# Calculation of Voltage Sag Indices for Distribution Networks

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*Abstract*— **This paper is presenting the main results of a work aimed at predicting the voltage sag performance of distribution networks by estimating voltage sag indices and using a time-domain simulation tool. The work is based on a Monte Carlo procedure developed by the authors using capabilities of the ATP package. The output of the procedure is the probability density function of voltage sag characteristics and the number of sags per year at every phase of each node of the system under study. The document includes a discussion on modeling guidelines for representing distribution system components in voltage sag calculations, a short summary of the Monte Carlo procedure for stochastic prediction of voltage sags and a detailed analysis of a test system.**

*Index Terms*—**Power Quality, Voltage Sags, Simulation, Modeling, Power Distribution.**

## I. INTRODUCTION

DIGITAL simulation is an alternative to monitoring for analysis and prediction of the voltage sag performance of a power system. Voltage sag characteristics can be accurately reproduced by means of a simulation tool based on a time-domain technique, and using a stochastic prediction that could incorporate the random nature of the voltage sag causes [1] - [4].

When using computer simulation for voltage sag assessment, the following aspects should be taken into account:

- Sags are transient events caused by faults, transformer energizing, motor starting and sudden load changes; all these phenomena can be classified as low- or mid-frequency transients.
- Faults are the main cause of voltage sags at transmission and distribution levels, and their characteristics (location, duration, resistance, type) are random.
- The load shows a time variation and the demand is diversified (i.e. non-coincident), so the impact of a voltage sag will depend on the instant at which the event is caused and on the diversity between loads.

The aim of this work is to predict the voltage sag performance of a distribution network by estimating voltage sag indices and using a time-domain simulation tool. The work is based on the application of a Monte Carlo procedure developed using capabilities of the ATP package [4], [5].

The tasks needed to obtain voltage sag indices can be summarized as follows: internal capabilities of the transients program are used for the development of power components and load models; capabilities of the program are linked to external routines for assessment of voltage sags using a Monte Carlo method; the output results are post-processed to obtain voltage sag indices.

The study will be performed by assuming that voltage sags are caused only by faults (i.e. short-circuits), there is no distributed generation in the system, and mitigation devices are not installed. Section II presents a summary of the modeling guidelines used in this work for voltage sag simulations. Section III summarizes the capabilities of a routine developed for random generation of distribution networks. Section IV provides a short summary of the procedure for stochastic prediction of voltage sags. Section V presents the results derived from a stochastic prediction of voltage sags, with emphasis on the effect of the protection system. Section VI details the calculation of voltage sag indices.

## II. MODELING GUIDELINES

Fig. 1 shows an example of voltage sag and the characteristics that could be needed to assess its effect: the magnitude, the duration and the phase angle jump. The point on wave is another aspect to be considered [1]. Note that to obtain the duration a threshold voltage must be set [6].

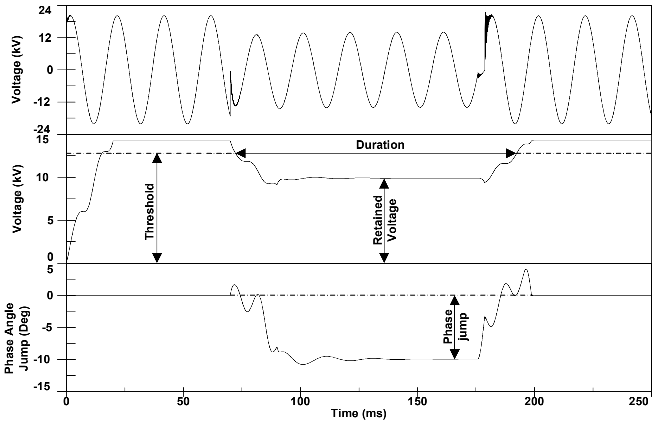

Fig. 1. Example of voltage sag and its characterization.

The simulation of such transient events and the determination of the voltage sag characteristics can be performed by means of many current time-domain simulation tools, e.g.

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EMTP-like programs. Built-in capabilities available in these tools can be used to reproduce very accurately most transients in power systems. However, an accurate representation of some components is not easy; e.g. a transformer model may require the representation of its nonlinear and frequency-dependent behavior. In addition, the user can be forced to choose between an accurate model and a feasible model, e.g. a detailed model of a dynamic voltage restorer (DVR) requires a very small time step size (about 1 μs) and would be time consuming in probabilistic studies.

The representation of the equipment involved in a transient process is usually chosen taking into account the range of frequencies that are associated to the simulated phenomenon. In general, transients associated to voltage sag causes can be classified as low and mid-frequency transients. If only voltage sags caused by faults are simulated, the frequency range of transients is in general below 5 kHz; therefore, models to be developed should be capable of reproducing very accurately transients below that frequency.

Load modeling is an important and complex issue [7]; all simulations performed in this work have been based on a constant impedance representation.

Table I shows a summary of modeling guidelines to be used by default in voltage sag simulations when using a tool based on a time-domain solution [8], [9]. When a stochastic prediction is to be performed and the test system must be simulated several thousand times, it is recommendable to use a maximum time-step size, e.g. 100 μs, and some simplified models.

TABLE I
MODELING GUIDELINES FOR VOLTAGE SAG STUDIES

| COMPONENT | MODELING GUIDELINES |
|---|---|
| Network equivalents | The most accurate representation should be deduced from the frequency response of the transmission system that is feeding the distribution network; however, a three-phase Thevenin equivalent model deduced from the short-circuit capacity will be good enough in most cases. |
| Lines and Cables | Lumped-parameter models are usually acceptable; however, distributed-parameter models should be used to obtain very accurate simulation results with any voltage sag transient. |
| Transformers | Saturable models are needed when transformer energization is the voltage sag cause; however, when the event has a different cause, e.g. a short-circuit, linear models can produce accurate enough results. |
| Protection devices | Circuit breakers, reclosers and any type of disconnectors can be represented as ideal switches. A more sophisticated model (nonlinear resistance) is generally needed to represent fuses. Protective relay models should only incorporate delays and reclosing times. |
| Loads | Although a constant impedance (i.e. a parallel R-L) model can be good enough in many cases, an accurate load model could also show voltage dependence, dynamic behavior and voltage sag sensitivity. In addition, for stochastic studies, the load model could incorporate a daily variation and a random nature. |

## III. RANDOM GENERATION OF DISTRIBUTION NETWORKS

### A. *Summary of capabilities*

A routine for random generation of distribution networks has been developed. Its main goal is to make available a tool for a fast and easy edition of systems of any size. The model of a distribution network must include all components needed for voltage sag calculations. Monitoring devices are also needed to capture voltage sag characteristics at both medium and low voltage sides of the distribution transformers.

The present version of the routine allows users to specify the number of feeders and the number of load nodes per feeder. By default, a load node model consists of a distribution transformer plus a LV load, and a monitor model at both MV and LV sides. Users can also select the seeds required by the algorithms implemented for random generation of those quantities needed to obtain the parameters of the system; this can guarantee that two systems will be either the same or different every time the routine is run. The main limitation of the routine is related to protective devices since both the location and the characteristics of each device must be manually selected by the user.

### B. *Test System*

Fig. 2 shows the diagram of the test system and the characteristic of the protective devices. The lower voltage side of the substation transformer is grounded by means of a zig-zag reactor of 75 Ω per phase.

## IV. STOCHASTIC PREDICTION OF VOLTAGE SAGS

The procedure for voltage sag assessment is based on the Monte Carlo method and assumes that sags are due only to faults caused within the distribution network. The test system is simulated as many times as required to achieve the convergence of the Monte Carlo method. Every time the system is run, fault characteristics are randomly generated using the following distributions:

- The fault location is selected by generating a uniformly distributed random number, since it is assumed that the probability is the same for any point of the distribution system.
- The fault resistance has a normal distribution.
- The initial time of the fault is uniformly distributed within a power frequency period.
- The duration of the fault has also a normal distribution.
- Different probabilities are assumed for each type of fault.

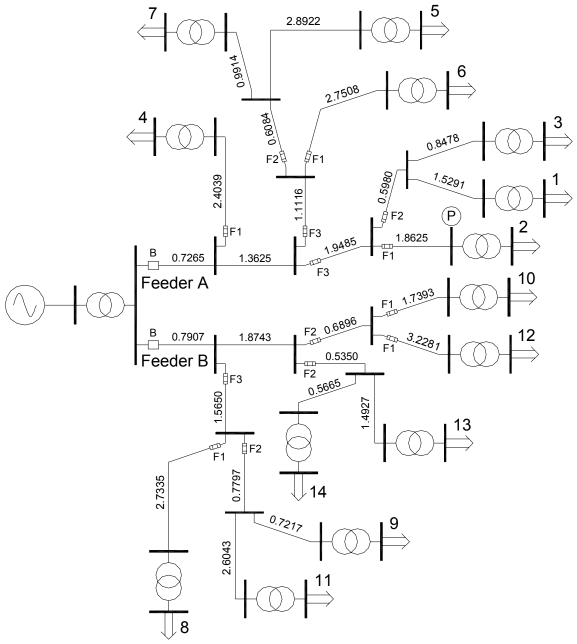
A constant resistance model is used for representing the fault impedance.

As mentioned above, loads are represented as constant impedances. If only the characteristics of voltage sags caused by faults are of concern, this representation is good enough.

## V. VOLTAGE SAG ASSESSMENT

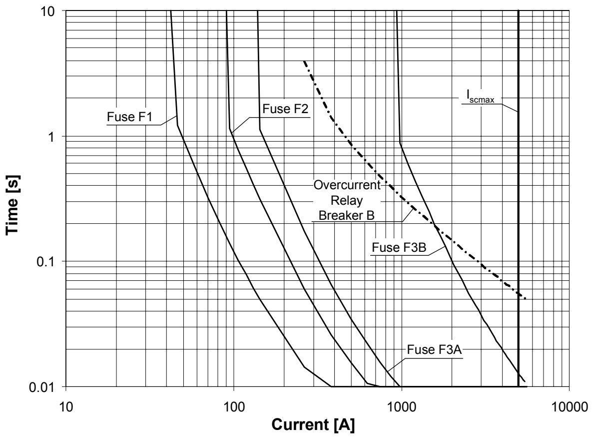
### A. *Introduction*

The following studies considering a different coordination between protective devices were performed:



HV equivalent: 110 kV, 1500 MVA, X/R = 10
Substation transformer: 110/25 kV, 20 MVA, 8%, Yd
Distribution transformers: 25/0.4 kV, 1 MVA, 6%, Dy
Lines: $Z_{1/2} = 0.61 + j0.39$, $Z_0 = 0.76 + j1.56$ Ω/km

a) Diagram of the test system.



b) Time-current characteristics of protective devices.

Fig. 2. Test system.

a. Protective devices do not operate; that is, the fault condition disappears before any device could open.
b. Circuit breakers operate faster than fuses and their relays have one reclose operation, being the reclosing time 200 ms. The simulations are performed without including fuse models.
c. The coordination between overcurrent relays and fuses allows fuses to operate. Curve labeled F3A in Fig. 2b is selected for fuses F3. Relays will have one 200 ms reclose operation.
d. The same as for the previous study, but allowing feeder relays to have two 200 ms reclose operations, and selecting fuse curve F3B.

The fault characteristics were randomly generated according to the following distributions:

- The fault location was selected by generating a uniformly distributed random number.
- The fault resistance had a normal distribution, with a mean value of 5 Ω and a standard deviation of 1 Ω, for each faulted phase.
- The initial time of the fault was uniformly distributed between 0.05 and 0.07 s.
- The mean value of the fault duration was varied, and by default the standard deviation was 10% of the mean value.
- The probabilities of each type of fault were LG = 75%, 2LG = 17%, 3LG = 3%, 2L = 3%, 3L = 2%.

In all studies the test system is run 1000 times and the following information is recorded during every run:

- fault characteristics (location, initiation time, duration, resistance, faulted phases, type of fault)
- voltage sag characteristics (retained voltage, duration) on every phase of all load nodes.

If it is assumed a probability of occurrence of 12 faults per year and 100 km of overhead lines, 1000 runs are equivalent to analyze the performance of the test system during 214 years.

### *B. Simulation results*

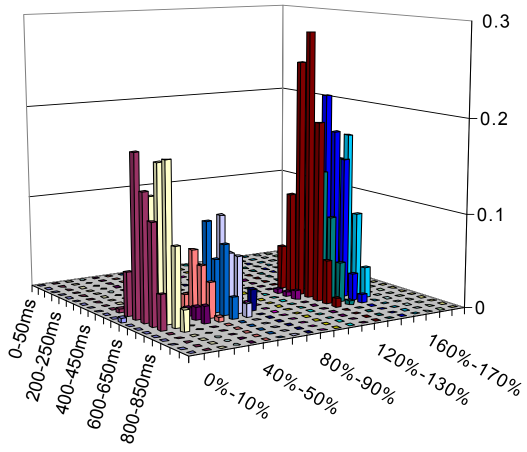
Fig. 3 shows respectively the number of sags per year caused at a MV node and the severity of these sags as compared to the ITIC curve. These results were deduced by assuming that the average fault duration was 600 ms and the standard deviation was 60 ms. Only sags caused at one phase are shown; since the probability of occurrence of faults is assumed to be the same at each phase in all lines, the charts will be very similar for each phase of the same node.

Acceptability curves can be used to predict equipment maloperation. Although not much information is available in the literature about equipment performance in front of three-phase unbalanced sags, acceptability curves for single-phase equipment are widely used. Voltage sag indices can be easily calculated when only single-phase equipment is installed.

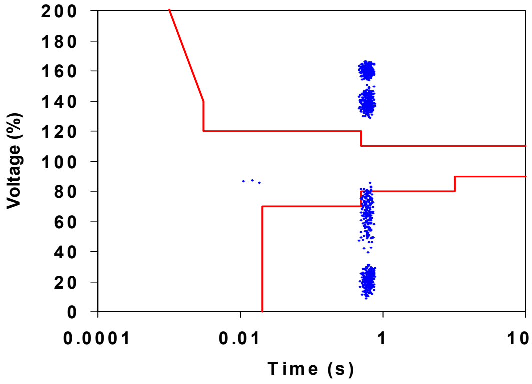
As a consequence of the protective device operation, voltage sags will not be always rectangular, since the coordination between protective devices can produce multiple events with different retained voltages. An important aspect is the characterization of these events.

According to IEEE P1564 [6], multiple events should be merged in a single event since the effect on end-user equipment will be the same. For instance, if the first event causes the equipment to trip, the process will be still down when the second event occurs.

The effect of voltage sags on sensitive equipment can be easily deduced from the results presented in Fig. 3b. However, it is important to emphasize that the sags caused at the LV side of a transformer can be very different from those caused at the MV side. Since it is assumed that only LV single-phase equipment is installed, only sags caused at the LV side of load transformers will be analyzed.



a) Number of sags per year



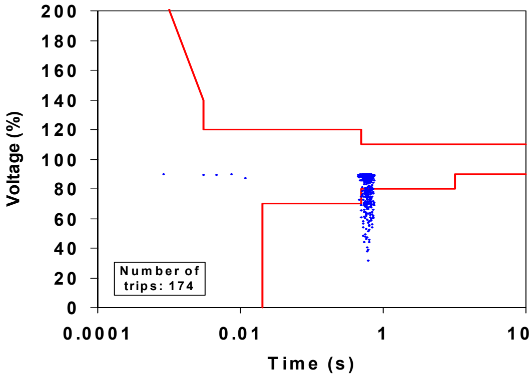
b) Voltage sag characteristics – ITIC curve

Fig. 3. Simulation results – 1000 runs
(Node 6, MV side, phase A – Mean fault duration = 600 ms).

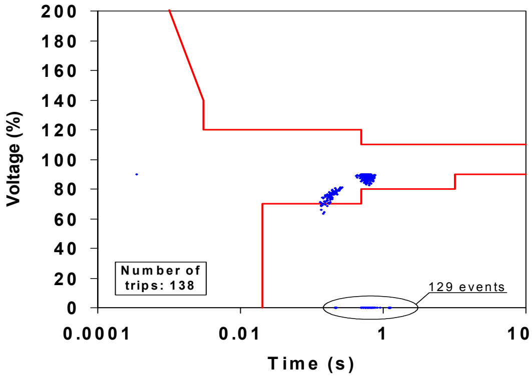
Fig. 4 shows some simulation results that were derived from the four studies mentioned above. All these results correspond to the voltage sags caused at the LV side of the load node 6 and were deduced by assuming that the mean fault duration was 600 ms. The plots show the number of trips per phase (i.e. the number of voltage sags below the lower ITIC curve) caused to sensitive equipment, and were obtained after removing events with a retained voltage between 90 and 110% of the rated voltage.

From these results one can deduce that the number of trips shows an evident dependence with respect to the design of the protection system and that the quantity of dangerous sags is very different at each side of a distribution transformer, see Fig.3b. The performance can be improved by including recloser and fuse operations, but the optimal design should also consider the characteristic curves of protective devices.

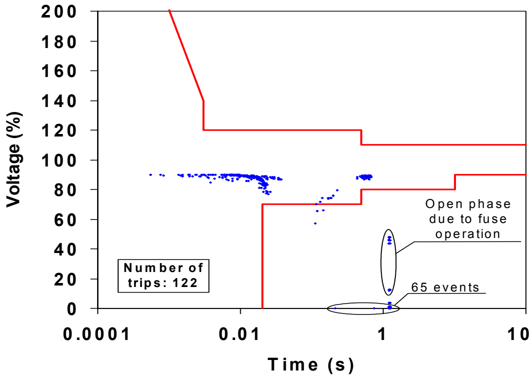
Table II shows a summary of the main results obtained in this work. The results were derived after 1000 runs, and using the same random generation of fault characteristics for all cases. Equipment sensitivity was represented by the ITIC curve and only single-phase equipment was installed at the LV level. The quantities within parenthesis indicate the number of trips that will experience sensitive equipment, taking into account that the test system is analyzed during 214 years, see Section V.A.



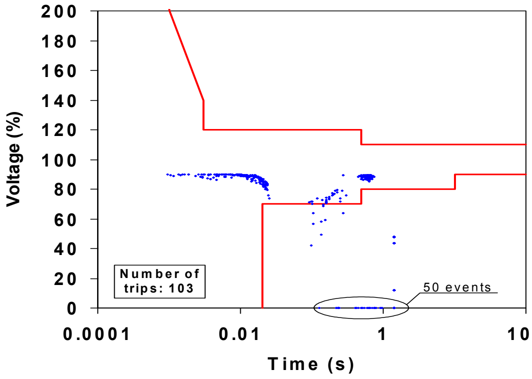
a) Protective devices do not operate



b) Breaker operation (Reclosing interval = 200 ms)



c) Fuse operation (Reclosing interval = 200 ms)



d) Breaker and fuse operation (Two reclosing intervals)

Fig. 4. Simulation results (Node 6, LV side, phase A – Mean fault duration = 600 ms).

TABLE II
AVERAGE NUMBER OF EQUIPMENT TRIPS PER PHASE AND YEAR
(NODE 6 - LV LEVEL)

| Protection system | Fault duration | | |
|---|---|---|---|
| | 200 ms | 600 ms | 1 s |
| No operation | 64<br>(0.30) | 169<br>(0.79) | 176<br>(0.82) |
| Breaker operation<br>($t_R$ = 200 ms) | 111<br>(0.52) | 133<br>(0.62) | 500<br>(2.34) |
| Breaker and fuse operation<br>($t_R$ = 200 ms) | 121<br>(0.56) | 122<br>(0.57) | 159<br>(0.74) |
| Breaker and fuse operation<br>(change fuse F3)<br>($t_R$ = 200 + 200 ms) | 82<br>(0.38) | 85<br>(0.40) | 194<br>(0.91) |

One can see that the number of trips can decrease by increasing the fault-clearing time and by introducing reclose operations. The improvement achieved by increasing the fault-clearing time is evident after comparing results derived with a mean fault duration of 200 ms: the lowest number of trips is obtained when protective devices do not operate (i.e. its operation is delayed), but a significant improvement can be also achieved if they operate and the fault-clearing time is increased for small fault currents, i.e. for single-phase-to-ground faults. The second conclusion is deduced after comparing the results obtained with a mean fault duration of 600 ms: the lowest number of trips is achieved when reclose operations are allowed. Both results are simultaneously confirmed with the last study, when reclosing is allowed and a fuse operation is delayed (see curve F3B in Fig. 2).

## VI. CALCULATION OF VOLTAGE SAG INDICES

Several indices have been proposed to assess the voltage sag performance of a power system [6], [10] - [13]. They can provide a count of event frequency and duration, the undelivered energy during events, or the cost and severity of the disturbances. For a thorough analysis of power quality indices, their advantages and limitations, see [10].

Only the first type of index, i.e. count of events, is analyzed in this paper. The above procedure is used to predict voltage sag characteristics on each phase of each load node at both MV and LV sides. This information is manipulated to obtain the number of trips per year in combination with an acceptability curve.

*SARFI* is an acronym that stands for "System Average RMS Variation Frequency Index" [6]. It gives the average number of events (sags, swells, short-interruptions) over the assessment period, usually one year, per customer served. A *SARFI* value is obtained by means of the following expression

$$SARFI = \frac{\sum_{i=1}^{n_s} N_i}{N_T} \qquad (1)$$

where $n_s$ is the number of events, $N_i$ is the number of customers experiencing an event and $N_T$ is the number of customers served from the section to be assessed.

When the index is derived from simulations, since only events caused at the MV distribution level are analyzed, the number of costumers that will experience an event at a load node is the number of costumers served from that node. Therefore, for a single site the *SARFI* value is the number of sags over the assessment period calculated as shown in [6]. The index for an entire system can be obtained as follows

$$SARFI = \frac{\sum_{j=1}^{n_n} N_j \cdot SARFI_{(j)}}{N_T} \qquad (2)$$

where $n_n$ is the number of load nodes, $N_j$ is the number of costumers served from node $j$, and $SARFI_{(j)}$ is the value for node $j$.

There are two types of *SARFI* indices: $SARFI_x$ and *SARFI-Curve*. $SARFI_x$ is defined with respect a voltage threshold $x$, and gives the number of events, with a duration between half-cycle and 1 minute, whose retained voltage deviates from the voltage threshold, while *SARFI-Curve* gives the event rate outside the operating area surrounded by an acceptability curve.

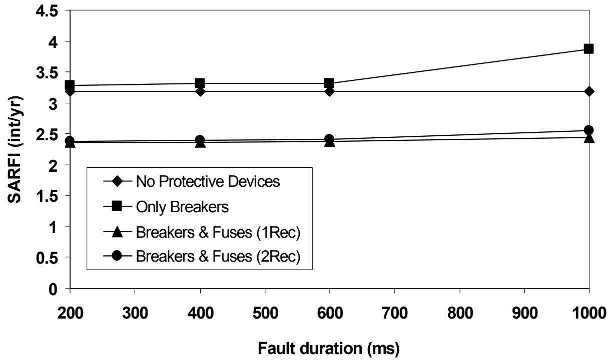
Since the acceptable power supply area is usually surrounded by an upper and a lower curve, the way in which a $SARFI_x$ index is calculated depends on the voltage threshold. For instance, $SARFI_{110}$ gives the rate of events whose retained voltage is greater than 110% of the reference voltage, while $SARFI_{60}$ gives the rate of events whose retained voltage is below 60% of the reference voltage.

The calculations in this paper are based on the assumption that there is only LV demand. However, some additional information is needed to obtain voltage sag indices: *SARFI* indices are based on the number of customers served from every load node; it will be assumed that this number is the same at every load node, i.e. at every distribution transformer, therefore equation (2) becomes

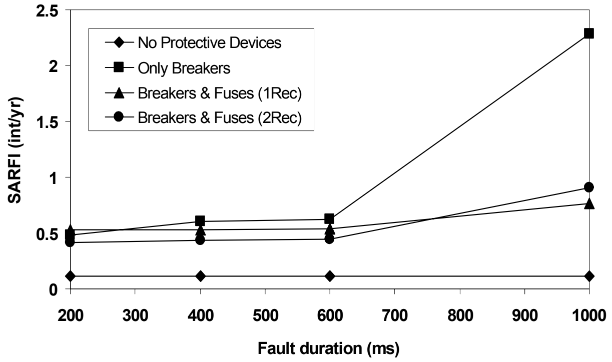
$$SARFI = \frac{\sum_{j=1}^{n_n} SARFI_{(j)}}{n_n} \qquad (3)$$

Fig. 5 shows the index values deduced for the entire test system. The main conclusions derived from the simulation results can be summarized as follows:

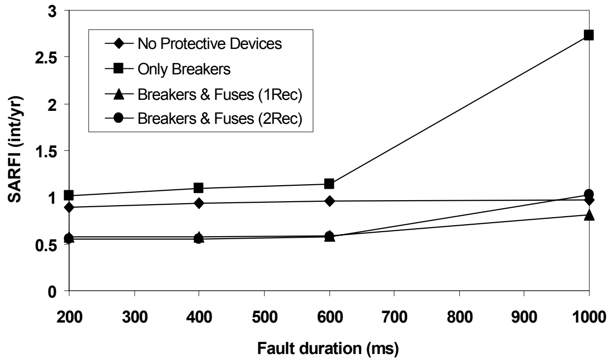
- The performance obtained with each protection scenario is not the same for every $SARFI_x$ index and some differences can be easily noted when comparing indices derived for different sites. *SARFI* values corresponding to different thresholds can show different behavior; for instance, at load node 6, the best $SARFI_{90}$ performance is achieved when all protective devices can operate, while the best $SARFI_{60}$ performance corresponds to a different scenario, i.e. when fuses are saved.
- *SARFI* values as a function of the mean fault duration do not show very significant changes for a given protection system, except when the fault duration is about 1 second or longer.
- ITIC equipment has a better performance that CBEMA equipment; however, when fuses operate the performance is very similar. Results deduced when loads are represented by means of ITIC and SEMI curves were the same.



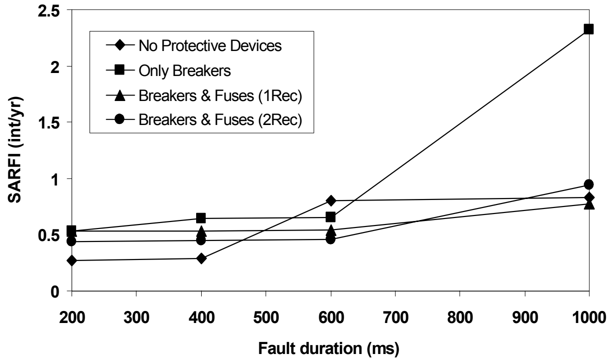
a) *SARFI*$_{90}$



b) *SARFI*$_{60}$



c) *SARFI-CBEMA*



d) *SARFI-ITIC*

Fig. 5. *SARFI* calculations for the whole system – LV nodes.

## VII. CONCLUSIONS

This paper has presented the application of a computer procedure for stochastic prediction of voltage sags to the calculation of voltage sag indices in distribution networks. The study was performed by assuming that sags are caused only by faults. The results deduced from the procedure are used to obtain the average number of events experienced by each customer served over the assessment period. The results presented in this paper have shown the influence that the design of the protection system and the equipment vulnerability can have on these indices.

## VIII. REFERENCES

[1] M.H.J. Bollen, *Understanding Power Quality Problems. Voltage Sags and Interruptions*, IEEE Press, 2000, New York.

[2] S.O. Faried and S. Aboreshaid, “Stochastic evaluation of voltage sags in series capacitor compensated radial distribution systems,” *IEEE Trans. on Power Delivery*, vol. 18, no. 3, pp. 744-750, July 2003.

[3] P. Heine and M. Lehtonen, “Voltage sag distributions caused by power system faults,” *IEEE Trans. on Power Systems*, vol. 18, no. 4, pp. 1367-1373, November 2003.

[4] J.A. Martinez and J. Martin-Arnedo, “Voltage sag stochastic prediction using an electromagnetic transients program,” *IEEE Trans. on Power Delivery*, vol. 19, no. 4, pp. 1975-1982, October 2004.

[5] J.A. Martinez and J. Martin-Arnedo, “Expanding capabilities of EMTP-like tools: From analysis to design,” *IEEE Trans. on Power Delivery*, vol. 18, no. 4, pp. 1569-1571, October 2003.

[6] IEEE P1564, “Recommended Practice for the Establishment of Voltage Sag Indices,” Draft 6, January 2004.

[7] J.A. Martinez and J. Martin-Arnedo, “Advanced load models for voltage sag studies in distribution networks,” *IEEE PES General Meeting*, June 2004, Denver.

[8] A.M. Gole, J.A. Martinez and A.J.F. Keri (Ed.), “Modeling and Analysis of System Transients Using Digital Programs,” IEEE PES Special Publication, TP-133-0, 1999.

[9] J.A. Martinez and J. Martin-Arnedo, “Modeling of protective devices for voltage sag studies in distribution systems,” *IEEE PES General Meeting*, June 2004, Denver.

[10] Joint WG CIGRE c4.7/CIRED, “Power Quality Indices and Objectives, CIGRE Report, Draft, March 2004.

[11] D.L. Brooks, R.C. Dugan, M. Waclawiak and A. Sundaram, “Indices for assessing utility distribution system RMS variation performance,” *IEEE Trans. on Power Delivery*, vol. 13, no. 1, pp. 254-259, January 1998.

[12] R.S. Thallam and G.T. Heydt, “Power acceptability and voltage sag indices in the three phase sense,” *IEEE PES Summer Meeting*, July 2000, Seattle.

[13] G-J. Lee, M.H. Albu and G.T. Heydt, “A power quality index based on equipment sensitivity, cost, and network vulnerability,” *IEEE Trans. on Power Delivery*, vol. 19, pp. 1504-1510, July 2004.

## IX. BIOGRAPHIES

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