Frequency Domain Transient Analysis Applied to Transmission System Restoration Studies

Pablo Gómez, Pilar Arellano, Ricardo O. Mota

Abstract--In this work, a frequency domain method to evaluate transient overvoltages produced in the restoration process of transmission systems is described. During this process, long simulation times are necessary given that several switching operations are performed to interconnect different parts of the network. The method is applied to analyze a particular transmission system, for which maximum overvoltages derived from the sequential energization of transmission lines at different restoration stages are evaluated. For the frequency domain analysis, the Numerical Laplace Transform (NLT) is applied, comparing its results with those obtained directly in time domain using the ATP-EMTP.

Keywords: Frequency domain analysis, restoration, switching transients.

I. INTRODUCTION

SEVERAL types of disturbances can produce power system complete blackout or partial outage. Restoration process must be performed at the minimum possible time and with the minimum number of operations.

Results from several types of analysis of the system, e.g., power flows, small disturbance stability, transient stability and electromagnetic transients are fundamental in the restoration process. Many analytical tools are available to perform these studies; however, electromagnetic transient analysis is usually neglected or greatly simplified. Therefore, large transient overvoltages due to inadequate switching operations are some of the main causes of restoration delay and equipment damage, being of particular concern the transmission line energization [1].

Over the last decades, switching overvoltages related to line energization have been studied with different methods. At the present time, time domain methods are preferred for transient analysis, given their simplicity to simulate changes in network topology and the inclusion of non-linear elements. Among these methods, the Electromagnetic Transient Program (EMTP), initially introduced by Dommel [2], is nowadays the most widely known and applied tool for the analysis of electromagnetic transients in power systems.

The inclusion of frequency dependent elements, such as transmission lines, has always been an inherent difficulty of time domain methods. Several approaches have been applied to overcome this problem since early 70s [3]-[8]. However, in a recent paper [12], it has been shown that two of the most advanced time domain line models used nowadays, namely the J. Marti model [7] and the Phase Domain model [8], can still present errors when simulating systems with strong frequency dependence.

On the other hand, when using frequency domain methods for electromagnetic transient studies [9]-[12], frequency dependence of the line parameters can be included in a straightforward manner. An important shortcoming of these methods is its difficulty to deal with changes in the network topology and with non-linear elements. This has been dealt with in previous works through the application of the superposition principle with good results [11], [12].

In this work a frequency domain method, based on the Numerical Laplace Transform (NLT) [13], [14], is applied to evaluate switching transient overvoltages produced in the restoration process of a particular transmission system, for which maximum overvoltages derived from the sequential energization of transmission lines at each restoration stage are evaluated and comparisons with ATP-EMTP are provided.

II. GENERAL METHODOLOGY

This section reviews the methodology applied to analyze switching transients in the frequency domain, previously described in [12].

A. Transmission Line Model

A multiconductor transmission line is considered as a distributed parameter model having series impedance matrix $\mathbf{Z}=\mathbf{R}+s\mathbf{L}$ and shunt admittance matrix $\mathbf{Y}=s\mathbf{C}$ per unit length, being s the Laplace variable. Taking into account skin and ground return effects, both resistance and inductance are considered as frequency dependent and computed from Gary's formulae [15]. Applying nodal analysis, a multiconductor transmission line can be represented in frequency domain as

$$\begin{bmatrix} \mathbf{I}_0 \\ \mathbf{I}_L \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_0 \coth(\mathbf{\Psi} l) & -\mathbf{Y}_0 \operatorname{csch}(\mathbf{\Psi} l) \\ -\mathbf{Y}_0 \operatorname{csch}(\mathbf{\Psi} l) & \mathbf{Y}_0 \coth(\mathbf{\Psi} l) \end{bmatrix} \begin{bmatrix} \mathbf{V}_0 \\ \mathbf{V}_L \end{bmatrix} \qquad (1)$$

where $\mathbf{V}_0$ and $\mathbf{I}_0$ are voltage and current vectors at the sending end, $\mathbf{V}_L$ and $\mathbf{I}_L$ are the respective values at the receiving end, l is the line length, $\mathbf{Y}_0$ and $\mathbf{\Psi}$ are the characteristic admittance and voltage propagation matrices given by

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$$\mathbf{Y}_0 = \mathbf{Z}^{-1}\,\mathbf{\Psi}, \qquad \mathbf{\Psi} = \mathbf{M}\,\sqrt{\boldsymbol{\lambda}}\,\mathbf{M}^{-1} \qquad \text{(2a), (2b)}$$

being **M** and **λ** the eigenvector and eigenvalue matrices of the product **ZY**. Nodal form allows easy inclusion of several lines in the complete electric network. The complete nodal form of the system can be defined in Laplace domain as

$$\mathbf{I}(s) = \mathbf{Y}_{bus}\mathbf{V}(s) \qquad (3)$$

where **V**(*s*) is the nodal voltage vector and **I**(*s*) is the injected current vector. Expression (3) is solved for **V**(*s*) and time domain waveforms are obtained using the inverse algorithm of the NLT, as described in the following section.

B. Numerical Laplace Transform Algorithm

Considering a finite integration range, the direct and inverse Laplace transforms can be written as:

$$F(c+j\omega) \cong \int_0^T \left[f(t)e^{-ct}\right]e^{-j\omega t} \qquad (4a)$$

$$f(t) \cong \mathrm{Re}\left\{\frac{e^{ct}}{\pi}\int_0^{\Omega} \sigma(\omega)F(c+j\omega)\,e^{j\omega t}\right\} \qquad (4b)$$

where $f(t)$ is real causal function and $F(s)$ its image in the Laplace domain; ω is the angular frequency, T is the observation time and Ω is maximum frequency of the spectrum. Term $\sigma(\omega)$ is a weighting function, also known as window function, used to attenuate the Gibbs errors produced by the truncation of the frequency range. In this work, Hanning window is applied, which is given by

$$\sigma(\omega) = \frac{1}{2}\left[1+\cos\left(\frac{\pi\varpi}{\Omega}\right)\right] \qquad (5)$$

Also, since time domain function $f(t)$ obtained by numerical evaluation of (4b) will necessarily be distorted by aliasing, the Laplace stability constant c can be used to attenuate the associated errors by "smoothing" the frequency response. A value of $c=2\Delta\omega$, obtained empirically by Wilcox [13], is used here. Finally, a numerical form of equations (4) that allows using the Fast Fourier Transform (FFT) [16] is obtained (see Appendix).

C. Switch Model for Closure

In the case of a switched network, the problem of topology changes that turns the network into a time variant system is addressed in frequency domain via the superposition principle [11], [12].

Assuming the switch as initially open, the potential difference between its terminals can be represented by a voltage source V_{sw}. Closure is performed by a series connection of another source V_{sw2} with equal magnitude than V_{sw} but opposite polarity, as shown in Fig. 1. The voltage source required to close the switch at time $t_c \geq 0$ is given by

$$V_{sw2} = L\{-v_{sw}(t)\,u(t-t_c)\} \qquad (6)$$

where $v_{sw}(t)$ is the time domain waveform of V_{sw} and L indicates the Laplace transform.

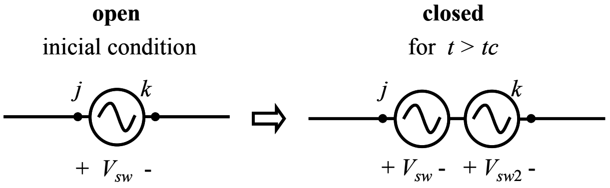

Fig. 1. Superposition principle for switch closure

Since nodal analysis is applied in this work, instead of the ideal voltage source V_{sw2} a Norton equivalent with current source $I_{sw2}=V_{sw2}/R_{sw}$ is used, being R_{sw} a resistance needed to perform the source transformation, which must be small enough to approximate the ideal source or it can take some particular value to represent a contact resistance. Alternatively, the modified nodal analysis (MNA) can be used to allow the direct insertion of ideal voltage sources [17].

The complete voltage response is obtained by adding the system response before switch closure to that resulting from applying the current source I_{sw2}. Therefore, the complete solution corresponding to a closure between nodes *j* and *k* of the system can be expressed as follows

$$\mathbf{V} = \mathbf{V}^{(0)} + \mathbf{Y}_{bus}^{(1)}\mathbf{I}^{(1)} \qquad (7)$$

where $\mathbf{V}^{(0)}$ is the node voltages vector before switching, $\mathbf{Y}_{bus}^{(1)}$ is the admittance matrix modified by the inclusion of R_{sw} and $\mathbf{I}^{(1)}$ is an injected current vector containing only the elements corresponding to source I_{sw2} connected between nodes *j* and *k*.

III. TRANSMISSION LINE ENERGIZATION DURING RESTORATION PROCESS

In order to analyze transient overvoltages related to transmission line energization during a restoration process, the frequency domain methodology described in Section II is applied to the test system shown in Fig. 2, extracted from [18]. Conductors arrangement is equal for all transmission lines and is shown in Fig. 3. Simulation is carried out using two different approaches:

1. In the first approach each switch operation is performed in separated simulation processes to obtain more accurate results.
2. To verify accuracy of the frequency domain analysis, the second approach is based on performing the complete restoration sequence in only one simulation, i.e., closing all switches in one simulation.

In both cases, results are compared with those obtained in time domain using the ATP-EMTP. Frequency dependence of the lines electrical parameters is accounted for in this program using the J. Marti line model [8]. Besides, to analyze the most severe overvoltages, each sequential energization is considered as critical, i.e., each switch pole closes at the maximum voltage value present, and neither pre-insertion resistors nor arresters are included.

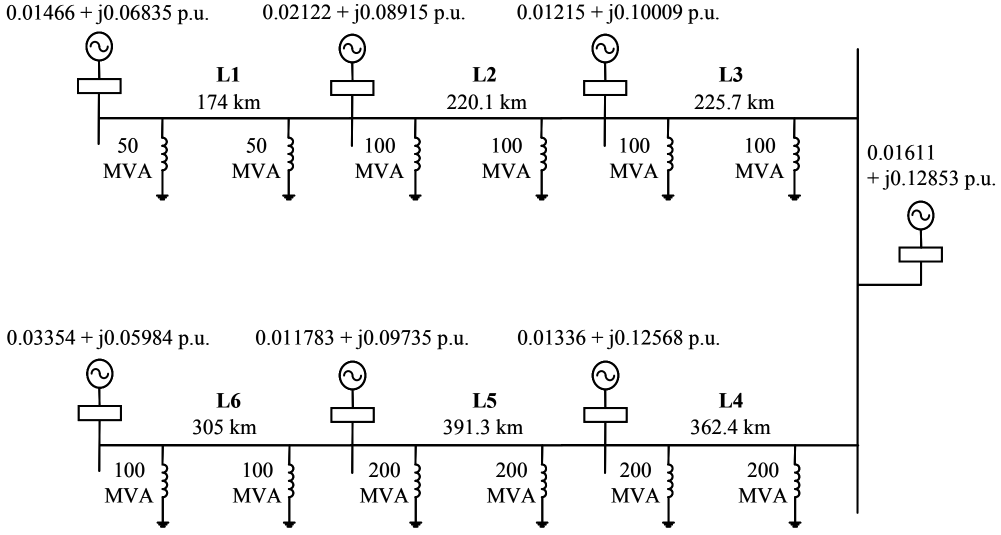

Fig. 2. One-line diagram of the test system

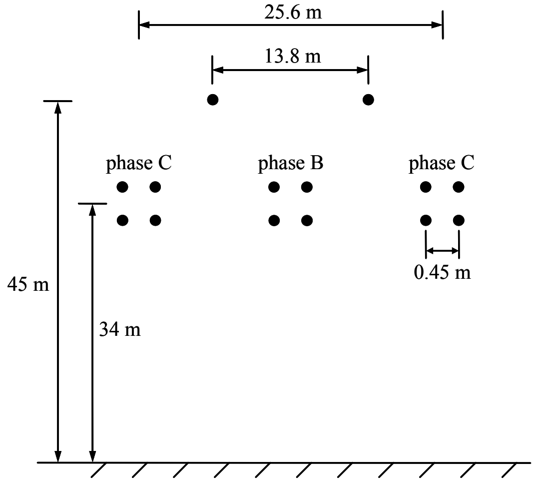

Fig. 3. Conductors arrangement

A. First approach - Restoration in Separated Simulations

As an example, Fig. 4 shows transient overvoltage at phase A of the open end of the line L2 when energized from its left hand side end, with line L1 previously connected. For both ATP and NLT simulations, N=2048 samples where used. An important difference in waveforms obtained with the frequency and time domain methods can be noticed.

Closing times of switch poles where 0.475, 0.47222 and 0.47777 s for phases A, B and C, respectively, considering a damping time of 0.4666 s (28 cycles) for the transient produced by previous connection of line L1.

Figure 5 shows transient overvoltage at phase A of the same line, when frequency domain analysis is performed with N=2048 but 15N points are considered in ATP-EMTP. This gives very similar results, showing that in this case the frequency domain method is much more accurate than ATP-EMTP.

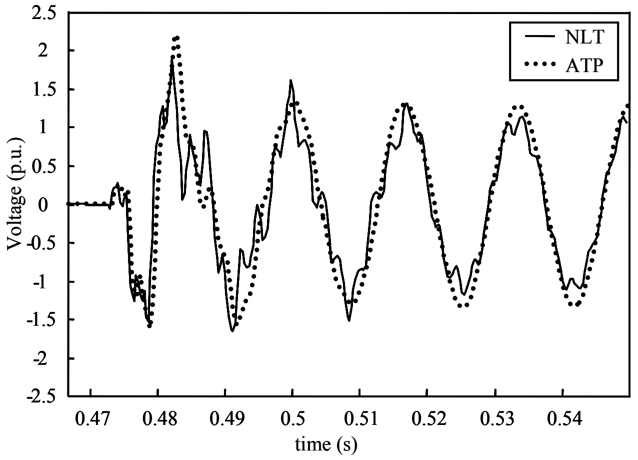

Fig. 4. Voltage at phase A of the open end of the line L2
Same number of samples for NLT and ATP.

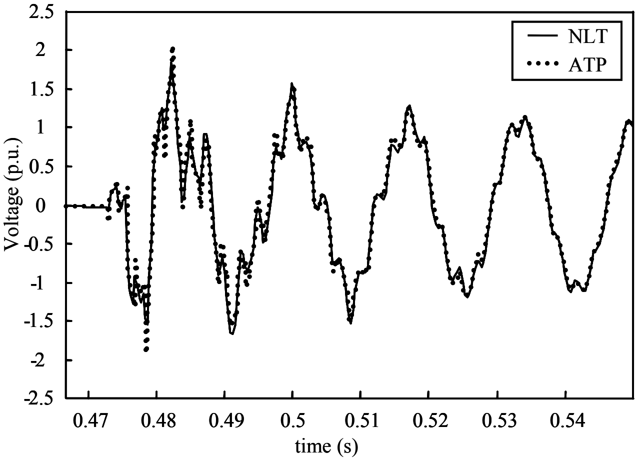

Fig. 5. Voltage at phase A of the open end of the line L2
Number of samples for ATP 15 times greater than with NLT.

B. Second Approach – Complete Restoration in One Simulation.

Accuracy of the frequency domain analysis for transmission system restoration studies is verified by considering the closing of all switches related to the test system in one simulation process.

Complete observation time was T=1.38 s taking into account 12 maneuvers, with N=2048 samples. The same simulation was performed in ATP-EMTP but, since in this program time step Δt must be greater that the travel time of the largest line, sampling was limited to 1.5N=3072.

Behavior of overvoltages throughout the restoration in a single simulation performed in ATP was compared with that obtained with the restoration in separated simulations using the NLT (first approach), getting differences between 6 and 13%. On the other hand, differences between separated and single simulation with the NLT were in the order of 0.01 to 6%. This is shown in Fig. 6 for the different restoration stages.

IV. CONCLUSIONS

In this article, a frequency domain method based on the Numerical Laplace Transform and the superposition principle is applied to evaluate transient overvoltages related to line switching in the restoration of a transmission system.

Unlike typical transient analysis, observation time for this type of studies is in the order of seconds, given that several switching operations occur and a damping time for the previous transient must be considered between each operation. It has been shown that in these cases the NLT gives identical results than the ATP, but the latter requires a much greater number of samples in the simulation (15 times greater for the example).

Determination of switching overvoltages associated to a given restoration sequence can be determined in a more accurate and reliable manner using the frequency domain method (NLT) than with the time domain program ATP.

V. APPENDIX

The numerical forms of (1), with odd sampling in the frequency domain, that allow using the Fast Fourier Transform algorithm [16] to get computer time savings is as follows:

$$F_m = \sum_{n=0}^{N-1} f_n D_n \exp\left(-\frac{j2\pi mn}{N}\right), \quad m = 1, 2, \ldots, N-1 \quad \text{(8a)}$$

$$f_n = \text{Re}\left\{C_n \sum_{m=0}^{N-1} F_m \sigma_m \exp\left(\frac{j2\pi mn}{N}\right)\right\}, \quad n = 1, 2, \ldots, N-1 \quad \text{(8b)}$$

where

$$F_m = F\left[c + j(2m+1)\Delta\omega\right] \quad \text{(9a)}$$

$$f_n = f(n\Delta t) \quad \text{(9b)}$$

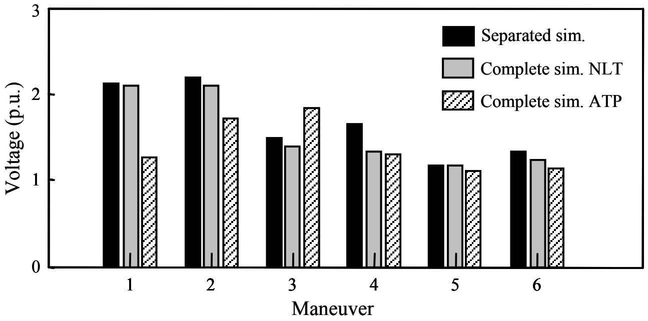

Fig. 6. Behavior of overvoltages throughout the restoration process

$$D_n = \Delta t \exp\left(-cn\Delta t - \frac{j\pi n}{N}\right) \quad \text{(9c)}$$

$$C_n = \frac{2\Delta\omega}{\pi} \exp\left(cn\Delta t + \frac{j\pi n}{N}\right) \quad \text{(9d)}$$

$$\sigma_m = \sigma\left[(2m+1)\Delta\omega\right] \quad \text{(9e)}$$

$$\Delta t = \frac{T}{N}, \quad \Delta\omega = \frac{\pi}{T} \quad \text{(9f),(9g)}$$

$$\Delta t = \frac{T}{N}, \quad \Delta\omega = \frac{\pi}{T} \quad \text{(9h),(9i)}$$

being $\Delta\omega$ the spectrum integration step and Δt the time discretization step. The application of the FFT algorithm to a Laplace-type integration was originally proposed by Ametani [19].

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