Transmission Line Modeling of Grounding Electrodes and Calculation of their Effective Length under Impulse Excitation

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Abstract—Grounding electrodes, being the basic component of any grounding system, need to be accurately modeled in transient analysis studies. An easy-to-apply methodology for calculation of voltage and current distribution along the electrode has been presented and validated in [1]. It is based on a distributed parameters transmission line model of the electrode. In the middle stages, closed–form mathematical expressions are used for the solution of telegraphy equations that describe the propagation of voltage and current waves along the electrode. The final expressions include few terms, so they are easy to apply. In this paper per unit length parameters of the electrode are comparatively calculated using various approaches. Calculation of the impulse response of grounding electrodes shows that there is a limit in their length that seriously contributes in lowering raised potentials. This limit is the “effective length” of grounding electrodes and is calculated in the paper.

Keywords: Grounding Electrode, Transient Response, Effective Length, Ground Potential Rise (GPR).

I. INTRODUCTION

MODELING of grounding systems and grounding electrodes is of primary importance for the analysis of the transient behavior of any electrical installation. Many attempts to accurately determine the transient response of a grounding electrode are presented in literature [2]-[5],[12],[15],[16]. These attempts are either based on computer models, solved numerically or on analytical expressions for current and voltage distributions based on simplifications or special initial conditions [2],[3],[17].

In this paper a novel methodology applicable on the calculation of the behavior of a grounding electrode under surge conditions is used. Attempts made in the past involve techniques for Laplace inversion or prediction of the results based on experimental data. Advantages of the new methodology are simplicity and accuracy, because it is based on a distributed model of the electrode and closed form mathematical expressions. This methodology is validated in literature [1],[18]. Grounding electrode is treated as in series connected π-circuits tending to the open-ended transmission line when the number of circuits increases. Analytical formulae are presented for current and voltage distributions along the electrode. Impulse injection current has been described by a double exponential waveform function.

Grounding electrode model is constructed under the assumption that the soil is homogeneous. In this paper per unit length inductance conductance capacitance and resistance of the electrode are comparatively calculated using various approaches and mathematical formulas.

Long grounding electrodes have lower grounding resistance and better behavior under low and high frequency excitation. In case a sinusoidal current source is applied at a point of a grounding conductor, maximum GPR (Ground Potential Rise) decreases as its dimensions increase. However, when high frequency components are injected as in the case of lightning, there is an upper limit in the length of the electrode that substantially affects the maximum GPR value. This is called here “effective length”. In this paper, the effect of effective length of grounding electrodes is demonstrated, for various soil and impulse excitation characteristics.

II. THEORETICAL BACKGROUND

A. Grounding Electrode Model

Correct calculation of per unit length parameters of the electrode is essential for the accuracy of the results. Special attention is given to the assumptions done in the middle stages of the development of calculation formulas.

A.1. Circuit model of the electrode

As shown in literature at low frequencies the impedance of a grounding electrode can be represented by a single resistance, while at high frequencies by a lumped R-L-C circuit. Three sets of formulas for the parameters of the circuit model of the electrode, resistance $R_e$, inductance $L_e$, conductance $G_e$ and capacitance $C_e$ are often used in the existing literature.

One is from the reference work by Rudenberg [6] and is used for the vertical rod:

$$G^{-1} = \frac{\rho}{2\pi} \log \frac{2\ell}{\alpha}, \quad C = 2\pi\ell \log \frac{2\ell}{\alpha} \quad \text{and} \quad L = \frac{\mu_0\ell}{2\pi} \log \frac{2\ell}{\alpha} \quad (1)$$

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and the other [7][8] is used for horizontal electrodes:

\[ G^{-1} = \frac{\rho}{2\pi l} \left[ \log \frac{4l}{\alpha} \right] , C = 2\pi \left[ \log \frac{4l}{\alpha} \right] , \]

\[ L = \frac{\mu_0}{\alpha} \left[ \log \left( \frac{2l}{s} \right) \right] \]  

(2)

where \( \alpha \) is the radius of the conductor replaced by \( \sqrt{2} \cdot \alpha \cdot z \) when the conductor is placed at depth \( z \).

Alternatively the following equation proposed by Dwight et al. [9] can be used to calculate \( G \):

\[ G^{-1} = \frac{\rho}{2\pi} \left[ \ln \left( \frac{2l}{\alpha} \right) + \left( \frac{2l}{s} \right) - 2s + \frac{s^2}{l} - \frac{s^4}{16(0.5l)^2} + \frac{s^6}{512(0.5l)^4} \right] \]

(3)

where \( s = 2z \) is two times the burial depth and \( L \), \( C \) can be calculated from the equations found in [4]:

\[ L = \frac{l}{c_0^2 \cdot \varepsilon \cdot \rho \cdot G} \]  

\[ C = \varepsilon \cdot \rho \cdot G \]  

(4)

Other, less commonly used calculation formulas can be found in literature [3][17].

The electrode can then be considered as a transmission line open at the lower end. The input impedance at the energization end is [11]:

\[ Z = Z_0 \coth \gamma l \]  

(5)

A.2. Frequency dependent transmission line model

For given conductor and soil data an electrodynamic model can be evaluated that results in the frequency dependent characteristic impedance \( Z_0(\omega) \) and propagation constant \( \gamma(\omega) \) [13][14]. Sunde [7] developed the mathematical equations for the unit length admittance and self-impedance of the grounding electrode:

\[ Y(\Gamma) = \left( Y_i + \frac{1}{\pi(\sigma + j\omega \varepsilon)} \log \left( \frac{11.2}{\Gamma \alpha} \right) \right)^{-1} \]

\[ Z(\Gamma) = Z_i + \frac{j\omega \mu}{2\pi} \log \left( \frac{1.18}{\alpha(\gamma + \Gamma)^2} \right) \]  

(6)

where \( \alpha \) is replaced by \( \sqrt{\alpha^2 + 4z^2} \) when the conductor is placed at a depth \( z \). The above equations (6) are solved iteratively for \( \Gamma = \sqrt{Z(\Gamma) Y(\Gamma)} \) at every single frequency in the range of interest. In practical examined cases the internal impedance \( Z_0 \) that represents energy within the conductor is nonzero. The internal impedance of cylindrical conductor is derived in a classical paper by Schelkunoff [19] where for a solid conductor with radius \( \alpha \) and internal characteristics \( \sigma_c \), \( \mu_c \) and \( \varepsilon_c \) it is for p.u. length:

\[ Z_i = \frac{j\omega \mu_{0c}}{2\pi \alpha \gamma_c} \int_1 (\gamma_c \alpha) \]  

(7)

where \( \gamma_c = [j\omega \mu_c (\sigma_c + j\omega \varepsilon_c)]^{1/2} \)

and \( \omega \) is the angular frequency. A simplified equation for easier and accurate calculation of \( Z_i \) can be found in [20]. At very low and zero frequencies this impedance reduces to the classical dc resistance of a cylindrical conductor:

\[ Z_i = \frac{1}{\pi \alpha^2 \sigma_c} \]  

(8)

A.3. Comparative evaluation of calculation formulas

A drawback of most calculation formulas for estimation of the lumped parameters \( R_c \), \( L_c \), \( C_c \) and \( G_c \) is that they are based on quasi-static approximation that limits their validity to some upper frequency that depends on the size of the grounding system and the electrical characteristics of the earth.

More particularly, Sunde’s assumptions in derivation of the electrode parameters are that the potential due to current leaving a conductor element is the same for a point source at the axis of the conductor. Thus is acceptable when dealing with long conductors. In order to solve the equations of the potential versus the leakage current along the entire conductor it is assumed that the resistance of the conductor is negligible, so that the voltage drop may be disregarded and the boundary condition to be satisfied at the surface of the conductor \( y = \alpha \) is that \( dV(x, \alpha)/dx = 0 \).

Considering the accuracy of (6), as noted in [7], when the current varies rapidly, as for example in the case of lightning strike, these are an improved approximation of the solution in terms of an exponential mode of propagation. There is a logarithmic approximation of the variation vs. conductor length of the main integral functions \( q, p \) which are added to the internal impedance and admittance terms to give (6).

Grounding resistance values calculated from formula (3) are closer to the results of finite elements method calculation and well known commercial software [21] than the results calculated from formula (2).

In Fig.1, the magnitude of the input impedance \( |Z| \) at the energization end obtained from the various calculation formulas, vs. electrode length is plotted. Curves 1 and 2 show the results of (2) and (3) when the parameters \( R, L, C, G \) are calculated for the whole length of the electrode and then they are divided to find the p.u. values. Curve 3 shows the results from calculation formulas (6). Curves 4, 6 are the input \( |Z| \) of n-pi-circuit model of the electrode where the parameters \( R, L, C, G \) are calculated for the whole length of the electrode and then they are divided to find the p.u. values for each pi-circuit. Curves 5, 7 are obtained as 4, 6 however the parameters put in every elementary pi-circuit are calculated for a 1m long electrode.

In order to avoid drawbacks caused by assumptions in the middle stages of the development of the mathematical equations (2) it is proposed here to use (3) and (4) for
calculation of R, L, C, G, for the whole length of the electrode and then divide to find the per unit length values.

B. Voltage and Current distributions along the grounding electrode

In this paragraph the transient response of grounding electrode in terms of voltage and current distributions along its length is calculated solving the equations of propagation directly in time domain.

B.1. Impulse excitation model

Transient response of grounding electrodes is calculated considering impulse current excitation of a double exponential waveform. It is:

\[ I_{\text{source}}(t) = I_o \cdot (e^{at} - e^{bt}) \]  

where \( a \) and \( b \) are real negative constants.

B.2. Derivation of Mathematical Expressions for Calculation of the Impulse Response of the electrode

Derivation of mathematical expressions for the proposed methodology is presented in detail in [1]. In this paper the mathematical background will be briefly described.

Time domain solution of the propagation equations equals to the sum of a partial solution and the solution of the homogeneous equation for the total network.

B.3. Partial solution of the differential equations

By expressing voltages and currents in the middle nodes of the electrode in Fig. 1, as a function of voltages and currents at their right side, a differential equation of \( k^{th} \) order is obtained:

\[ i_0 = a_1 D^k V_n + a_2 D^{k-1} V_n + a_3 D^{k-2} V_n + ... + a_k V_n \quad (10) \]

where \( A = R + LD \), \( B = G + CD \), \( D = d/dt \), \( k > n \), and \( a_i \) are real constants. Differential equations of the form (10) can be written for every unknown current \( i_1, ..., i_n \) or voltage \( V_0, V_1, V_2, ..., V_{n-1} \) shown in the circuit of Fig. 1.

For known current \( i_0(t) = I_0(e^{at} - e^{bt}) \), equation (10) accepts the partial solution \( V_n = I_0 (C_{a,n} e^{at} - C_{b,n} e^{bt}) \) where \( C_{a,n}, C_{b,n} \) are real constants. Assuming the current distribution along the electrode \( i_k = C_{a,k} e^{at} - C_{b,k} e^{bt} \) we can uniquely determine constants \( C_{a,k}, C_{b,k} \) in order to satisfy Kirchoff’s laws. The following expressions are obtained from calculation of the limit of expressions for
current and voltage distributions as the number of segments \( n \) tends to infinity:

\[
I_p(x,t) = \frac{\sinh(\gamma_c(\ell-x))}{\sinh(\gamma_c \ell)} e^{\alpha \ell} - \frac{\sinh(\gamma_c(\ell-x))}{\sinh(\gamma_c \ell)} e^{\alpha x} \quad (11)
\]

\[
V_p(x,t) = -\frac{\cosh(\gamma_c(\ell-x))}{\sinh(\gamma_c \ell)} e^{\alpha \ell} - \frac{\cosh(\gamma_c(\ell-x))}{\sinh(\gamma_c \ell)} e^{\alpha x} + \text{Const} \cdot e^{\alpha x} \quad (12)
\]

where \( \gamma_c = \sqrt{(R_c + aL_c)(G_c + aC_c)} \)

### B.4. General solution of the homogeneous differential equations

Full expressions for voltage and current distributions along the electrode are the (11) plus the general solutions of the homogeneous differential equations. The differential equation that relates source current to the voltage at the end of the electrode is of the form:

\[
i_0 = \frac{a_2}{\beta - \alpha} V_n \quad (12)
\]

where \( D = \frac{d}{dt}, A = R + LD, B = G + CD, \beta_2 = B - \alpha_2 \),

\[
a_2 = \frac{-AB + \sqrt{4AB + A^2}}{2A}
\]

The general solution of (4) where the right part tends to \( -\sqrt{(G_c + xC_c/R_c + xL_c)} \cdot \sinh(\gamma_c \ell) \), is a linear combination of exponential terms. Each of them corresponds to a root of the characteristic polynomial. Roots of (12) are:

\[
x = -G_c/\ell \quad \text{and} \quad \gamma_c \ell = k\pi, k = \pm1,2,3,\ldots \Rightarrow
\]

\[
r_{kz}(\ell) = \frac{-R_c \ell^2 - L_c G_c \ell^2 \pm \sqrt{[R_c \ell^2 - L_c G_c \ell^2]^2 - 4L_c G_c \ell^2 \pi^2 k^2}}{2L_c \ell^2}
\]

The general solution to the homogeneous equation for the voltage at the end is:

\[
V_h(\ell,t) = \sum_{k=1}^{\infty} \left( C_1(k) \cdot e^{\gamma_c(k) \ell} + C_2(k) \cdot e^{-\gamma_c(k) \ell} \right)
\]

where \( C_1(k), C_2(k) \) are real constants and \( k \) is integer. Considering the propagation of each of these exponential terms we obtain:

\[
I_h(x,t) = \sum_{k=1}^{\infty} C_1(k) \cdot \frac{\sinh(\gamma_c(k)(\ell-x))}{Z_{\gamma_c(k)}} \cdot e^{\gamma_c(k) \ell}
\]

\[
+ C_2(k) \cdot \frac{\cosh(\gamma_c(k)(\ell-x))}{Z_{\gamma_c(k)}} \cdot e^{-\gamma_c(k) \ell}
\]

\[
V_h(x,t) = \sum_{k=1}^{\infty} C_1(k) \cdot \cos(\gamma_c(k)(\ell-x)) \cdot e^{\gamma_c(k) \ell}
\]

\[
+C_2(k) \cdot \cos(\gamma_c(k)(\ell-x)) \cdot e^{-\gamma_c(k) \ell}
\]  

\[
(14b)
\]

\( C_1(k) \) and \( C_2(k) \) are suitably calculated to satisfy the boundary condition \( V(x, x\sqrt{L_c/C_c}) = 0 \) at any point \( x \) of the electrode. That is to assure velocity of propagation \( 1/\sqrt{L_c/C_c} \). Furthermore \( C_1(k) = C_2(k) \), so the expressions (14a) and (14b) are further simplified.

### B.5. Complete time-domain solution of the differential equations

Complete time domain solution of the equations of propagation of voltage and current waves along the electrode, is:

\[
I(x,t) = I_h(x,t) + I_p(x,t)
\]

\[
V(x,t) = V_h(x,t) + V_p(x,t)
\]

Only a few of the infinite terms in (14) are needed to form the solution (15) with high accuracy.

### C. Determination and estimation of the Effective Length

In case a sinusoidal current source is applied at the energization end of a grounding electrode, when its length increases, maximum GPR decreases almost continuously. However, there is an upper limit in the length of the electrode that seriously effects the max GPR value observed in case of a lightning strike. The length of the conductor value, beyond which no serious decrement is observed, is called here “effective length”. The max GPR value decrement is constant versus length increment, and it is rapid for lengths below the effective length and slow when conductors are longer than the effective length.

Another definition of the effective length determines it as the length of the horizontal conductor, beyond which the maximum transient voltage at the injection point is length independent.

An example of effective length is given in Fig. 3. The maximum GPR produced by a 9kA 1.4/17µs impulse current strike and also by a sinusoidal current with the same magnitude at the injection end of horizontal conductors of various lengths are comparatively plotted. The 1.4/17µs impulse current wave shape has short rising time to its peak. For this reason there is larger deviation of the results from those obtained when the sinusoidal current is injected.
Effective length is dependent on frequency or the rise time of the impulse energization. It can be estimated using a diagram of $|Z|$ vs. length or $|Z|$ vs frequency. Effective length of an electrode in 500 Ohm-m soil in 100 kHz is 20 m as it can be seen in Fig. 4a. In Fig. 4b, $|Z|$ of a 20 m long electrode vs. frequency has been plotted. It remains to a low value until 100 kHz where it begins to increase. Plot of $|Z|$ of a longer electrode would show the expected increase to a lower frequency value, as it would be more reactive. Consequently it appears that 20m corresponds to the longer electrode with low impedance in 100 kHz, so that is the effective length value in 100 kHz.

III. APPLICATION EXAMPLES

The proposed model is applied to the transient analysis of a 140m long electrode with radius 1.5mm, buried in 0.9m in 300 Ohm-m soil. Injection current has a 7/28 µs waveform. Voltage profiles at various points of the electrode are shown in Fig. 5. The soil relative permittivity is considered low so the effect of the capacitive component weakens. In this case, the electrode shows a reactive behaviour. This results in faster appearance of the voltage peak at the injection point.

In Fig. 6 the transient responses of horizontal grounding conductors with different lengths buried in soil with $\varepsilon_r=4$, $\rho=1000\Omega\text{m}$, and $\mu=1$ are simulated. Impulse current is given from the formula $I(t)=I(e^{-500000t} - e^{-1000000t})$. The lengths of the conductors are 20m, 40m, 50m, 80m, 100m, and 280m. The radius of the conductors is 4mm and the burial depth is 0.75m. In Fig. 6 the magnitude of the transient voltage at the injection point is decreasing and finally goes to an asymptotic value when the length of the conductor is increasing. According to the definitions of the effective length of horizontal grounding conductor, the maximum transient voltage at the injection point will not decrease further, when the length of the conductor exceeds a certain value. It is observed from the simulations that the effective length of the horizontal grounding conductor for the given current impulse and soil conditions is approximately equal to 40m.

The ratio of peak voltage to peak current has been plotted when a fast 1/4µs (Fig. 7) and a slow 8/20µs (Fig. 8) impulse current has been injected to show the influence of effective length of electrodes placed in soil with various characteristics.

In Figs 7 and 8 it can be observed that the faster the injection current rises to its maximum value, the shorter the effective length is.
IV. CONCLUSIONS

In this paper transmission line modeling of grounding electrodes is presented and used for calculation of the transient response of grounding electrodes. The method used to solve for voltage and current distribution along the electrode is based on closed form solution of the telegraphy equations. Solution is achieved directly in time domain, so any transformation to and from the frequency domain is not needed. The main advantage is that results are obtained using only a few terms in the final expressions, making the method simple and easy to apply.

In the paper special attention is given to the accuracy of calculation of per unit length parameters of the electrode.

Finally the effect of effective length of grounding electrodes that limits their performance in higher frequency energization is demonstrated, for various soil and impulse excitation characteristics.

V. REFERENCES


VI. BIOGRAPHIES

M.I. Lorentzou received a PhD from the Electrical and Computer Engineering Department, NTUA in 2001. Her research activity concerns analysis of the transient behavior of grounding systems, Lightning Protection of Wind Turbines, and calculation of power system transients. She works at the Power System Planning Department of the Hellenic Transmission System Operator. She is member of Technical Chamber of Greece.

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