Transformer leakage flux modeling

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Abstract – **The conventional transformer model separates the magnetizing impedance from the leakage inductances. One of the leakage inductances becomes negative for some three-winding transformers and may result in an unstable time domain circuit model. This paper shows how to avoid this problem by modifying the transformer model. Various alternatives are presented and the selection among them depends on the geometrical data available for the transformer. It is for transformers with concentric cylindrical windings better to connect the magnetizing branch to the terminal of the inner winding than to the internal node in the conventional model. The location of the magnetizing branch may have a significant influence when the transformer is in heavy saturation**.

Keywords: Transformer modeling, negative leakage inductance, unstable circuit, physically based model, EMTP.

I. INTRODUCTION

Fig. 1 shows the Saturable Transformer Component in EMTP [1]. This model corresponds to the conventional transformer model and its main characteristic compared to other models is that it separates the magnetizing impedance from the short-circuit (leakage) inductances. The leakage inductances are normally determined from measurements. Only one such measurement can be performed for a two-winding transformer and it is common practice to assume the two leakage inductances to the equal (in p.u.). Three independent measurements can be made for a three-winding transformer. The three leakage inductances are thus determined uniquely. For transformers with four or more windings the number of independent measurements is higher than the number of inductances and the model (Fig. 1) can in general not reproduce correctly all the measured values.

The transformer model in Fig. 1 includes winding resistances and ideal transformers. The winding resistances are neglected in this paper, but they can easily be included as a part of the network connected to the transformer's terminals. The ideal transformers are omitted as well. This implies that all impedances are referred to the same voltage level.

It is well known that one of the leakage inductances in Fig. 1 normally becomes negative for three-winding transformers. Ref. [2] shows that this negative value may result in an unstable circuit that gives completely unrealistic time domain responses. One example of such a case is found

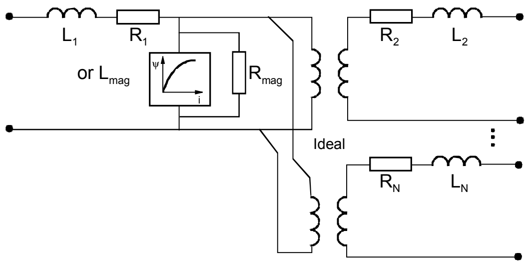


Fig. 1. Saturable Transformer Component.

when an ideal voltage source is connected to the winding with the negative inductance (e.g. L_2) and the other windings are open. Assuming a constant magnetizing inductance L_m gives an eigenvalue equal to

$$\lambda = -R_m\left(L_2 + L_m\right)/\left(L_2 \cdot L_m\right) \qquad (1)$$

This eigenvalue becomes **positive** and the numerical value is typically in the range 10^8 s^{-1}. This implies that a part of a transient response shows an exponential increase with a time constant in the range 10^{-8} s. Ref. [2] shows some examples of unstable responses obtained with a resistive load connected to the winding with the negative inductance.

Ref. [2] shows that the problem due to the unstable circuit can be avoided by representing the magnetizing losses by resistances connected to the transformer's terminals or by introducing a series inductance in the magnetizing branch. Those modifications were however not derived from physical conditions of the transformer. This is the main purpose of this paper.

II. PASSIVE NETWORK, INDUCTANCES WITH MUTUAL COUPLING

A passive network is a network that does not generate any power. It does therefore not cause any unstable condition. The transformer is a passive component and must therefore correspond to a passive network. One way of achieving this is to restrict the model to non-negative RLC components only. This approach is however not a good solution since it will not agree with the measured short-circuit inductances when they result in a negative value for one of the leakage inductances in Fig. 1.

A negative component may on the other hand be a part of a passive network. A positive resistance may for instance be modeled as a series connection of a negative resistance and a positive one.

Fig. 2 shows an equivalent for two inductances with mutual coupling.

The inductances correspond to a passive network when L_a and L_b are positive and the coupling factor is less or equal to one, i.e.:

$$M^2 \le L_a \cdot L_b \qquad (2)$$

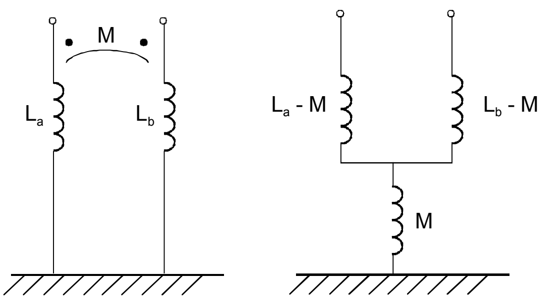
It is worth noting that $L_b - M$ may become negative as long as condition (2) is fulfilled.
(Numerical example: $L_a = 2L_b$ and $M = 1.3 \cdot L_b$ (coupling factor 0.92) gives $L_b - M = -0.3\ L_b$).

III. TRANSFORMER MODEL DERIVED FROM A PHYSICAL BASIS

Fig. 3 shows a cross section of the transformer that is assumed to have three concentric cylindrical windings. The core is assumed cylindrical as well. All windings are assumed to have one turn only. (The actual number can be accounted for by ideal transformers).

A model for the transformer will now be derived assuming that the magnetic flux density outside the core is vertical (i.e. end effects are neglected).

The flux linkages of the windings can be expressed as a contribution from the core (Ψ_m) plus a linear contribution due to the field outside the core.



Inductances with mutual coupling

Equivalent without mutual coupling

Fig. 2. Inductances with mutual coupling.

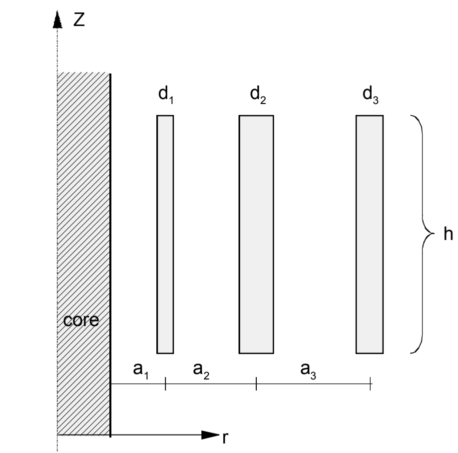


Fig. 3. Transformer with three cylindrical windings.

$$\begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \end{bmatrix} = \begin{bmatrix} \Psi_m \\ \Psi_m \\ \Psi_m \end{bmatrix} + \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{12} & L_{22} & L_{23} \\ L_{13} & L_{23} & L_{33} \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \qquad (3)$$

The elements in the inductance matrix (L) can be determined column by column by applying a current in one winding and determining the flux linkages with $\Psi_m = 0$. This last condition is achieved by introducing a fictitious winding (magnetizing winding) with zero thickness at the surface of the core and applying a current equal to the one in the first winding but with opposite sign.

The leakage inductance formula in the appendix can be used to determine the diagonal elements in the inductance matrix. The formula assumes homogenous flux density between the two windings involved (i.e. in the leakage channel) and it is possible to determine the off-diagonal elements in the inductance matrix utilizing this assumption. The matrix becomes:

$$L = k \cdot \begin{bmatrix} a_1 - d_1/6 & a_1 & a_1 \\ a_1 & a_1 + a_2 - d_2/6 & a_1 + a_2 \\ a_1 & a_1 + a_2 & a_1 + a_2 + a_3 - d_3/6 \end{bmatrix} \qquad (4)$$

where

$$k = \mu_0 \frac{\pi \cdot D_m}{h} \qquad (5)$$

D_m is the mean diameter of the windings.

The flux in the core must be produced by a magnetizing current i_m. This current can be considered to be supplied by the fictitious magnetizing winding. The total current in this winding is thus $i_m - i_1 - i_2 - i_3$. However, this current must be zero, which implies that i_m is the sum of the currents in the other windings. The relation between Ψ_m and i_m is non-linear and is normally represented by a parallel connection of a resistance and a non-linear inductance.

Fig. 4 shows an equivalent circuit that can be derived from (3) and (4) and the magnetizing impedance. The decoupling technique in Fig. 2 has been applied as well.

The dotted lines show how the equivalent can be extended to take a fourth winding into account.

It should be noted that the following minimum values applies (see Fig. 3):

$$a_{1\ min} = d_1/2 \qquad (6)$$

$$a_{2\ min} = (d_1 + d_2)/2 \qquad (7)$$

$$a_{3\ min} = (d_2 + d_3)/2 \qquad (8)$$

Fig. 4 is based on a geometrical model but several simplifications have been introduced and discrepancies must be expected between the model and a real transformer. It is however important to note that Fig. 4 represents a passive network. This can be verified from (4) by considering the coupling factor for any pair of windings.

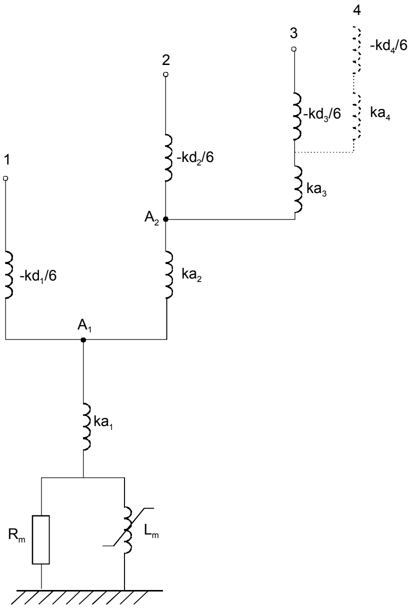


Fig. 4. Transformer equivalent based on (3) and (4).

IV. ADAPTING TRANSFORMER MODEL TO MEASURED DATA

The magnetizing current has normally no influence on the short-circuit impedances. Ignoring this current in Figs. 1 and 4 shows that the two models are equivalent when:

$$L_1 = k\,(a_2 - d_1/6) \qquad \textbf{(9)}$$

$$L_2 = -k\,d_2/6 \qquad \textbf{(10)}$$

$$L_3 = k\,(a_3 - d_3/6) \qquad \textbf{(11)}$$

Fig. 1 is equivalent to Fig. 4 under these conditions if the inductance ka_1 is ignored and the magnetizing branch in Fig. 4 is connected to node A_2 instead of node A_1.

R_m and L_m in Fig. 4 can be adjusted in such a way that the two magnetizing branches give the same impedance at steady state nominal voltage. The discrepancy between the two branches is then insignificant when the transformer is not saturated. Some discrepancy occurs during saturation but it is most probably insignificant compared to the error that is introduced by representing the magnetizing losses by a linear resistance.

The main difference between the two models is the influence on the magnetizing current from the voltage drop across the inductance $k \cdot a_2$. This influence may be significant when the transformer is saturated.

The inductance in Fig. 4 corresponding to L_1 can be decomposed into two components:

$$L_{11} = -k\,d_1/6 \qquad \textbf{(12)}$$

and

$$L_{12} = k \cdot a_2 \qquad \textbf{(13)}$$

Eq. (7) shows that L_{12} is at least $3 \cdot |L_{11}|$. L_{12} becomes at least $6 \cdot |L_{11}|$ if one assumes that d_1 is approximately equal to d_2. This implies that it is better to connect the magnetizing branch to terminal 1 than to the node A_2. Ref. [2] shows that the transformer model in Fig. 1 becomes a passive one when the magnetizing branch is connected to one of the terminals of the model or contains a sufficiently great series inductance.

The transformer model in Fig. 4 is based on physical considerations and it is therefore assumed to be a better basis for modeling the transformer than Fig. 1. The model should however agree with the short-circuit measurements. This implies that the leakage inductances L_1 , L_2 and L_3 are to be determined from measurements and not from geometrical data. Those data could be used to divide L_1 into the two parts L_{11} and L_{12}. The geometrical model gives

$$L_{11} / L_{12} = -d_1/(6\,a_2) \qquad \textbf{(14)}$$

This ratio is an estimate and it should be used only if the relative values of L_1 , L_2 and L_3 from the measurements show a reasonable agreement with values calculated from the geometrical data using (9) – (11). If no reasonable agreement is obtained, it is recommended to connect the magnetizing branch to the terminal of the inner winding.

Fig. 5 shows the various models that have been described so far (Note that the values of R_m and L_m are not the same for all models).

L_1 , L_2 and L_3 are determined from measurements and the numbering of the windings is assumed done in such a way that L_1 and L_3 are greater than (or equal) L_2.

The choice among the models depends on the value of L_2 and the additional geometrical data that are available.

Table 1 shows the proposed selection.

Table 1 Selection of transformer model

Case	Description		Select transformer model
I	$L_2 > 0$		A
II	$L_2 < 0$	Winding 2 is not the center winding	B or C
III	$L_2 < 0$	Position of the windings unknown	B or C
IV	$L_2 < 0$	Position of the windings known and winding 2 is the center winding	D (or E)
V	$L_2 < 0$	Distances known and relative values of L_1 , L_2 and L_3 are in reasonable agreement with the geometrical data	E

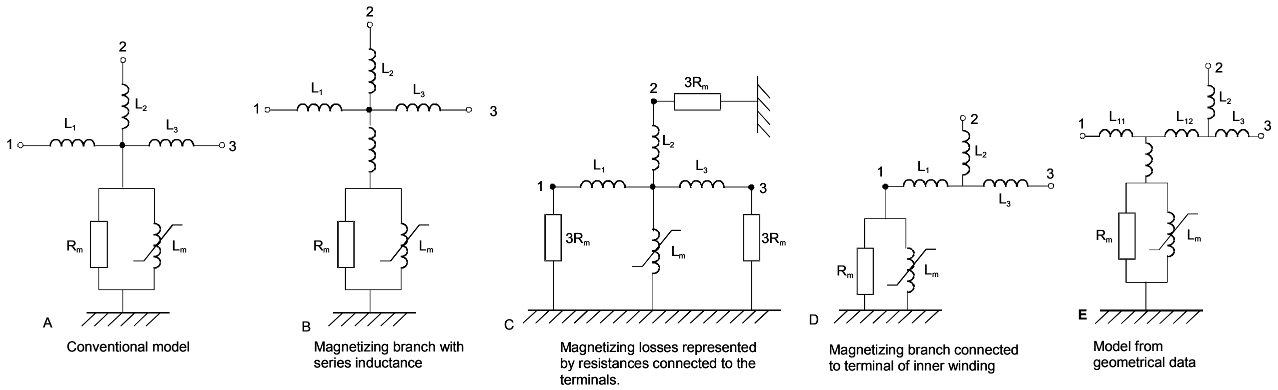


Fig. 5. Transformer models.

Comments to the table:

Case I: L_2 does not agree with Fig. 4. One possible reason is that the windings are not arranged as three concentric windings. The conventional model corresponds in this case to a passive network since all inductances are non-negative.

Case II: Fig. 4 shows that the negative inductance corresponds to the center winding, but the measured values give another result. That means that there is a fundamental disagreement between the measured values and the model based on geometrical data. Fig. 5 B or Fig. 5 C should then be applied since they correspond to passive networks that are derived without taking geometrical data into account. The series inductance in Fig. 5 B should be selected in such a way that it is greater than the absolute value of the parallel connection of L_2 and the minor one of L_1 and L_3 [2]. Fig. 5 B deviates in general less from Fig. 5 A in the frequency domain than Fig. 5 C. The difference is however not very significant and Fig. 5 C is often selected because it is easier to implement in existing programs that includes the transformer model in Fig. 5 A.

Case III: The negative inductance L_2 may or may not correspond to the center winding. A cautious approach is therefore to use the same model as in case II. It might however be worth to consider to apply the model in Fig. 5 D if it is reasonable to assume that L_2 corresponds to the center winding and $L_1 \approx L_3$.

Case IV: Fig. 5 D is the natural choice in this case. However, Fig. 5 E could be used if L_2 is used to estimate L_{11}. Eqs. (10) and (12) give:

$$L_{11} = L_2 \cdot d_1 / d_2 \qquad (15)$$

The ratio d_1 / d_2 can be estimated by assuming the same nominal current density in the windings and making some correction for the increased amount of insulation when the nominal voltage increases.

V. MAGNETIZING IMPEDANCE

The magnetizing impedance is very high compared to the short-circuit impedances when the transformer is not saturated. There is in this case no significant deviation among the various alternatives in Fig. 5 as long as they correspond to a passive network.

The situation may be quite different when saturation occurs. The magnetizing impedance in heavy saturation is of the same order of magnitude as the short-circuit impedances and the alternatives in Fig. 5 are not equivalent any more.

Ref. [3] (section 6.6.2) shows (in Table 6.2) a comparison between measurements and computations with the magnetizing branch connected as in Fig. 5 A or connected to the terminal of the inner winding. The transformer was in heavy saturation and was energized from one winding with the other windings open. The voltage was measured at all terminals and all windings were energized (one by one). Connecting the magnetizing branch to the terminal of the inner winding in the computation model gave differences less than $\pm$ 5%. Differences in the range 60% (one case even 128%) were obtained when connecting the branch as in Fig. 5 A. Ref. [3] concludes that the Saturable Transformer Component (in EMTP) could become more useful if the code were changed so that the magnetizing branch could be connected to any terminal. An EMTP-user can however overcome this obstacle by ignoring the magnetizing current in the Saturable Transformer Component and connecting a resistance and a non-linear inductance to the actual terminal.

The magnetizing impedance is normally determined by measurements, at least partly. The impedance in

Fig. 5 corresponding to the measurement may include a leakage inductance in series with the magnetizing branch. It is important to take this inductance into account when determining the magnetizing impedance to be used in the model.

The magnetizing branch includes a parallel resistance R_m and Fig. 4 shows that this resistance should not be in parallel with $k \cdot a_1$ that is a part of the magnetizing inductance. There is however no need distinguish between the two alternatives **except when one of them results in a transformer model that does not correspond to a passive network**.

VI. TWO-WINDING TRANSFORMERS

Section V showed that the location of the magnetizing impedance in the model might have a great influence even when there is no stability problem. It may therefore be of some interest to analyze a two-winding transformer. Fig. 6 a shows the model obtained from Fig. 4 for a two-winding transformer. The model normally applied is shown in Fig. 6 b.

The negative inductance between nodes 1 and A1 in Fig. 6 a can be calculated if the geometrical data are known. The inductance between nodes A1 and 2 should then be selected to give a leakage inductance that corresponds to the measured value. If it is not possible to determine the value of the negative inductance between nodes 1 and A1, then the best estimate is zero, i.e. to connect the magnetizing branch to the node corresponding to the inner winding (Fig. 6 c). This choice is much better than applying the conventional model (Fig. 6 b). **The conventional model should therefore be used only when it is impossible to identify the inner winding**.

VII. SOPHISTICATED FIVE-LEGGED TRANSFORMER MODEL

Ref. [4] presents a topologically correct model for a five leg three-phase transformer with two windings per phase. Fig. 7 corresponds to Fig. 2 in [4] and shows a cross section of core and winding assembly as well as the derived magnetic circuit. The modeling of the leakage inductances is not discussed in [4] and it is therefore analyzed in this paper.

The current sources I_{H1} and I_{X1} in Fig. 7 correspond to the current in phase 1 of the high voltage and the low voltage winding respectively. The low voltage winding is the inner winding. It is found from the figure that L_{H1X1} equals the leakage inductance between the two windings. The inductance L_{X1C} represents the leakage flux between the inner winding and the core. Comparing Figs. 6c and 7 it is found that the leakage inductances L_{H1X1} and L_{X1C} are connected in the same way as L (corresponding to L_{H1X1}) and $k \cdot a_1$ (corresponding to L_{X1C}). The same model is used for the leakage flux of all phases in Fig. 7. The model in [4] corresponds therefore to Fig. 6 c and not to the conventional model (Fig. 6 b). Modeling the linkage fluxes as in Fig. 6 a may give a more accurate model provided that the relevant geometrical data are known.

VIII. CONCLUSIONS

One of the leakage inductances in the conventional transformer model becomes normally negative for

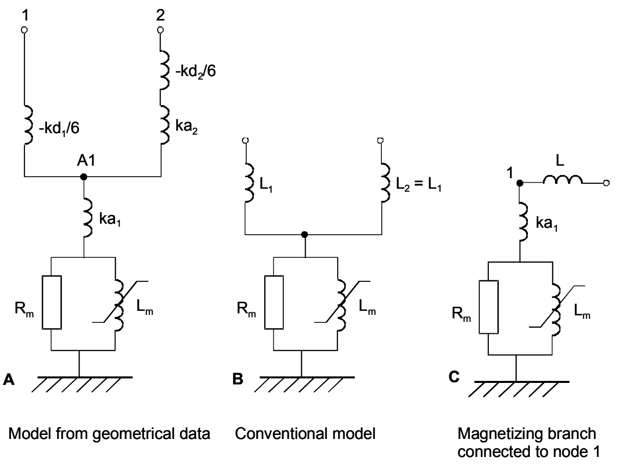


Fig. 6. Two-winding transformer models.

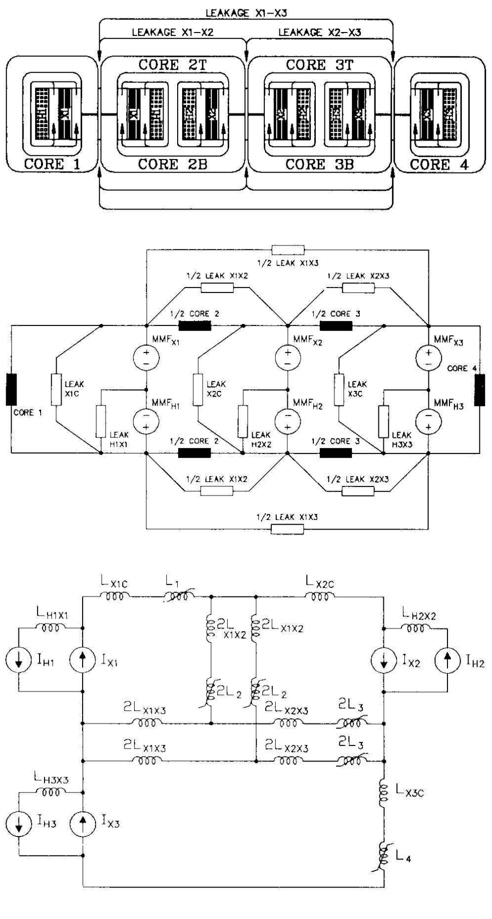


Fig. 7. Model presented in [4] (Fig. 2).
Top: Cross section of core and winding assembly. Leakage flux tubes are labeled. Center: Corresponding lumped-parameter magnetic circuit. Bottom: Duality derived equivalent circuit.

three-winding transformers and could cause an unstable time domain circuit model. This paper shows how to avoid this problem.

Alternative modifications of the original model are proposed and the choice should be made depending on geometrical data available for the actual transformer. The main deviation among the various alternatives is the location of the magnetizing branch. This location does not have any significant influence except when the transformer is saturated. The influence may be very significant in heavy saturation and it is in general better to connect the magnetizing branch to the terminal of the inner winding than to the internal node in the conventional model.

The leakage inductances in the two-winding version of the conventional transformer model are normally assumed to be equal. A more accurate model is obtained for transformers with concentric windings if the magnetizing branch is connected to the terminal of the inner winding.

IX. REFERENCES

[1] Canadian / American EMTP User Group, *Alternative Transients Program (ATP) – Rule Book*, 1987 – 1998

[2] T. Henriksen; "How to Avoid Unstable Time Domain Responses Caused by Transformer Models", paper no. 2000 TR 428 submitted (May 2000) to IEEE for publication in *IEEE Transactions on Power Delivery*.

[3] H. W. Dommel; *Electromagnetic Transients Program (EMTP). Theory Book*, Bonneville Power Administration. Portland, Oregon 1987

[4] B. A. Mork; "Five-Legged Wound-core Transformer Model: Derivation, Parameters, Implementation and Evaluation", *IEEE Transactions on Power Delivery*, Vol. 14, No. 4, pp 1519-1526, October 1999

APPENDIX. LEAKAGE INDUCTANCE FORMULA

Ref. [A1] presents in (6a) a formula for the short-circuit reactance between two concentric windings. The formula is based on the flux density variation shown in Fig. A1.

The leakage inductance corresponding to the windings with thickness d_2 and d_3 in Fig. A1 can be found from (6a) [A1]: (Each winding is here assumed to have one turn).

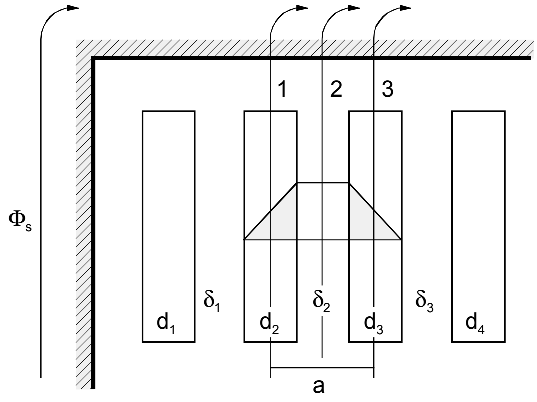


Fig. A1. Leakage flux density variation, from Fig. 16 [A1].

$$L_{23} = \frac{\mu_0 \cdot \pi \cdot D^1}{\ell^1}\left(\frac{d_2}{3}+\delta_2+\frac{d_3}{3}\right) \qquad \textbf{(A1)}$$

where μ_0 is permeability of vacuum, D^1 is the mean diameter and ℓ^1 is the high (h) of the windings plus a contribution $\left(d_2+\delta_2+d_3\right)/3$. Ignoring this last contribution and substituting the distance δ_2 by a (see Fig. A1) gives:

$$L_{23} = \frac{\mu_0 \cdot \pi \cdot D^1}{h}\left(a-\frac{d_2}{6}-\frac{d_3}{6}\right) \qquad \textbf{(A2)}$$

This formula ignores the variation in the circumference as the distance from the core increases. One version of the formula was presented by Kapp in 1898 [A2]. Experience has shown that (A1) is reasonably correct. Ref. [A3] presents same comparisons between the formula and measurements and the deviation is typically in the range $\pm$ 10%.

REFERENCES

[A1] K. Schlosser; "Eine auf physikalischer Grundlage ermittelte Ersatzschaltung für Transformatoren mit mehreren Wicklungen", Brown Boveri-Nachrichten, vol. 45, pp. 107-132, March 1963 (in German)

[A2] G. Kapp; "Ein Beitrag zur Vorausberechnung der Streuung in Transformatoren", ETZ bd. 19 (1898), H15, pp. 244-246 (in German)

[A3] K. Schlosser; "Anwendung der Ersatzschaltung eienes Transformators mit mehreren Wicklungen", Brown Boveri-Nachrichten, vol. 45, pp- 318-333, June 1963 (in German)