UMEC Transformer Model for the Real Time Digital Simulator

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Abstract **-- This paper introduces the UMEC (Unified Magnetic Equivalent Circuit) transformer model recently implemented for the RTDS® Simulator. The UMEC transformer algorithm was reorganized to meet the requirement of real time simulation. The detailed implementation and optimization of the algorithm are described. Validation of the model is also presented. The validation was conducted by comparing the real time output with that of analytical calculations and off-line simulation results from EMTDC™.**

Keywords **-- UMEC Transformer, Matrix sparsity, Numerical oscillation, Real time digital simulation**

I. INTRODUCTION

A new transformer model based on the Unified Magnetic Equivalent Circuit (UMEC) algorithm has been implemented for the RTDS® Simulator. The purpose of developing the model was to represent the inter-phase coupling of 3 limb and 5 limb power transformers as part of real time simulations.

The RTDS Simulator is a real time power system simulator operating continuously in real time. The system performs electromagnetic transient simulations and is often used for closed loop testing of physical devices (e.g. controllers and protective relays). It is therefore not only challenging, but also critical to maintain hard real time execution.

The RTDS Simulator is widely used for closed loop testing of protective relays and the effect of transformer inter-phase coupling can in some cases be important. In particular, the effect is important when considering the zero sequence impedance of a two winding wye-wye connected transformer. In this configuration, a 3-limb transformer presents a higher zero sequence impedance than three single phase banks, because part of the zero sequence flux is forced out of the core.

Prior to the implementation of the UMEC model, transformers were represented as single phase banks with no magnetic coupling between the phases of the transformer. The conventional three phase transformer model in electromagnetic transient (EMT) type programs assumes that the three phases are independent. This means three phases are exactly balanced and the flux in core legs and yokes are as in three single-phase transformers. Moreover the primary and secondary winding leakages are combined and the magnetizing current is placed on one side. The physical structure is basically ignored using this conventional modeling approach.

In reality, the saturation of the iron in a three-phase transformer is not uniform. Particular limbs may saturate more or less depending on the currents flowing in the transformer. This makes it difficult to provide a more detailed representation of three-phase transformer saturation. Fortunately, there has been considerable effort placed on providing these more detailed models [1][2][3]. The UMEC transformer model was developed based on magnetic circuit theory, which represents the core's physical structure as well as the mutual coupling of the electrical windings. Therefore it is a more accurate model to represent three phase transformers, especially those of three-limb construction.

Since the fundamental algorithm was already established [1], the challenge of implementing the UMEC model on the RTDS Simulator was to solve the algorithm in real time.

The basic theory of the UMEC transformer model implemented in RTDS and EMTDC is the same. The paper will introduce the details in the implementation of the UMEC transformer, such as the application of matrix sparsity techniques and compensation of the numerical oscillations.

Finally detailed verification will be presented. The UMEC transformer model for the RTDS Simulator was verified using two different approaches. The first approach was to test the transformer model at different operating conditions such as short circuit, open circuit and rated load and compare to analytical calculation. The other approach was to compare the UMEC transformer simulation results against the results obtained from EMTDC. It will be presented that both the analytical calculations and comparison with EMTDC prove the performance and validity of the new UMEC transformer model.

II. MODEL FORMULATION

First, the mathematic model of a multi limb transformer, based on the unified equivalent magnetic circuit theory is derived. The three-phase, three-limb transformer shown in Fig 1 is used as an example.

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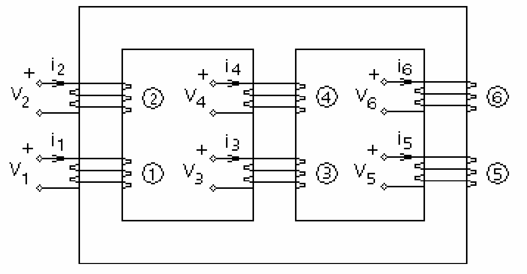


Fig. 1 Three phase three limb transformer model

In vector form, the relationship of the branch flux and the magnetic motivation force (MMF) is

$$\phi_k = [P_k]([N]i - \theta) \tag{1}$$

whereby ϕ_k is the flux vector, P_k is the permeance matrix, i is the winding current vector, N is the vector for the number of windings, θ is the MMF branch vector, and the subscript k denotes the number of the magnetic branch.

At each node the flux must sum to zero as follows:

$$[A]^T \phi = 0 \tag{2}$$

Application of the branch-node connection matrix A to the vector of the node MMF gives the branch MMF:

$$[A]^T \theta_{node} = \theta \tag{3}$$

Combining (1), (2) and (3) gives,

$$\phi = [M][N]i\,, \tag{4}$$

where

$$[M] = [P] - [P][A]([A]^T[P][A])^{-1}[A]^T\,. \tag{5}$$

The winding voltage vector V_s is related to the branch flux ϕ_s by Farady's Law of magnetic flux. The relationship in trapezoidal discrete format is,

$$\phi_s(t) = \phi_s(t-\Delta t) + \frac{\Delta t}{2}(V_s(t) + V_s(t-\Delta t))\,. \tag{6}$$

The subscript "s" denotes the subset of all the magnetic branches on which windings are mounted. Solving for the current i in (4) and (5), the standard trapezoidal discrete format of the transformer equations is obtained as,

$$i_s(t) = [Y_{ss}]V_s(t) + I_{hist}\,, \tag{7}$$

in which the equivalent admittance matrix is,

$$[Y_{ss}] = ([M_{ss}][N_{ss}])^{-1}\frac{\Delta t}{2}[N_{ss}]^{-1}, \tag{8}$$

and history current injection vector is,

$$I_{hist} = ([M_{ss}][N_{ss}])^{-1}(\frac{\Delta t}{2}[N_{ss}]^{-1}V_s(t-\Delta t) + \phi_s(t-\Delta t)) \cdot \tag{9}$$

III. SATURATION MODELING

The saturation phenomenon is represented in the UMEC transformer model by updating the variable permeance matrix P for each time step. The saturation curve of flux vs. MMF is computed and stored on the processor. During the simulation, either flux or MMF can be obtained for each magnetic branch. The flux for branches with a winding can be obtained from (6) and the MMF for those without can be obtained from (3). Based on this, the permeance is determined from the saturation curve using (10) as shown in Fig. 2.

$$P_i = \frac{\phi_i}{\theta_i} \tag{10}$$

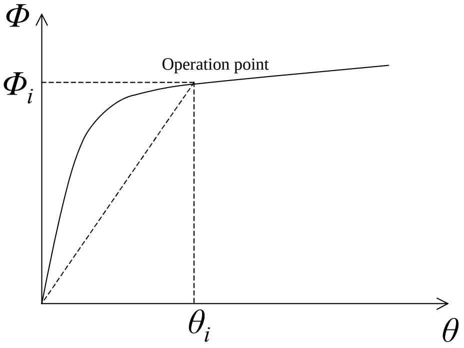


Fig. 2 Saturation Curve

IV. EXPLORATION OF THE MATRIX SPARSITY

The highest efficiency is always sought in designing a real time simulation algorithm. It can be observed in (5) and (7) that the computation of the equivalent admittance matrix Y_{ss} and history current injection I_{hist} requires significant matrix manipulation, including the inversion of a 6×6 matrix two times. Further investigation of the matrix manipulations reveals that the matrix A is highly sparse while the matrix P is diagonal. In order to explore the matrix sparsity and reduce the computation load, the algorithm was reorganized as follows.

Substituting (5) into (8) yields:

$$[Y_{ss}] = \{\big[[P] - [P][A]([A]^T[P][A])^{-1}[A]^T\big][N_{ss}]\}^{-1}\frac{\Delta t}{2}[N_{ss}]^{-1} \tag{11}$$

Matrix *[A]*^{T}*[P][A]* can be manually derived and inverted as,

$$[B] = ([A]^T[P][A])^{-1}\,. \tag{12}$$

Combining (11) and (12), [Y_{ss}] can be derived as,

$$\begin{aligned}
[Y_{ss}] &= \{\big[[P] - [P][A][B][A]^T\big][N_{ss}]\}^{-1}\frac{\Delta t}{2}[N_{ss}]^{-1} \\
&= \{[P][N_{ss}] - \big[[P][A][B][A]^T\big][N_{ss}]\}^{-1}\frac{\Delta t}{2}[N_{ss}]^{-1} \qquad (13) \\
&= \{[P][N_{ss}] - \big[[P][A][B]\big][A]^T[N_{ss}]\}^{-1}\frac{\Delta t}{2}[N_{ss}]^{-1} \\
&= \{[E] - [C]\cdot[D]\}^{-1}[F]
\end{aligned}$$

in which,

$$[E] = [P][N_{ss}]\,, \tag{14}$$

$$[C] = [P][A][B]\,, \tag{15}$$

$$[D] = [A]^T[N_{ss}]\,, \tag{16}$$

and $$[F] = \frac{\Delta t}{2}[N_{ss}]^{-1}. \tag{17}$$

It should be noted that matrices [*B*], [*C*], [*D*], [*E*], and [*F*] and the relationships among them can be handled analytically such that only non-zero elements need be considered. This approach takes full advantage of the matrix sparsity and provides a large reduction in the computational load.

A comparison of the approximate numbers of multiplications and additions to obtain Y_{ss} according to the original and improved algorithm is listed in Table 1. The comparison demonstrates that the efficiency of the algorithm has been vastly improved by taking advantage of the matrix sparsity. Without the use of the sparsity techniques applied, the computational load would not have been acceptable for real time simulation.

TABLE I
COMPARISON OF COMPUTATION LOAD

	Number of multiplying	Number of Adding
Original algorithm	11016	9720
Improved algorithm	320	380
Saving	95.1%	93.9%

V. COMPENSATION OF THE NUMERICAL OSCILLATIONS

The modeling of saturation causes a one time step numerical oscillation on the transformer terminal voltage. In non-real-time simulations such as EMTDC, the problem is solved by applying interpolation and chatter removal. In real time simulation however, these algorithms cannot be used due their heavy computation burden. Therefore alternate compensation methods had to be utilized to remove the numerical oscillation.

After investigating different approaches, the numerical oscillation was damped by connecting the compensation circuit shown in Fig. 3 to the terminals of the UMEC transformer. The circuit applies injections based on the voltage from the last time step and proved effective in removing the numerical chatter.

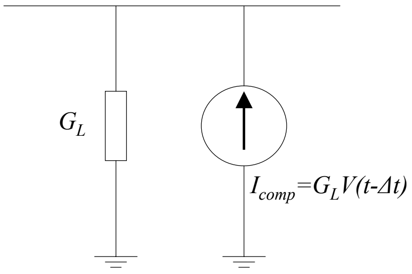


Fig. 3 Compensation circuit

VI. VALIDATION OF UMEC TRANSFORMER

The UMEC transformer model for the RTDS Simulator was verified using two different approaches. The first approach was to test the transformer model at different operating conditions (i.e. short circuit, open circuit and steady state load) and compare the results with those of analytical calculation. The other approach was to compare the UMEC transformer simulation results against the results obtained from EMTDC. Some of the validation study results are shown in this section. The simulation time step in all the cases is 50 microseconds.

VI.I Model Test

These tests are intended to show correct and proper simulation of the RTDS UMEC transformer model. Although, a number of different transformer configurations are available, only the three-phase, three-limb, two-winding model is documented in this section. In order to simplify the comparison of results against theoretical calculation, a Y-Y winding configuration was assumed.

The following test cases are documented below:

Case 1a Short Circuit Test with 10% Magnetizing Reactance
Case 1b Short Circuit Test with 25% Magnetizing Reactance
Case 1c Open Circuit Tests with 1% Magnetizing Reactance
Case 1d Open Circuit Tests with 10% Magnetizing Reactance
Case 1e Steady State Load Test
Case 1f Saturation Test

A. Short Circuit Tests

This test is used to confirm proper representation of the transformer leakage reactance. The secondary terminals of the transformer are shorted to ground and rated voltage was applied to the primary. The primary current was monitored and compared to the expected (calculated) values shown in table II.

TABLE II

RESULTS COMPARISON OF SHORT CIRCUIT TEST

Case Name	I_{1RMS} (kA)	
	Expected	Measured
Case1a	2.51029	2.510
Case1b	1.0041	1.005

B. Open Circuit Test

This test is used to confirm proper representation of the transformer magnetizing reactance. The secondary terminals of the transformer are open circuited and rated voltage was applied to the primary. The primary current was monitored and compared to expected (calculated) values in table III.

TABLE III

RESULTS COMPARISON OF OPEN CIRCUIT TEST

Case Name	I_{1RMS} (kA)	
	Expected	Measured
Case1B1	0.00251	Changes around 0.0023
Case1B2	0.0251	Changes around 0.023

It should be noted that the measured currents shown in Table III were unsymmetrical due to the 3-limb core structure of the transformer. The relatively large asymmetry observed is expected with the small current levels indicative of open circuit tests. As a result of the asymmetry, the average 3-phase RMS meter displayed a value that varied around the expected value.

C. Steady State Load Test

This test is used to show that the expected operating conditions are achieved in steady state with rated load applied to the transformer terminals. The primary and secondary voltages and currents were monitored and compared to the calculated values shown in table IV.

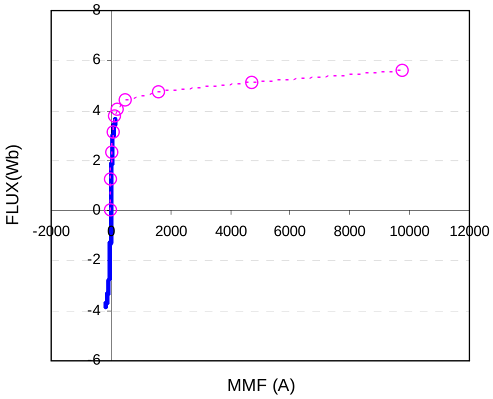
TABLE IV

RESULTS COMPARISON OF STEADY STATE LOAD TEST

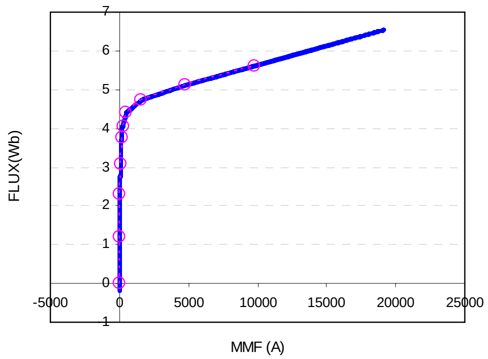
Case Name		V_{1RMS} (kV)	I_{1RMS} (kA)	V_{2RMS} (kV)	I_{2RMS} (kA)
Case1e	Expected	132.47	0.3283	65.661	0.6566
	Measured	132.4	0.3279	65.61	0.6561

D. Saturation Test

This test was used to confirm the proper representation of the saturation curve used for the transformer model. The verification was made by inspection of the x-y plot of MMF versus flux. Two different cases were tested and the results are plotted in Fig. 4. Fig. 4 (a) and (b) are the Flux-MMF plots for steady state operation and an inrush transient respectively. The small circles are the input data and the black curve represents the measured result. It can be seen that the saturation characteristic of the transformer performs as expected.



(a) Steady state load test



(b) Inrush transients

Fig. 4 Simulation results of saturation test

VI.II Comparison with EMTDC

In this section, results from the RTDS UMEC transformer model are compared with those from EMTDC. The basic theory of the UMEC transformer model implemented in the RTDS and EMTDC are the same. The only fundamental difference is that EMTDC uses chatter removal to combat numerical oscillation caused by the saturation nonlinearity. However since the RTDS must operate in real time, chatter removal could not be implemented and a compensation circuit was used instead. In order to verify the performance of the UMEC transformer model in the RTDS, identical cases were set up in EMTDC and on the RTDS Simulator. Extensive tests were conducted and the results from the two simulation programs compared. Results from three such comparisons are shown below. Fig. 5 compares the transient current during transformer energization with a load. Fig. 6 compares the inrush current when the transformer was energized with the secondary open circuited. Fig. 7 compares the transient current when a ground fault occurs on phase A of the secondary. The plots demonstrate an excellent correlation of results between the RTDS Simulator and EMTDC.

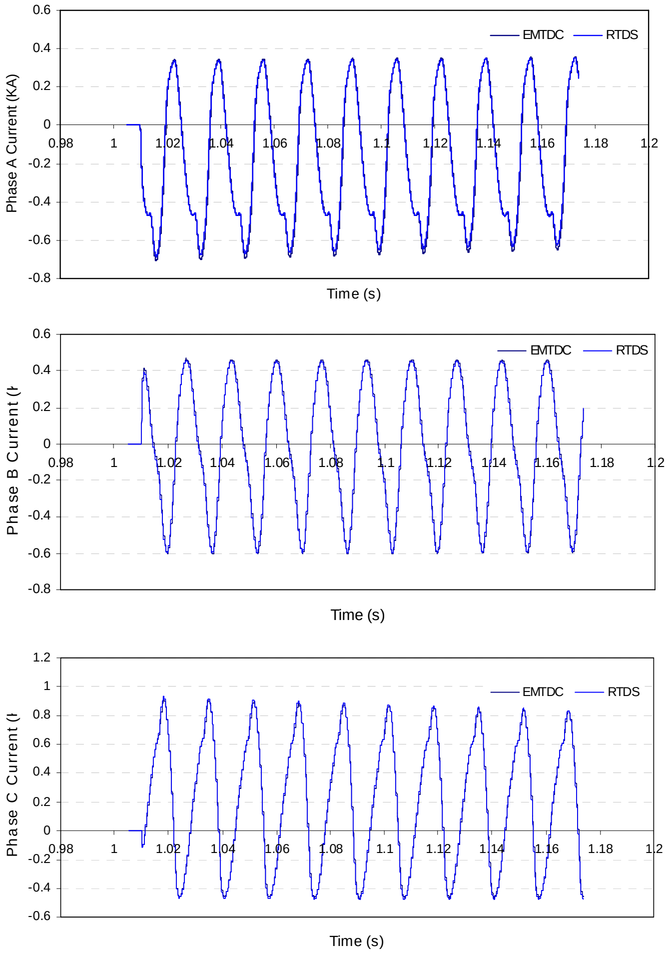


Fig. 5 Transients current for transformer energization

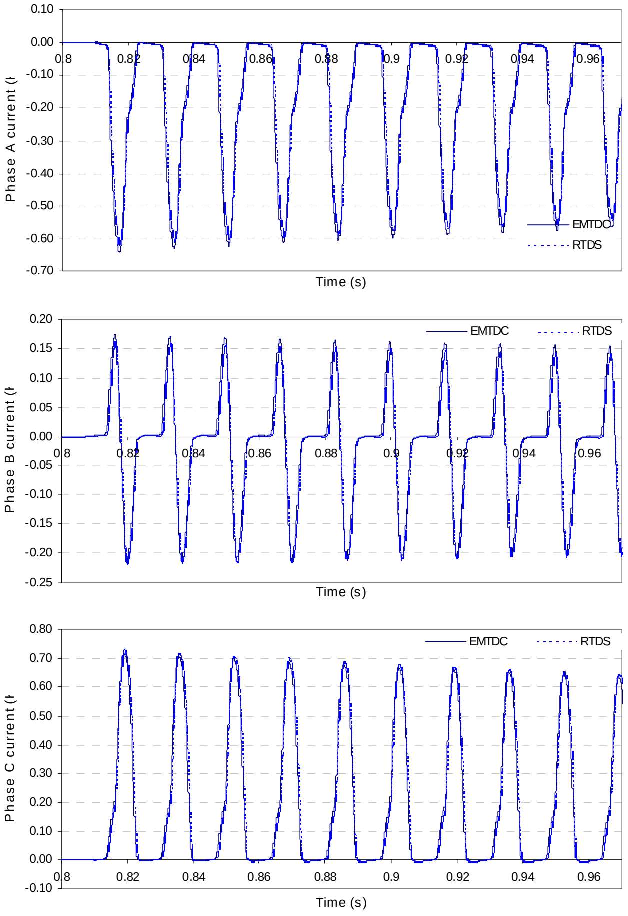


Fig. 6 Inrush current

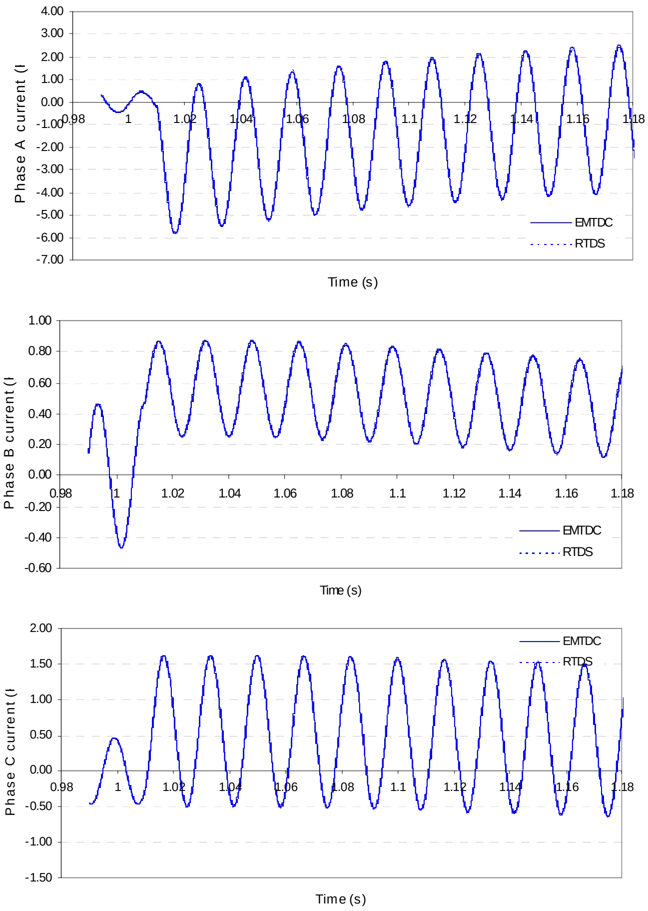


Fig. 7 Transients current when phase A is grounded

VII. Conclusion

This paper describes the implementation of the UMEC transformer model in the RTDS Simulator. Through the use of optimization techniques, including taking advantage of matrix sparsity, it was possible to achieve real time simulations that included the UMEC model. The accuracy of the real time model was verified by comparing simulation results to analytic calculations and to EMTDC simulation results. Subsequently the new model was included in the RTDS Simulator component library.

VIII. REFERENCE

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