10}$$

where K is the length of the admittance matrix.

Consider a single-phase element fitted as an improper function with 3 poles, one real and one complex (conjugate) poles. Therefore,

$$y(s) = \frac{r}{s-a} + \frac{r'-jr''}{s-(a'-ja'')} + \frac{r'+jr''}{s-(a'+ja'')} + d + se \tag{11}$$

To synthesize (11) we must calculate an RLC network as shown in Fig. 1, where

$$C_0 = e \ \ ; R_0 = \frac{1}{d} \tag{12}$$

The real pole is represented by the RL circuit as

$$L_r = \frac{1}{r} \ \ ; R_r = -\frac{a}{r} \tag{13}$$

while the complex conjugate pair is given by RLCG network where

$$\begin{aligned} L_c &= \frac{1}{2r'} \\ R_c &= 2L_c\left(L_c\left(r'a' + r''a''\right) - a'\right) \\ C_c &= \frac{1}{L_c\left(a'^2 + a''^2 + 2R_c\left(r'a' + r''a''\right)\right)} \\ G_c &= -2L_cC_c\left(r'a' + r''a''\right) \end{aligned} \tag{14}$$

(Note: G_c is a conductance). The values for any element in the RLGC or in RL branches, can be negative even though the numerical stability of the system has been ensured by the passivity enforcement.

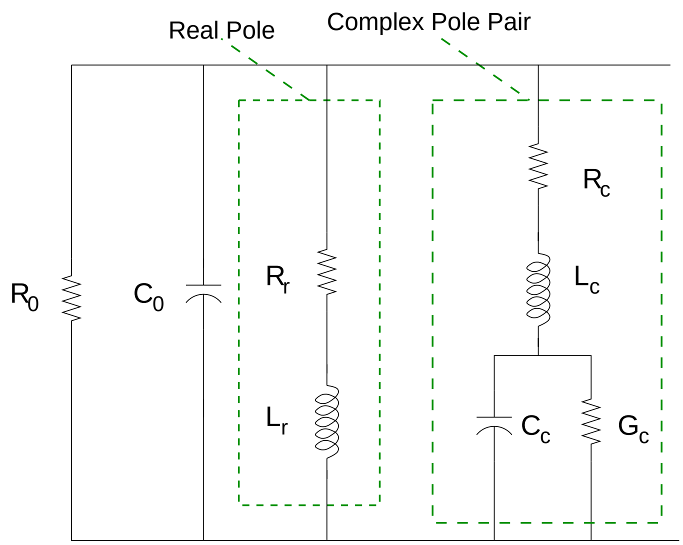

Fig. 1. Basic structure of the synthesized electrical network.

IV. Rational Fitting

To illustrate the inaccuracy problem of the network realization, we investigate the modeling from a power transformer self admittance that has been measured from 10 Hz up to 1 MHz. The transformer is a single-phase unit, 225 MVA, 345/230 kV.

Fig. 2 shows the measured transformer admittance (98 samples), as well as the result of the fitting by the relaxed VF. The responses appear somewhat "jagged", which is a result of irregular (non-logarithmic) sampling. The fitting was done using five iterations with 76 poles and inverse magnitude weighting, including non-zero terms *d* and *e*.

The main reason for the discrepancy in Figs 2 and 3 is inaccuracy in the data. Figs 4 and 5 show the same result when allowing unstable poles in the fitting process. It is observed that a much better fitting is now achieved.

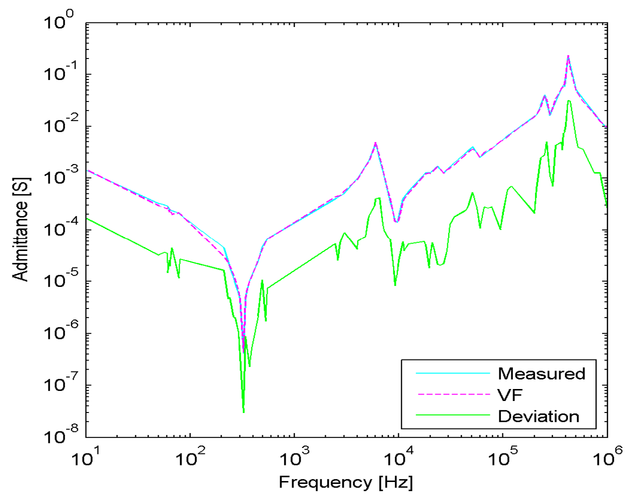

Fig. 2. Measured transformer admittance and fitting result.

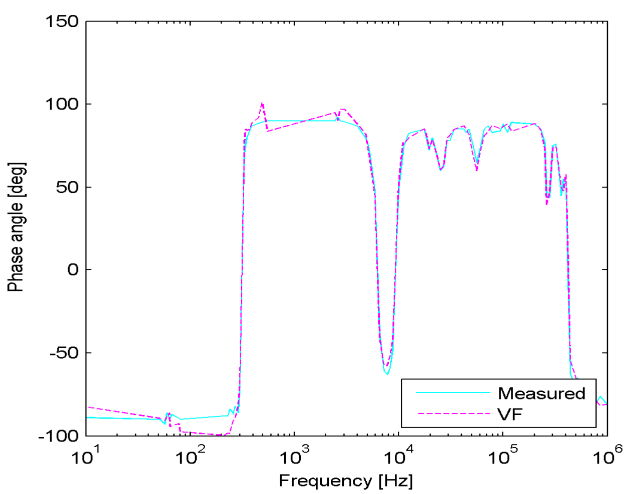

Fig. 3. Measured transformer phase angle and fitting result.

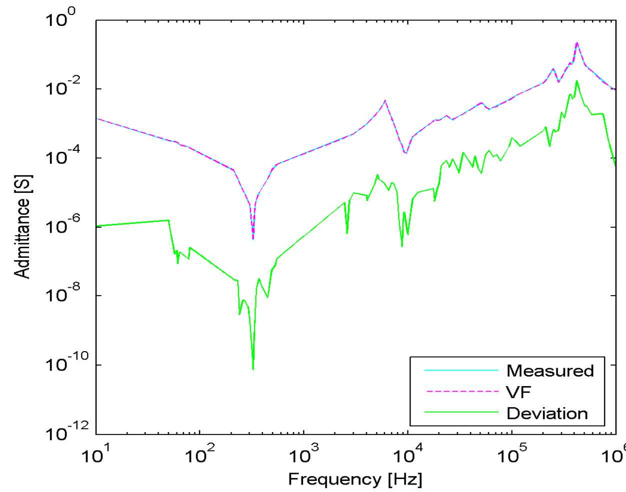

Fig. 4. Measured transformer admittance and fitting result. Unstable poles allowed.

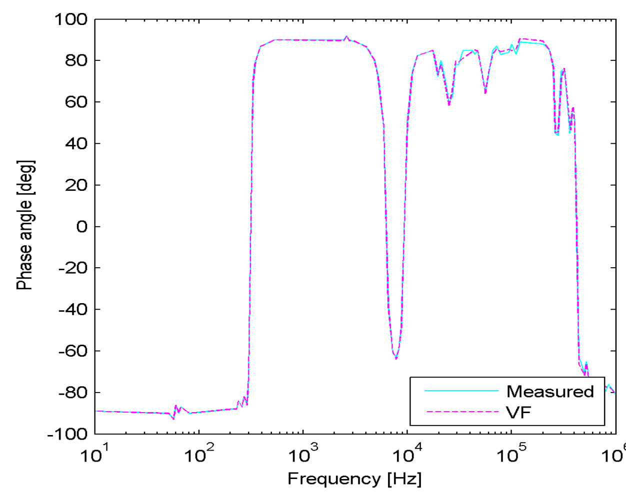

Fig. 5. Measured transformer phase angle and fitting result. Unstable poles allowed.

V. ERROR CAUSED BY THE TRUNCATION

We now go back to the rational fitting with stable poles (Figs 2 and 3). Using the procedure described in section III we obtain an equivalent circuit for ATP. The circuit element values are written to file in format 14.6e (Matlab notation). It is, however, possible to use a wider format (16.8e), see Table I.

TABLE I
ALTERNATIVE OUTPUT FORMATS

Format	
14.6e	-1.579269e+004
16.8e	-1.57926902e+004

Fig. 6 compares the rational function with the frequency response of the lumped circuit (ATP Frequency Scan). This plot uses 501 logarithmically spaced samples, in order to resolve the behavior. It can be seen that there is a significant mismatch between the frequency response of the rational fitting and that of the ATP circuit. (The shown deviation is the magnitude of the complex deviation). In addition, we emulate the truncation effect as follows: the circuit elements are written to file in the 14.e format and are read back into the program. From the truncated circuit element values we calculate back the rational model as described in the Appendix. The deviation between this modified rational function and the original one is included in the plot. This deviation is seen to exactly match that by ATP Frequency Scan. This means that we are able to emulate the effect of alternative output formats.

Fig. 7 shows the deviation with alternative output formats. It is seen that the deviation goes quickly down with wider formats.

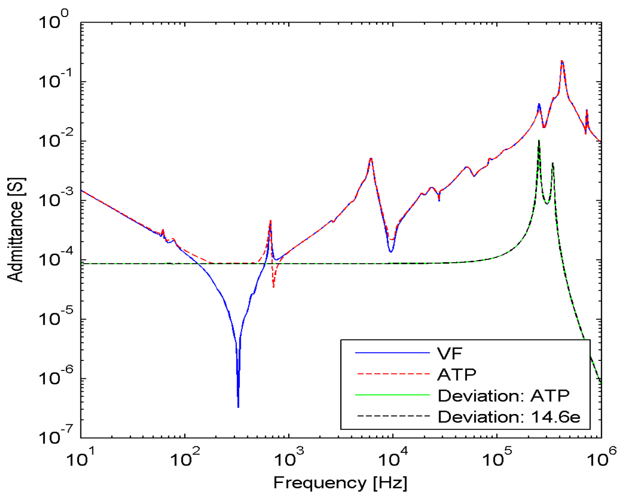

Fig. 6. Comparison of rational model, ATP, and perturbed model.

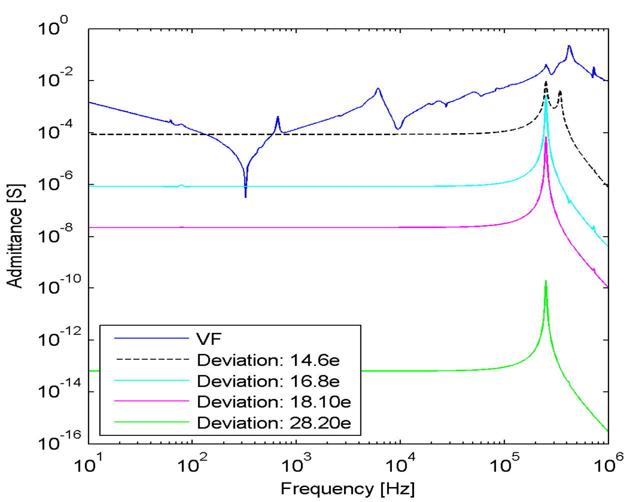

Fig. 7. Impact of output format on frequency response.

VI. Analysis

The reason for the mismatch in Fig. 6 was pinpointed by comparing the frequency response of the individual pole-residue terms. It was found that one of the complex pairs was causing the problem, see Fig. 8.

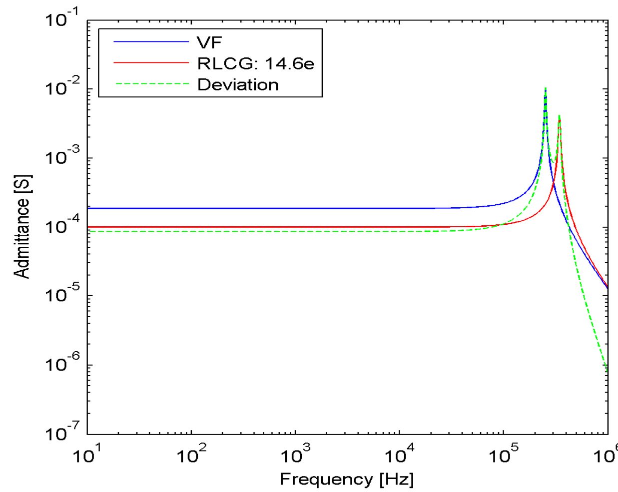

Fig. 8. Frequency response of complex pair (sum of two fractions).

Table II shows the circuit values of this RLCG branch. At DC, the current through the RLCG branch is equal to R=1/G. The terms R and 1/G are nearly equal (with opposite sign) and the difference in the last digit is due to truncation. Since the two terms have opposite sign, the error of the series impedance is about equal to their sum, which is 1E4. The associated admittance is the inverse, i.e. 1E–4. And we can indeed see that the error is about 1E–4 in both Fig. 8 (sum of conjugate pair components) and in Fig. 6 (complete response). It is further seen that the error in Fig. 7 is with format 16.8e about 1E–6 since the round-off occurs two digits later.

TABLE II
RLCG Branch Elements (format: 14.6e)

R	1.522074e+010
L	5.734329e+003
C	2.475228e-014
1/G	-1.522075e+010

VII. Discussion

As a first remedy, the widest available output format should be used (16.8e). In the given example, the error caused by truncation would then be barely observable, see Fig. 7. This, however, does not guarantee that intolerable deviations will not show up in other cases. It would therefore be a better solution to modify the ATP input format to allow branch cards with full precision. Alternatively, one could look for a way of modifying the rational model so that the cancellation problem will not occur. Still, the most elegant solution is surely to use a convolution based approach [1],[12],[13] since the truncation problem will then cease to be an issue. Also, the convolution based approach is computationally more efficient than usage of an equivalent circuit. It is remarked that convolutions has for many years been the standard way of representing transmission line models with frequency dependent parameters [14],[15].

VIII. CONCLUSIONS

This paper has shown that the representation of a rational model by an equivalent electric circuit can lead to inaccurate results due to usage of finite precision branch cards. In one example, an RLCG branch was found to cause significant round off errors due to the series connection between R and $1/G$, where the two terms were nearly equal in magnitude but with opposite sign. It is therefore important to use the widest output format available. The best solution is probably to make use of a convolution based approach.

IX. APPENDIX – RECOVERING RATIONAL MODEL FROM ELECTRIC CIRCUIT

Consider that we have given an RLCG branch. The complex conjugate pair in (11) is recovered from this branch components as follows,

$$\gamma = \frac{-(G^2L^2 + C^2R^2 - 2CL(2+RG))}{L^2} \quad \text{(A.1)}$$

$$r' = 1/(2L) \ , \ r'' = \frac{(LG - RC)}{(2L^2\sqrt{\gamma}} \quad \text{(A.2)}$$

$$a' = \frac{-(RC + LG)}{(2LC)} \ , \ a'' = \frac{-\sqrt{\gamma}}{2C} \quad \text{(A.3)}$$

The same expressions apply also to *RL* branches. With G and C zero, the imaginary parts r'' and a'' result zero.

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XI. BIOGRAPHIES

Antonio C S Lima was born in Rio de Janeiro, Brazil, in 1971. He received the B.Sc., M.Sc., and Ph.D. degrees in electrical engineering from the Federal University of Rio de Janeiro (UFRJ) in 1995, 1997, and 1999, respectively. Currently, he is with the Electrical Engineering Department, Federal University of Rio de Janeiro. In 1998, he was a Visiting Scholar in the Department of Electrical and Computer Engineering, the University of British Columbia, Vancouver, BC, Canada. From 2000 to 2002, he was with the Brazilian Independent System Operator, ONS, Rio de Janeiro, Brazil, dealing with electromagnetic transient studies for the Brazilian National Grid. His main interests are electromagnetic transient analysis in power systems and analysis of electromagnetic fields.

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