Transformer Modeling for Low Frequency Transients - The State of the Art

Juan A. Martinez-Velasco[1], Bruce A. Mork[2]

(1) Dept. d'Enginyeria Elèctrica, Universitat Politècnica de Catalunya, 08028 Barcelona, Spain (e-mail: martinez@ee.upc.es), (2) Dep. of Electrical Engineering, Michigan Technological University, Houghton, MI 49931, USA (e-mail: bamork@mtu.edu)

***Abstract* – This paper presents a summary of a literature search performed by the authors as part of an ongoing research project aimed at developing more reliable models for representing transformers in low- and mid-frequency transients. The document includes a summary of models proposed to date, as well as modeling guidelines for nonlinear and frequency dependent phenomena associated with low- and mid-frequency transients.**

Keywords – Transformers, Modeling, Low Frequency Transients, EMTP, Saturation, Hysteresis, Eddy Currents.

I. INTRODUCTION

Transformer representation can be very complex due to the large number of core designs and to the fact that some of the transformer parameters are both nonlinear and frequency dependent. Physical attributes whose behavior, depending on frequency, may need to be correctly represented by a transformer model are core and coil configurations, self- and mutual inductances between coils, leakage fluxes, skin effect and proximity effect in coils, magnetic core saturation, hysteresis and eddy current losses in core, and capacitive effects. Models of varying complexity have been implemented in transients tools to duplicate the transient behavior of transformers. This paper is aimed at presenting the state-of-the-art on transformer models for simulation of low frequency transients; that is, phenomena well below the first winding resonance (several kHz). They include ferroresonance, most switching transients, and harmonic interactions.

The document is organized as follows. A summary of the main models proposed for representation of power transformers in low frequency power system transients is presented in Section 2. The representation of nonlinear and frequency-dependent parameters, which can have a significant influence on transformer behavior during low frequency transients, is analyzed in Section 3. The specification of some parameters can be a bottleneck due to the lack of reliable procedure for their determination; this issue is discussed in Section 4. Finally, a discussion of all of these issues is included in Section 5.

II. TRANSFORMER MODELS

Several criteria can be used to classify transformer models developed for simulating low frequency transients: number of phases, behavior (linear/nonlinear, constant/ frequency-dependent parameters), mathematical models. This section presents a summary of the main principles and the assembly equations of the most popular low-frequency transformer models. They have been classified into three groups. The first group uses either a branch impedance or admittance matrix. The second group is an extension of the Saturable Transformer Component model to multi-phase transformers. Both types of models have been implemented in the EMTP and both of them have important limitations for simulating some core designs. Topology-based models form a larger group for which many approaches have been proposed. Their derivation is performed from the core topology and can represent very accurately any type of core design in low frequency transients if parameters are properly determined.

A. Matrix Representation (BCTRAN model) [1]

The steady state equations of a multi-phase multi-winding transformer can be expressed using the branch impedance matrix

$$[V] = [Z]\,[I] \tag{1}$$

In transient calculations, (1) must be rewritten as

$$[v] = [R]\,[i] + [L]\,[di/dt] \tag{2}$$

where *[R]* and *jω[L]* are respectively the real and the imaginary part of *[Z]*, whose elements can be derived from excitation tests. This approach includes phase-to-phase couplings, models terminal characteristics, but does not consider differences in core or winding topology, since all core designs get the same mathematical treatment.

There could be some accuracy problems with the above calculations since the branch impedance matrix *[Z]* can become ill-conditioned for very small exciting currents or when they are totally ignored. In addition, the short-circuit impedances, which describe the more important transfer characteristics of a transformer, get lost in such excitation measurements. To solve these problems an admittance matrix representation should be used

$$[I] = [Y][V] \tag{3}$$

[Y] does exist and its elements can be obtained directly from standard short-circuit tests. For transient studies, *[Y]* must be split up into resistive and inductive components.

The transformer can be also described by the following equation

$$[di/dt] = [L]^{-1}[v] - [L]^{-1}[R][i] \tag{4}$$

All of these models are linear; however, for many transient studies it is necessary to include saturation and hysteresis effects. Exciting current effects can be linearized and left in the matrix description, which can lead to simulation errors when the core saturates. Alternately, excitation may be omitted from the matrix description and attached externally at the models terminals in the form of nonlinear elements. Such an externally attached core equivalent does not have the same topology as the duality-derived core. However, although not topologically correct, is good enough in many cases.

Although these models are theoretically valid only for the frequency at which the nameplate data was obtained, they are reasonably accurate for frequencies below 1 kHz.

B. Saturable Transformer Component (STC Model) [2]

A single-phase N-winding transformer model can be based on a star-circuit representation, see Fig. 1, whose equation has the following form

$$[L]^{-1}[v] = [L]^{-1}[R][i] + [di/dt] \qquad (5)$$

The matrix product $[L]^{-1}[R]$ is symmetric, which is not true in the general case. Saturation and hysteresis effects can be modeled by adding an extra nonlinear inductor at the star point. This model can be extended to three-phase units through the addition of a zero-sequence reluctance parameter, but its usefulness is limited. The input data consists of the *R-L* values of each star branch, the turn ratios and the information for the magnetizing branch. This model has some important limitations: it cannot be used for more than 3 windings, the star circuit is not valid for N > 3, the linear/nonlinear magnetizing inductance L_m, with resistance R_m in parallel, is connected to the star point, which is not always the correct topological connecting point, and numerical instability has been reported for the 3-winding case.

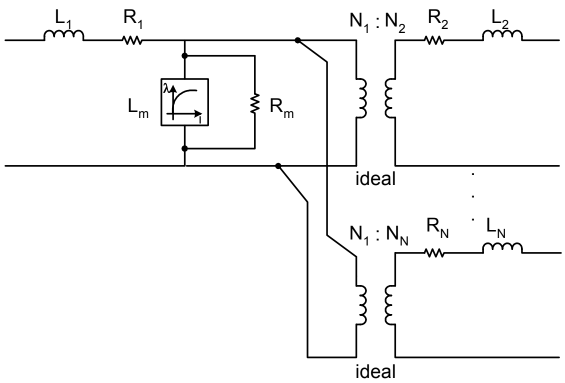

Fig. 1. Star-circuit representation of single-phase N-winding transformers.

C. Topology-Based Models

This group has been split up into two subgroups: models derived using duality, that is models constructed with a circuit-based approach without any previous mathematical description, and geometrical models, for which core topology is considered, but their solution pass through a mathematical description.

C1. Duality-based models : Topologically correct equivalent circuit models can be derived from a magnetic circuit model using the principle of duality [3], [4]. This approach results in models that include the effects of saturation in each individual leg of the core, interphase magnetic coupling, and leakage effects. In the equivalent magnetic circuit, windings appear as MMF sources, leakage paths appear as linear reluctances, and magnetic cores appear as saturable reluctances. The mesh and node equations of the magnetic circuit are duals of the electrical equivalent node and mesh equations respectively. To make models practically useful, the current sources resulting from the transformation are replaced with ideal transformers to provide primary-to-secondary isolation and coupling to the core, while preserving the overall primary to secondary turns ratios. Turns ratios are chosen so that core parameters are referenced to the low voltage winding. The portion of a model inside the coupling transformers represents the core and leakages. Winding resistances and interconnection of the windings appears external to the coupling transformers. The advantage of this is that the derived core equivalent functions independently of winding configuration. Winding resistances, core losses, and capacitive coupling effects are not obtained directly from the transformation, but can be added to this equivalent electrical circuit. Keywork performed during the last years is listed below.

- In 1981 Dick and Watson presented the derivation of the model of a three-legged stacked core transformer [5]. The main contributions of their work was the proposal of a new hysteresis model and the determination of transformer parameters from measurements.
- In 1991 Arturi applied this technique for representing a five-legged step-up transformer working in highly saturated conditions [6].
- In 1994 De León and Semlyen proposed a very complete transformer model that was derived from a hybrid approach, a combination of duality, which was used to obtain the iron core model, and their own technique for calculation of leakage inductances [7].
- In 1994 Narang and Brierley used duality to derive the equivalent circuit of the magnetic core, which is interfaced by means of a three-phase fictitious winding to an admittance matrix that represents the correct magnetic coupling among windings [8].
- In 1999 Mork presented the derivation of a five-legged wound core transformer model, which was validated by duplicating ferroresonant phenomena [9].

C2. Geometric models: Topologically correct models can be based on the following formulation

$$[v] = [R][i] + [d\lambda / dt] \qquad (6)$$

in which the coupling between magnetic and electrical equations is made taking into account the core topology.

A short summary of the main models is presented below.

- The coupled magnetic model was developed by Yacamini and Bronzeado for simulating inrush transients [10]. Because the permeability of the ferromagnetic elements varies with flux density, the magnetic material is divided into sections, each of which has a substantially uniform flux density. The link between magnetic equations, $F = \Re\Phi$, and (6) is the Ampere's circuital law, $F = Ni$.

- The Unified Magnetic Equivalent Circuit model was proposed by Arrillaga et al. [11]. This model uses the normalized core concept for derivation of the inductance matrix. Leakage permeances can be obtained from open and short-circuit tests; the effective lengths and cross-sectional areas of their leakage paths are not required.
- GMTRAN was developed by Hatziargyriou, Prousalidis and Papadias [12]. The magnetic equations were included in (6) by means of the inductance matrix, $[\lambda]=[L][i]$. The most important contribution of this model was the derivation of the $[L]$ matrix from the core topology.
- SEATTLE XFORMER was developed and implemented in the ATP by Chen [13]. Flux linkages were chosen in this model as state variables. That is, the magnetic equations in (6) are included by means of the relationship $[i]= [\Gamma][\lambda]$. The main contribution of this model is therefore the derivation of the $[\Gamma]$ matrix.

III. NONLINEAR AND FREQUENCY-DEPENDENT PARAMETERS

Some transformer parameters are nonlinear and/or frequency-dependent due to three major effects: saturation, hysteresis and eddy currents. Saturation and hysteresis introduce distortion in waveforms; hysteresis and eddy currents originate losses. Saturation is the predominant effect in power transformers, but eddy current and hysteresis effects can play an important role in some transients.

A. Modeling of Iron Cores

Iron core behavior is usually represented by a relationship between the magnetic flux density B and the magnetic field intensity H. A general difficulty in modeling the magnetization curves is the fact that each of the magnetic field values are related to an infinity of possible magnetizations depending on the history of the sample. To characterize the material behavior fully, a model has to be able to plot numerous associated curves, see Fig. 2. A major hysteresis loop is the largest possible loop whose ends enter into technical saturation. Any other closed loop is called a minor loop with a distinction also being made between symmetric and asymmetric minor loops.

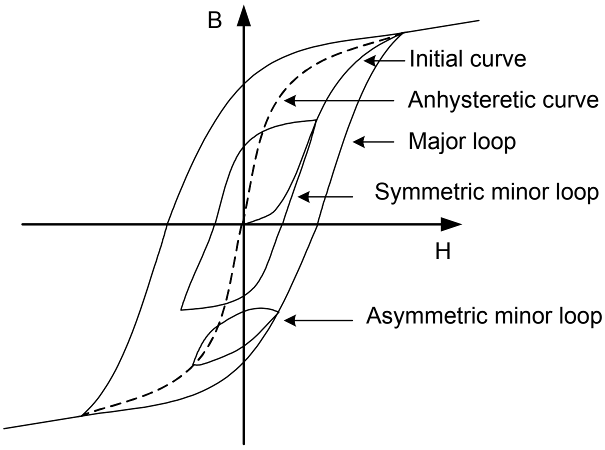

Fig. 2. Magnetization curves and hysteresis loops.

Hysteresis can be caused by three types of phenomena: interaction between domains; anisotropy; and internal frictional type pinning forces caused by interstitials, dislocations, impurities [14]. The dominant cause varies with the material. Many of the approaches to modeling ferromagnetic hysteresis loops are curve fits, which ignore the underlying physics of the material behavior. Micromodels based on a rigorous physical basis are too time consuming to be useful for macroscopic scale applications for real engineering materials. An intermediate solution is concerned with models able to relate microstructural parameters to the macroscopic responses of the material to outside fields, as recorded by the magnetization curves [14]. For a review of hysteresis models see also [15].

Hysteresis loops usually have a negligible influence on the magnitude of the magnetizing current, although hysteresis losses can have some influence on some transients. The residual flux has a major influence on the magnitude of inrush currents. Starting a simulation from a known residual flux is relatively easy, but determining its value is more complicated.

The inductance of a nonlinear inductor depends on the operating point. Assume that its behavior is defined by a set of piecewise linear segments

$$\lambda = a_k + b_k i \tag{7}$$

being k the segment number. Combining (7) with the relationship between voltage v and flux linkage λ, and using trapezoidal integration yields

$$i(t) = Gv(t) + I \tag{8}$$

where

$$G = \frac{\Delta t}{2b_k} \quad ; \quad I = \frac{1}{b_k}\left[\lambda(t-\Delta t) + \frac{\Delta t}{2} v(t-\Delta t)\right] - \frac{a_k}{b_k}$$

The hysteresis element can be represented by the equivalent circuit shown in Fig. 3 [16]. It is similar to the equivalent circuit of a linear inductor; however, the resistance depends on the segment slope and needs to be updated with changes of the operating segment. This requires partial retriangularization of the nodal conductance matrix. The current source consists of the past recorded history and must be updated every time step.

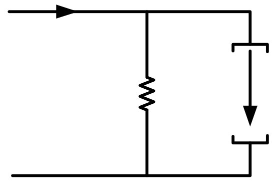

Fig. 3. Equivalent circuit for the hysteresis element.

Magnetic saturation of an iron core can be represented by the anhysteretic curve, the B–H relationship that would be obtained if there was no hysteresis effect in the material. Hysteresis is a history-dependent phenomenon, while saturation is a single-valued relationship. The saturation characteristic can be modeled by a piecewise linear inductance with two slopes, since increasing the number of slopes does not generally improve the accuracy. However, there are some cases, e.g. ferroresonance, for which a more detailed representation of the saturation characteristic is usually required. The slope in the saturated region above the knee is the air-core inductance, which is almost linear and very low compared with the slope in the unsaturated region. The specification of this inductor requires a curve relating the flux linkage, λ, to the current, i. The information usually available is the rms voltage as a function of the rms current.

B. Eddy Current Effects

Several physical phenomena occur simultaneously in a loaded transformer that result in a nonuniform distribution of current in the conductors. These effects manifest themselves as an increase in the effective resistance and winding losses with respect to those for direct current.

The magnetization curves presented in Section 3.2 are only valid for slowly varying phenomena, as it has been assumed that the magnetic field can penetrate the core completely. In general this is not always true. A change in the magnetic field induces eddy currents in the iron. As a consequence of this, the flux density will be lower than that given by the normal magnetization curve. As frequency changes, flux distribution in the iron core lamination changes. For high frequencies the flux will be confined to a thin layer close to the lamination surface, whose thickness decreases as the frequency increases. This indicates that inductances representing iron path magnetization and resistances representing eddy current losses are frequency dependent [17]. The circulation of these eddy currents adds losses. To limit their influence, a transformer core is built up from a large number of parallel laminations.

Excitation losses are mostly iron-core losses. These losses consist of hysteresis and eddy current losses. They cannot be separated, although in modern transformers hysteresis losses are much smaller than eddy current losses.

B1. <u>Eddy current models for transformer windings</u>. There exists a number of analytical expressions for the calculation of losses in transformer windings. Those derived by De León and Semlyen [18] were based on the following assumptions: the magnetic field has only an axial component parallel to the axis of the windings, the conductors have a rectangular cross section, all conductors carry the same total current, there is no gap between conductors, and the surface field intensity is assumed to be undisturbed by the eddy currents. These assumptions imply that the magnetic field at the lateral surfaces of the conductors is known and it can be used to specify the boundary conditions. Foster equivalent circuits have been selected to represent the frequency dependence of the windings, see Fig. 4. These circuits must be of infinite order to exactly reproduce the impedance at all frequencies; however, a computationally efficient circuit can be obtained by fitting only at certain pre-established frequencies. Several fitting procedures to determine circuit parameters have been developed [18].

For practical studies a series model of order 2 or 3 is adequate. Such a model neglects displacement currents, so it is valid at frequencies below the first winding resonance, that is up to tens of kHz, and it should be used only when currents are uniformly distributed.

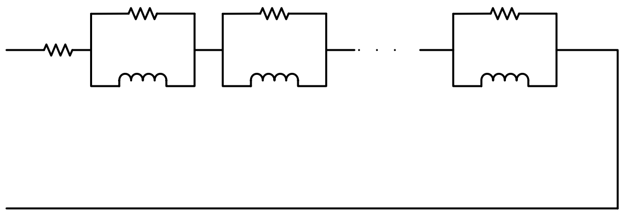

Fig. 4. Series Foster equivalent circuit for windings.

B2. <u>Eddy current models for iron laminated cores</u>. Eddy current models intended for simulation of the frequency dependence of the magnetizing inductance as well as losses can be classified into two categories obtained respectively by the realization of the analytical expression for the magnetizing impedance as a function of frequency, and by subdivision of the lamination into sublaminations and the generation of their electrical equivalents [19].

Computationally efficient models have been derived by synthesizing a Foster or a dual Cauer equivalent circuit to match the equivalent impedance of either a single lamination or a coil wound around a laminated iron core limb. By using a continued fraction expansion a standard Cauer equivalent circuit is derived form the Foster equivalent circuit, so the final result is a Cauer type in both cases. The accuracy of the standard Cauer representation over a defined frequency range depends on the number of terms retained in a partial fraction expansion, and therefore on the number of sections. To represent the frequency range up to 200 kHz with error less than 5%, only four terms are required, see Fig. 5 [19]. The first section governs its characteristics at frequencies up to a few kHz, each subsequent section comes into play as the frequency increases.

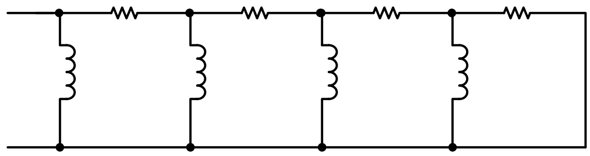

Fig. 5. Four section continued fraction model.

Another form of the Cauer model would have shunt resistances and series inductances, see Fig. 6 [18]. The inductances represent (using duality) the flux paths and the resistances are in the path of eddy currents. The high frequency response is defined by the blocks near the terminals, while in the standard Cauer circuit, the high frequency behavior is governed by the inner blocks. The blocks of this model can be thought as being a discretization of the lamination. The parameters of this circuit can be calculated by means of an iterative method that could be seen as an optimization of the discretizing distances for the

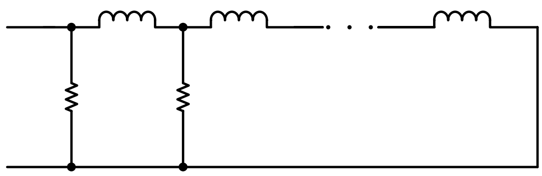

Fig. 6. Cauer model for half lamination.

used fitting frequencies. Errors smaller than 1% up to 200 kHz can be obtained with model order of 4. Inductive components of the models representing the magnetizing reactances have to be made nonlinear to account for the hysteresis and saturation effects. Since the inductances and resistances in this model do not represent any physical part of the iron lamination it is not obvious how to incorporate the effects. However, since the high frequency components do not contribute appreciably to the flux in the transformer core, it can be assumed that only low frequency components are responsible for driving the core into saturation. It may, therefore, be justifiable to represent as nonlinear only the first section of the model.

An alternative approach to model eddy current effects is based on subdividing each lamination into a number of sublaminations which are sufficiently narrow so that a uniform flux distribution within each sublamination can be assumed [20]. The corresponding equivalent circuit is obtained by connecting sections in a cascade. The accuracy of the model over a given frequency range depends on the thickness of the sublamination, and therefore, on the number of ladder sections in the resulting circuit representation. Considerable computational burden is introduced to the overall transformer model when this approach is used. In addition, the large number of nonlinearities that are needed makes it difficult to initialize and can result in a divergent iteration [19].

IV. Parameter Determination

Data usually available for any power transformer are: power rating, voltage rating, excitation current, excitation voltage, excitation losses, short-circuit current, short-circuit voltage, short-circuit losses, saturation curve, capacitances between terminals and between windings. Excitation and short-circuit currents, voltages and losses must be provided from both direct and homopolar measurements. Although procedures for determining transformer parameters from standard tests have been proposed, depending on the model selected for representing some effects, additional information will be usually required.

Although no agreement has been reached up to date on the best representation for three-phase core transformers, it is recognized that it should be based on the core topology, and include eddy current effects and saturation/hysteresis representation. In addition, a very careful representation and calculation of leakage inductances is required. Finally, coil-capacitances have to be included for an accurate simulation of some transients. Since no standard procedures have been developed, a parameter estimation seems to be required regardless of the selected model.

- A good reference for the calculation of leakage induc-tan-ces from standard test values is the work by Brandwajn et al [1]. Other contributions in this field can be found in [21] and [22].
- Important contributions on the calculation of parameters to be specified in duality-based models are the works by Dick and Watson [5], Stuehm [23] and Narang and Brieley [8]. See also references [6] and [9].
- The influence of eddy current losses and the determination of resistances as a function of frequency have been studied by Fuchs et al. [24].
- For the determination of the saturation characteristic see [25], [26], as well as the discussion of [26].
- The determination of hysteresis parameters is very dependent on the selected hysteresis model [27].

Temperature influence cannot be neglected, neither in lab tests nor in field measurements. Since resistances are the only parameters that are affected by temperature, an analysis based on a hand calculation of resistances at different temperatures can be easily performed.

V. Discussion

It is obvious from the previous sections that a model for representing transformers in low- and mid-frequency transients has to incorporate accurate representation of the transformer core, leakage inductances, eddy current effects in windings and core, saturation and/or hysteresis effects. Terminal capacitances can also have an important effect in some transients [28].

The development of a model that could include all these effects and could be used in any transient simulations for frequencies below 10 kHz is not immediately obvious. The computational burden for the most complex core design would be considerable, and the accurate determination of all parameters a very complex task. To date the most complete model was developed by De León and Semlyen [7].

An important issue for any transformer model is the nodes to which the core equivalent must be connected. For instance, it may be less important to which node an unsaturated inductance is connected to in the single-phase model shown in Fig. 1, but it may make a difference when the inductance is saturated, because of its low value. Ideally, the nonlinear inductance should be connected to a point where the integrated voltage is equal to the iron-core flux. To identify that point requires design details not normally available.

Simplified models could be accurate enough for simulating some transients. For instance, there are many low frequency transients in which terminal capacitances will not play any important role. Hysteresis can be reduced to a single-valued saturation curve in those cases for which the residual flux has no effect and the representation of losses is not critical. Eddy current effects in windings and iron core can be represented by low order equivalent circuits for a frequency range below 10 kHz, usually a model of order 2 or 3 will suffice.

Some guidelines for parameter specification are presented below.

- Winding resistances are frequency dependent and must incorporate eddy current and stray loss effects.
- Air core inductances of inner windings for concentric designs have smaller reactances than outer windings, and most often the inner winding is the lower-voltage winding. The leakage inductance must be divided among windings, the division is arbitrary for two-winding transformers or autotransformers without tertiary, and deter-

mined by air-core impedances. As a rule of thumb most of the leakage impedance, 75% - 90% of total impedance, should be placed on HV side

- Saturation representation can be incorporated from test data/manufacturer's curves, or estimating the key parameters from transformer geometry. Several factors need to be taken into account with the first approach, since the exciting current includes core loss and magnetizing components. Manufacturers usually provide only RMS currents; winding capacitance can significantly affect low-current data, and hysteresis biases the curve.
- A linear resistance is the default approach for core loss representation. A nonlinear resistor for core loss model can have some serious limitations since hysteresis loss is dependent on maximum flux, not voltage, and loss match for 50/60 Hz excitation does not mean that correct flux-current trajectory is followed.

VI. CONCLUSIONS

This paper has presented a summary of the most important issues related to transformer modeling for simulation of low- and mid-frequency transients. Although much effort has been dedicated to the development of transformer models, there is no agreement on the most adequate model.

Important difficulties are the great variety of three-phase transformer core designs, the nonlinear and frequency dependent behavior of many transformer parameters, the inadequacy for acquisition and determination of some transformer parameters.

The development of an accurate transformer model is a sophisticated work. However, several modeling levels could be considered since not all parameters have the same influence on all transient phenomena. For example, a simple representation of hysteresis loops will suffice for some transients, while a very simple model, or even no representation at all, of eddy current losses may be acceptable for ferroresonance studies.

ACKNOWLEDGMENT

Support for this work is provided by Bonneville Power Administration, part of the US Department of Energy. Additional collaboration and administrative assistance is being provided by the Keith Companies.

REFERENCES

[1] H.W. Dommel, *EMTP Theory Book*, Bonneville Power Administration, Portland, August 1986.

[2] V. Brandwajn, H.W. Dommel and I.I. Dommel, "Matrix representation of three-phase n-winding transformers for steady-state and transient studies," *IEEE Trans. on Power Apparatus and Systems*, vol. 101, no. 6, pp. 1369-1378, June 1982.

[3] E.C. Cherry, "The duality between interlinked electric and magnetic circuits and the formation of transformer equivalent circuits," *Proc. of the Physical Society, pt. B*, vol. 62, pp. 101-111, 1949.

[4] G. R. Slemon, "Equivalent circuits for transformers and machines including non-linear effects," *Proc. IEE, pt. IV*, vol. 100, pp. 129-143, 1953.

[5] E.P. Dick and W. Watson, "Transformer models for transient studies based on field measurement," *IEEE Trans. on Power Apparatus and Systems*, vol. 100, no. 1, pp. 401-419, January 1981.

[6] C.M. Arturi, "Transient simulation and analysis of a five-limb generator step-up transformer following an out-of-phase synchronization," *IEEE Trans. Power Delivery*, vol. 6, no. 1, pp. 196-207, January 1991.

[7] F. de León and A. Semlyen, "Complete transformer model for electromagnetic transients," *IEEE Trans. on Power Delivery*, vol. 9, no. 1, pp. 231-239, January 1994.

[8] A. Narang and R. H. Brierley, "Topology based magnetic model for steady-state and transient studies for three phase core type transformers," *IEEE Trans. on Power Systems*, vol. 9, no. 3, pp. 1337-1349, August 1994.

[9] B.A. Mork, "Five-legged wound-core transformer model: Derivation, parameters, implementation, and evaluation," *IEEE Trans. on Power Delivery*, vol. 14, no. 4, pp. 1519-1526, October 1999.

[10] R. Yacamini and H. Bronzeado, "Transformer inrush calculations using a coupled electromagnetic model," *IEE Proc.-Sci. Meas. Technol.*, vol. 141, no. 6, pp. 491-498, November 1994.

[11] J. Arrillaga, W. Enright, N.R. Watson and A.R. Wood, "Improved simulation of HVDC converter transformers in electromagnetic transient programs," *IEE Proc.-Gener. Transm. Distrib.*, vol. 144, no. 2, pp. 100-106, March 1997.

[12] N.D. Hatziargyriou, J.M. Prousalidis and B.C. Papadias, "Generalised transformer model based on the analysis of its magnetic core circuit," *IEE Proc.-C*, vol. 140, no. 4, pp. 269-278, July 1993.

[13] X. Chen, "A three-phase multi-legged transformer model in ATP using the directly-formed inverse inductance matrix," *IEEE Trans. on Power Delivery*, vol. 11, no. 3, pp. 1554-1562, July 1996.

[14] F. Liorzou, B. Phelps and D.L. Atherton, "Macroscopic model of magnetization," *IEEE Trans. on Magnetics*, vol. 36, no. 2, pp. 418-428, March 2000.

[15] F. de León and A. Semlyen, "A simple representation of dynamic hysteresis losses in power transformers," *IEEE Trans. on Power Delivery*, vol. 10, no. 1, pp. 315-321, January 1995.

[16] J.G. Frame, N. Mohan and T.H. Liu, "Hysteresis modeling in an electromagnetic transients program," *IEEE Trans. on Power Apparatus and Systems*, vol. 101, no. 9, pp. 3403-3412, September 1982.

[17] J. Avila-Rosales and F.L. Alvarado, "Nonlinear frequency dependent transformer model for electromagnetic transient studies in power systems," *IEEE Trans. on Power Apparatus and Systems*, vol. 101, no. 11, pp. 4281-4288, November 1982.

[18] F. de León and A. Semlyen, "Time domain modeling of eddy current effects for transformer transients," *IEEE Trans. on Power Delivery*, vol. 8, no. 1, pp. 271-280, January 1993.

[19] E.J. Tarasiewicz, A.S. Morched, A. Narang and E.P. Dick, "Frequency dependent eddy current models for nonlinear iron cores," *IEEE Trans. on Power Systems*, vol. 8, no. 2, pp. 588-597, May 1993.

[20] J. Avila-Rosales and A. Semlyen, "Iron core modeling for electrical transients," *IEEE Trans. on Power Apparatus and Systems*, vol. 104, no. 11, pp. 3189-3194, November 1985.

[21] D.J. Wilcox, W.G. Hurley and M. Conlon, "Calculation of self and mutual impedances between sections of transformer windings," *IEE Proc.*, vol. 136, Pt. C, no. 5, pp. 308-314, September 1989.

[22] F. de León and A. Semlyen, "Efficient calculation of elementary parameters of transformers," *IEEE Trans. on Power Delivery*, vol. 7, no. 1, pp. 376-383, January 1992.

[23] D. L. Stuehm, "Final report - Three phase transformer core modeling," Bonneville Power Administration Award no. DE-BI79-92BP26700, Feb. 28, 1993.

[24] E.E. Fuchs, D. Yildirim and W.M. Grady, "Measurement of eddy-current loss coefficient PEC-R, derating of single-phase transformers, and comparison with K-factor approach.," *IEEE Trans. on Power Delivery*, vol. 15, no. 1, pp. 148-154, January 2000.

[25] C.G.A. Koreman, "Determination of the magnetizing characteristic of three-phase transformers in field tests," *IEEE Trans. on Power Delivery*, vol. 4, no. 3, pp. 1779-1785, July 1989.

[26] W.L.A. Neves and H.W. Dommel, "On modeling iron core nonlinearities," *IEEE Trans. on Systems*, vol. 8, no. 2, pp. 417-425, May 1993.

[27] D.C. Jiles, J.B. Thoelke and M.K. Devine, "Numerical determination of hysteresis parameters for the modeling of magnetic properties using the theory of ferromagnetic hysteresis," *IEEE Trans. on Magnetics*, vol. 28, no. 1, pp. 27-35, January 1992.

[28] IEEE Slow Transients TF, "Modeling and analysis guidelines for slow transients – Part III: The study of ferroresonance," *IEEE Trans. on Power Delivery*, vol. 15, no. 1, pp. 255-265, January 2000.