An Identification Procedure for Return Stroke Characteristics

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Abstract - **In the paper an inverse identification procedure for the return stroke current is presented. The procedure allowed us to accurately identify return stroke characteristics for simulated data in presence of perfect conductive ground.**

Keywords: Lightning, Return stroke, Return Stroke models.

I. INTRODUCTION

In recent years, several studies on the physics of lightning have dealt with the so-called return stroke models [1-7]. We will concentrate our attention upon the engineering models. Engineering models are defined by means of mathematical expressions relating the longitudinal current *i*(*z'*,*t*) to the channel base current *i*(*0*,*t*), which can be measured or specified. Such models can be summarised [1] by relationships in which *i*(*z't*) is attenuated by a suitable height dependent function *P*(*z'*). The main aim of this kind of models is to create current waveforms that produce model predicted fields as close as possible to the measured ones. The usual approach to validate such models is based on a *direct procedure*: for an assigned return stroke model, the electromagnetic fields are calculated at one or more distances and then compared to the observed ones. A return stroke model is then considered suitable if there is a relatively good coincidence between calculated and measured fields.

Our purpose is to describe the possibility of identifying exactly the attenuation function *P*(*z'*), by means of an *inverse procedure*, solving the equations relating the measured field to the channel base current.

II. THE PROBLEM

The electromagnetic field radiated by the lightning channel, considered as a vertical antenna above a perfectly conducting plane (Fig.1), is given by the following relations in the frequency domain [8].

$$
\begin{aligned}
E_r(P) &= \int_{-H}^{+H} G_r(P,P',\omega)\, exp\left(-j\omega\frac{R}{c}\right) I(z',\omega)\,dz' \\
E_z(P) &= \int_{-H}^{+H} G_z(P,P',\omega)\, exp\left(-j\omega\frac{R}{c}\right) I(z',\omega)\,dz' \qquad (1)\\
H_\phi(P) &= \int_{-H}^{+H} G_\phi(P,P',\omega)\, exp\left(-j\omega\frac{R}{c}\right) I(z',\omega)\,dz'
\end{aligned}
$$

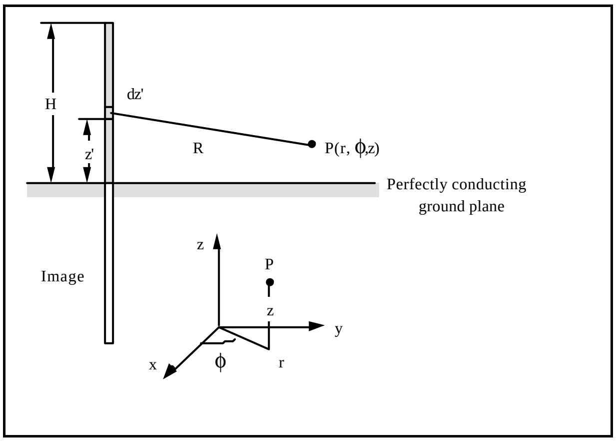

Fig. 1. Geometry of the problem.

where ù is the angular frequency, G_r, G_z and Gö are suitable Green functions that will be specified soon, and *I*(*z'*,ù) is the spectrum of the current along the channel, namely the return stroke current. Since this current is related to the channel base current by

$$
I(z',\omega) = I(0,\omega)\, exp\left(-j\omega\frac{|z'|}{v}\right) P(z'), \qquad (2)
$$

the electromagnetic field can be rewritten in the simplified form

$$
\begin{aligned}
\frac{E_r(P)}{I(0,\omega)} &= \int_{-H}^{+H} G_r(P,P',\omega)\, exp\left[-j\omega\left(\frac{R}{c}+\frac{|z'|}{v}\right)\right] P(z')\,dz' \\
\frac{E_z(P)}{I(0,\omega)} &= \int_{-H}^{+H} G_z(P,P',\omega)\, exp\left[-j\omega\left(\frac{R}{c}+\frac{|z'|}{v}\right)\right] P(z')\,dz' \qquad (3)\\
\frac{H_\phi(P)}{I(0,\omega)} &= \int_{-H}^{+H} G_\phi(P,P',\omega)\, exp\left[-j\omega\left(\frac{R}{c}+\frac{|z'|}{v}\right)\right] P(z')\,dz'
\end{aligned}
$$

and Green functions are

$$
\begin{aligned}
G_r(P,P',\omega) &= \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{j\omega}\frac{3r(z-z')}{R^5}+\frac{3r(z-z')}{cR^4}+j\omega\frac{r(z-z')}{c^2R^3}\right], \\
G_z(P,P',\omega) &= \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{j\omega}\frac{2(z-z')^2-r^2}{R^5}+\frac{2(z-z')^2-r^2}{cR^4}-j\omega\frac{r^2}{c^2R^3}\right], \\
G_\phi(P,P',\omega) &= \frac{1}{4\pi}\left[\frac{r}{R^3}+\frac{r}{cR^2}\right], \qquad (4)
\end{aligned}
$$

c and ε_0 are the speed of light and the permittivity in the free space, respectively, v is the current wave propagation speed, and the geometrical parameters are referred to the Fig. 1. In the form (3) we recognise a Fredholm integral equation of the first kind which is a classical example of a linear ill-posed problem [9,10]. The superior analytical tool for analysis of first-kind Fredholm integral equations is the *singular vale expansion* (SVE) of the kernel [11]. However, uniqueness is not a-priori guaranteed for equations like (3): it has to be proved in each problem. Moreover, analytic solution of the problem (3) is not available in general, and, even if available, is not useful in our context.

In order to identify the attenuation function $P(z')$, we should then look for a solution of an integral equation of the first kind by means of an adequate expansion, which will enables us to identify the unknown $P(z')$ with a given accuracy.

III. IDENTIFICATION PROCEDURE

Let us start to write a general expansion of the unknown function $P(z')$

$$P(z') = \sum_{1}^{\infty} p_i \varphi_i(z'), \quad \text{with } \frac{|z'|}{H} < 1 \tag{5}$$

where p_i are expansion coefficients and $\varphi_i(z')$, (for $i=1,2,3,..$) represents a complete basis in a certain functional space [12]. The vertical component of the electric field (3), which does not vanish on the ground, is therefore given by

$$\frac{E_z(P,\omega)}{I(0,\omega)} = \sum_{1}^{\infty} p_i \int_{-H}^{+H} G_z(P,P',\omega)\, exp\left[-j\omega\left(\frac{R}{c}+\frac{|z'|}{v}\right)\right]\varphi_i(z')dz', \tag{6}$$

which, truncating the expansion to the first N terms and sampling at N frequencies becomes

$$\frac{E_z(P,\omega_s)}{I(0,\omega_s)} = \sum_{1}^{N} p_i \int_{-H}^{+H} G_z(P,P',\omega_s)\, exp\left[-j\omega\left(\frac{R}{c}+\frac{|z'|}{v}\right)\right]\varphi_i(z')dz', \tag{7}$$

with $s = 1,2,..,N$. Equations (7) define a system of linear and complex equations in the unknown (real) expansion coefficients. The problem is to find the expansion coefficients p_i . In order to solve the system (7), we need the free term, which are the experimental data relative to the field component and to the channel base current measurements. We also need the velocity v, which can be obtained experimentally [3]. Finally, we need an expansion basis. In principle, any expansion basis which is complete in our functional space can be chosen (e.g. piecewise constant, Fourier trigonometrical functions, etc). However, if the selected basis can approximate the unknown function $P(z')$ using only few terms, the problem can be solved by simply working on the Single Value Decomposition (SVD) of the coefficient matrix of system (7).

Ill-posed problems are typically characterised by the coefficient matrix A having a cluster of small singular values, and there is a gap between large and small singular values. This implies that one or more rows and columns are nearly linear combinations of some or all of the remaining rows and columns. Therefore, the matrix A contains almost redundant information, and the key to numerical treatment of such problems is to extract the linearly independent information in A, to arrive at another problem with a well-conditioned matrix. The technique to implement such an idea is known as Truncated Singular Value Decomposition (TSVD) [12,13]. It consists in reducing the matrix dimension by retaining only the largest singular values (the ones in the upper part of the gap), truncating all the others. The technique will be shown in the next paragraph.

The problem, when the expansion basis is not very effective, which means that one needs a large number of expansion terms, must be solved in a different manner. In this instance the reconstruction of $P(z')$ can also be carried out accurately, but one needs to use more sophisticated regularisation techniques such as Thikonov regularisation [9,13].

In order to test the procedure, the free term of equation (7) has been simulated assuming a certain attenuation function as a reference and then the aim has been that of identifying it. We imagine (Fig. 2) to use only one sensor at a fixed distance, placed on the ground and spanning in a broad frequency range (we imagine to use an antenna placed at r = 500 m and operating in the range 50 Hz - 1 MHz for numerical simulations). We have simulated the data using as attenuation function the so called Modified Transmission Line (MTLE) model [5] because it is defined in terms of the most complicated $P(z')$, namely

$$P(z') = exp\left(-\frac{|z'|}{\lambda}\right) \tag{8}$$

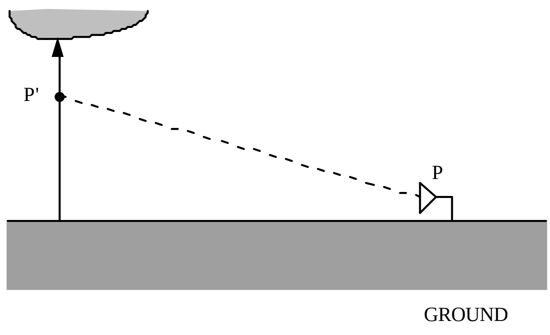

Fig. 2. Position of a sensor aimed at recording the electric or magnetic field.

As expansion basis we selected the following modified Chebyshev polynomials

$$\varphi_i(z') = T_{i-1}\left(\frac{|z'|}{H}\right) = cos\left[(i-1)arccos\frac{|z'|}{H}\right] \quad \text{with i=1,2,3,...} \tag{9}$$

We also considered the following parameters $v = 1.9108$ ´ 10^8 m/s ~ $2/3\ c$, $H = 15$ km, $z = 0$ m, and ë = 2 km.
Following the SVD procedure we firstly analyse the singular value set which is plotted in Fig. 3 ($N=30$). It is evident the gap between the large and the small singular values. As we stated the technique consists in retaining the largest ones (in the upper part of the gap). In our instance they are the first twelve singular values. In Fig. 4 we report the p_i values as solution of system (7). The identification of $P(z')$ is shown in Fig. 5.
The result is very accurate since with respect to the following definition for the relative error

$$Relative\ error = 100\left|\frac{P_v(z') - P_R(z')}{P_v(z')}\right|$$

we obtained a value in the order of 10^{-7}.
We want now show how important is the respect of the number of terms to retain suggested by the SVD analysis.
Let us analyse what happens for two opposite cases: three and thirty expansion terms obtained by respectively selecting the first three and all the thirty singular values (Fig. 6 and Fig. 7). The result is very inaccurate. This can be easily understood if we consider the plot of relative error as a function of the number of expansion terms used (Fig. 8). This “valley” shape is typical of ill-posed problems: in the left part the large relative error is due to the fact that the number of coefficients used in the expansion basis is too small and the base itself is not able to identify. In the right part we are in a region where the matrix is numerically singular and the reconstruction errors are unacceptable.

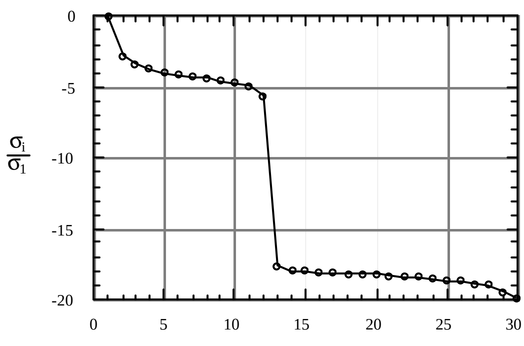

Fig.3. Singular values normalised to ó$_1$ ~ 0.41169 (logarithmic vertical scale).

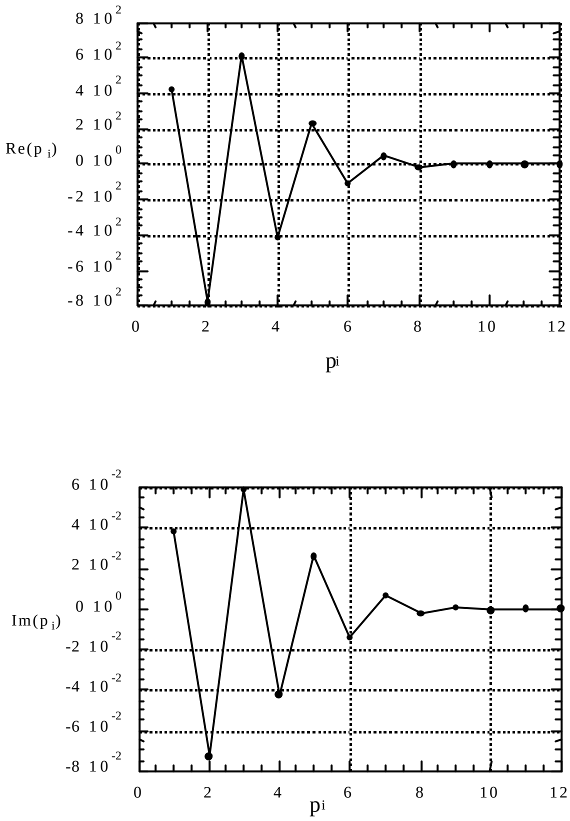

Fig 4: Values of the coefficient p_i

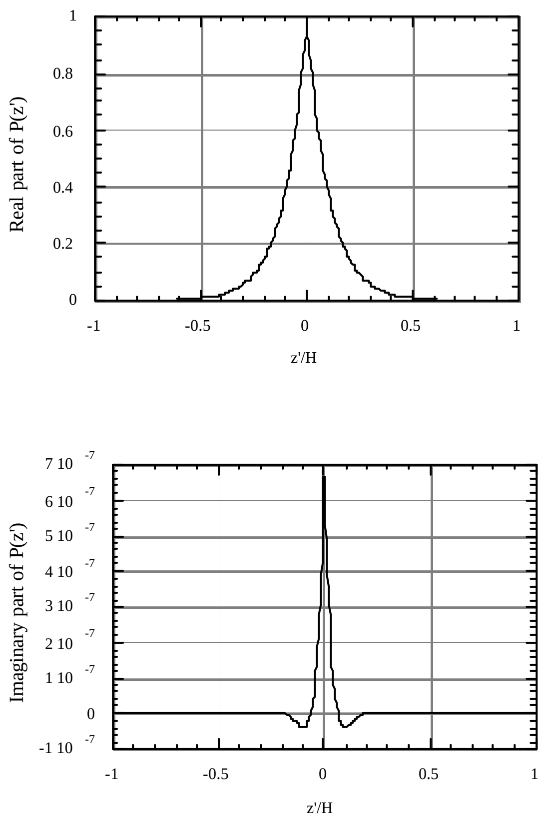

Fig 5. $P(z')$ identification obtained by retaining the twelve largest SVD components.

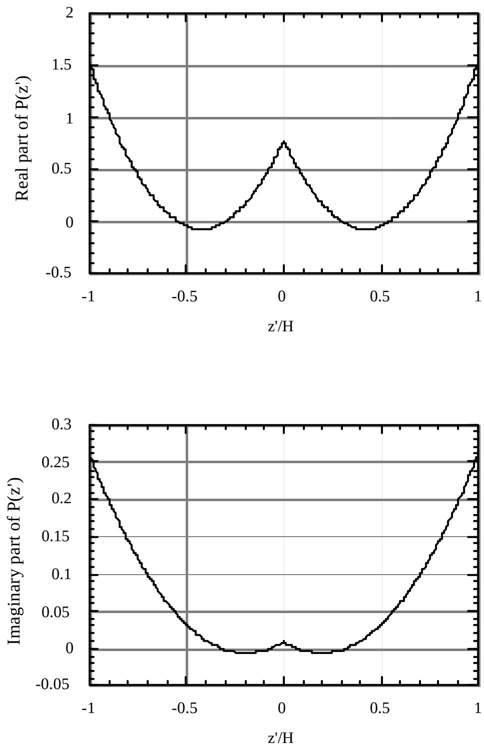

Fig 6. P(z’) reconstruction using only three terms

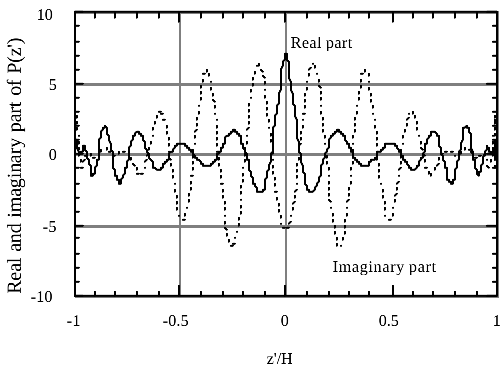

Fig 7. P(z’) reconstruction using 30 expansion terms

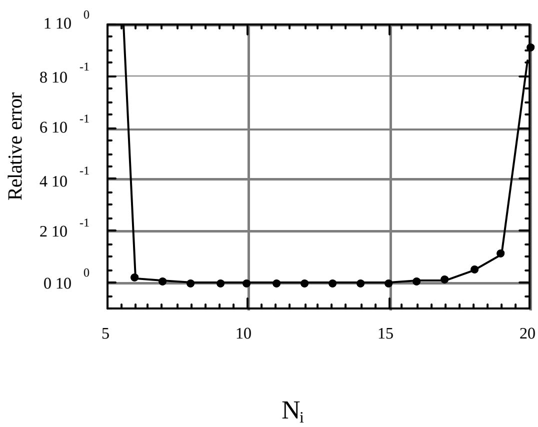

Fig 8. *P*(*z’*) reconstruction using 30 expansion terms

IV. CONCLUSIONS

We have shown a procedure to identify the lightning return stroke attenuation function in the frequency domain. The procedure has shown to be very effective, working with simulated data. We plan in a near future to work on experimental data. Once the algorithm will be completely tested, our aim is to use the proposed algorithm with measured data to perform a statistical characterisation of the attenuation function $P(z')$.

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