Simplified Modelling of Hysteresis for Power System Transformers Studies

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Abstract - **Power system Engineers frequently face problems related to hysteresis in power transformers that need a deeper understanding in order to analyse and solve these problems. Even though there are many accurate models available, these models could present some difficulties to those Engineers who a have practical profile. Therefore, this work has the aim of presenting a simplified modelling of the power transformers that readily allows the simulation of common problems in power system, like ferroresonance, magnetic losses, influence of harmonics components and inrush currents. Due to the simplicity of the proposed modelling, it can be easily implemented with the use of spreadsheets, which allow intense graphical interaction.**

Keywords: Hysteresis modelling; magnetic losses; inrush current; ferroresonance; harmonics.

I. INTRODUCTION

As hysteresis is a very complex phenomenon its modelling for power transformers has been constantly studied by many researchers, who have developed many models for simulation of its characteristics [1,2]. Recently, models of hysteresis have been developed which aimed the complexities reduction, what effectively have simplified the analyses of problems in power transformers related to hysteresis [3]. Nevertheless, present available modelling is still difficult for power system Engineers to deal with. This work presents a modelling that can be easily implemented in personal computers to help power system Engineers to analyse hysteresis effects. It can be applied, for example, in quite common problems in power system, like those related to ferroresonance, to magnetic losses under sinusoidal, with or without harmonics, and to inrush currents.

Even though proposed modelling has not yet been tested with experimental data, yet, its results are reasonably similar to experimental data presented in literature [2,3], yielding fast and relatively accurate evaluations on power transformer hysteresis analysis. Engineers. Nevertheless, experimental tests are in course and they will be later presented.

II. BASIC PRINCIPLES OF PROPOSED HYSTERESIS MODELLING

It is recognised that inside of the magnetic circuits of power transformers there are many complex interactions, that might initially suggest that power transformer is impossible to model. One of these complex interactions is the one related to unavoidable non-uniformity of magnetic flux distribution, that causes localised saturation, for example. However, from the point of view of practicity some necessary simplifications have been added to the models, along the time, since power transformer have been studied . Therefore, today it is quite possible to deal with hysteresis with few requirements of knowledge on physics of magnetism. This proposed modelling is intended to further simplification of the analysis of power transformers and it is based upon the following two principles :

a) dc magnetisation curve, with the presence of saturation, is mathematically fitted with any function that has signal coherence. It means that any change of signal in magnetisation current causes same change of signal in magnetic flux ;
b) magnetisation current will always present two components : Static and Dynamic. First component is obtained from dc magnetisation curve of a non-magnetised specimen and the second one is directly proportional to the time rate of magnetic flux. Therefore, by this principle, experimental dc magnetisation curves are only correct if variations of magnetisation current are quite slowly made and magnetic circuit is non-magnetised. Thus, the influence of the second component can be neglected.

Mathematically, second principle is so described :

$$i(\lambda,t) = i(\lambda(t)) + k\frac{d\lambda(t)}{dt} \qquad (1)$$

Where $\lambda(t)$ is the magnetic flux linkage along time and is directly related to the magnetic flux by the number of coils of the winding. k is a constant of proportionality, which is associated to the geometry of magnetic circuit and to its material composition [4]. From (1) it is possible to notice both components described on the second principle. It can also be seen that if the flux variation rate is negligible, only the first component appears and time can be omitted. This is the static regime of hysteresis.

III. THE PRESENCE OF HYSTERESIS LOOPS IN THE

SINUSOIDAL REGIME

Using (1) to model the circuit of a power transformer, traditional hysteresis loops are obtained if sinusoidal voltage is applied to the winding of this power transformer. The reason for the presence of loops lies in two facts : 1) The dynamic component cannot be negligible and 2) This component is related to energy dissipation, of a resistive nature, while first component is reactive, with its non-linear inductive nature. At this point, it is important to emphasise that an ellipse can be obtained on a oscilloscope if a linear inductor and a resistor are connected in parallel, sinusoidal voltage is applied to both and magnetic flux linkage and total current are input signals to x and y oscilloscope channels, respectively. Oscilloscope must be adjusted to xy mode. Therefore, steady-state hysteresis loops can be understood as being degenerated ellipsis. This shape change is due to saturation , which ias a non-linear inductor characteristic.

For computational simulation purposes, (1) must be simultaneously solved with equation derived from Ohm's and Lenz's laws applied to the circuit of the winding :

$$V(t) = r\, i(t) + \frac{d\lambda(t)}{dt} \qquad (2)$$

In this equation, the first term o the right side is the voltage drop along winding due to its electric resistance . $V(t)$ represents any kind of voltage source. In most of the problems, voltage source can be considered as an ideal one, with no internal impedance. But even in the problems that this internal impedance needs to be considered, only its resistance is required, in view of representing internal distortions on voltage. Nevertheless, actual internal impedance can be also considered, but it causes increased complexity to solve the set of equations, with small increase on results accuracy.

IV. EXAMPLES

In view of a better understanding of proposed modelling, some common problems that are constantly faced by power system Engineers are here analysed by computational simulation, based on the presented modelling :

A. *Basic example : Evaluation of magnetic losses for pure sinusoidal and for non- sinusoidal voltage.*

As a first example, consider a hypothetical small power transformer that has its experimental dc magnetisation curve adjusted with hyperbolic function, with metrical units, as described in Section II, principle (a).

$$i(\lambda) = 0.5\, sinh\, (3\lambda) \qquad (3)$$

As this expression is obtained from static regime, time has been omitted. As present modelling suggests, the dynamic equation of winding current, derived from (3) is :

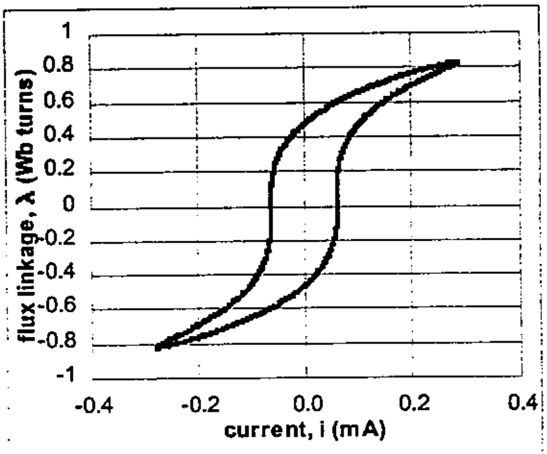
$$i(\lambda(t)) = 0.5\, sinh(\, 3(\lambda(t)) + k\, \frac{d\lambda(t)}{dt} \qquad (4)$$

Where the second component is due to the change of magnetic flux linkage in time. Now, a pure sinusoidal voltage source , 220 V*ef*, is connected to the winding of the transformer, which causes a magnetic flux inside magnetic circuit of this transformer:

$$V(t) = 220\sqrt{2}\, sin(120\pi t + \Phi)) = r\, i(t) + \frac{d\lambda(t)}{dt} \qquad (5)$$

Both (4) and (5) can be easily simultaneously solved by numerical methods. $\lambda(t)$ is the state variable, and the current is consequently obtained from (4). Fig. 1 shows hysteresis loop for steady state condition, calculated by Euler's method. For this calculation k is equal to 0.0002 Siemens and r = 1 Ω. It is important to notice that transient state inherently occurs as previous equations are solved. Nevertheless, as in the case of energising a linear RL-circuit, steady-state can be reached by adequate chose of initial voltage phase, Φ, present in (5). For the present example, transient state has been reached by trials of values of Φ, resulting in $\Phi = \pi/2$. Also, initial value for λ is null, even though it may have different values for representing the effects of residual magnetism. From this simulation, magnetic losses resulted in 9.54 W, and power factor resulted in 0.27. This last value has been obtained by traditional method of calculation.

Fig. 1. Steady-state hysteresis loop for purely sinusoidal voltage - Basic example.



Addition of harmonics to the expression of voltage source can be easily done to study the influences of a non-sinusoidal input voltage. By using the model, the influences of harmonics on magnetic losses is also easily evaluated. Fig. 2 shows steady-state hysteresis loop for a case of strong presence of third harmonic, with its voltage amplitude equal to the fundamental one. Even though this situation may be quite unlikely to occur, this is so done because in actual situations the presence of harmonics causes few influences on hysteresis, due to its mathematical dependence to voltage in the integral form.

Obtained results from proposed modelling gave magnetic losses equal to 4.79 W and power factor equal to 0.46. Many others results are possible, like those about the influence of simultaneous harmonics and also the influence of sustained over- and under-voltages.

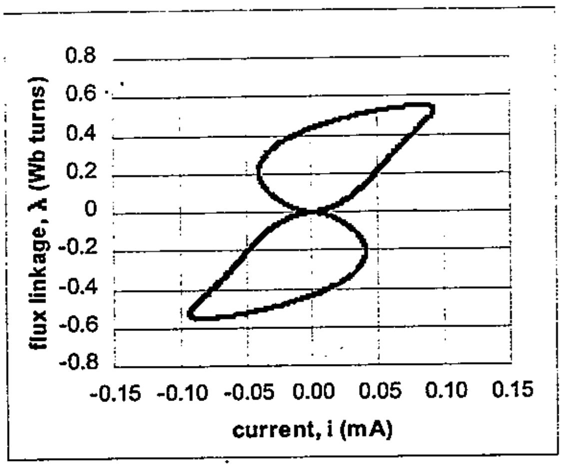


Fig. 2. Steady-state hysteresis loop for sinusoidal voltage with presence of third harmonic. $\Phi = \pi/2$.

B. *Inrush current*

For the same transformer of the basic example, with pure sinusoidal voltage source calculation is made for generic values of Φ, and critical values of inrush currents can be obtained. Fig. 3 shows the behaviour of inrush current for $\Phi = 0$. It can be seen that this transient current decays slowly, which explains problems associated to undesirable protection tripping when transformers are energised [5]. For a better analysis of this problem, effective current expected in the transient state is 163 mA.

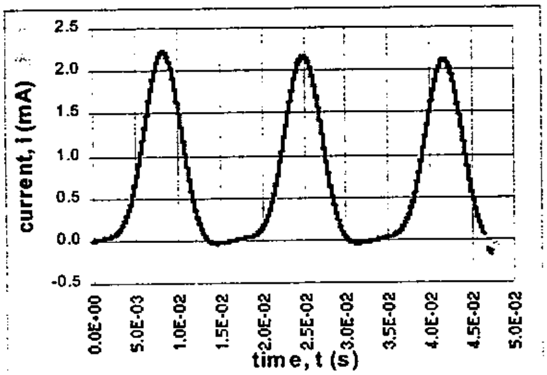
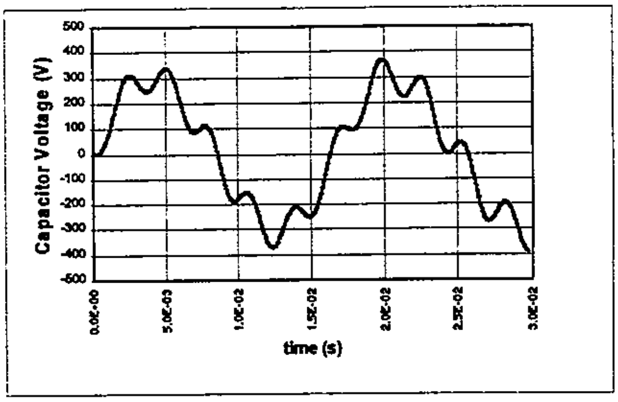


Fig. 3. Inrush current

C. *Ferroresonance*

As unloaded power transformers have reduced power factors, some Engineers are led to correct this situation by simply connecting capacitor banks. Thus, in some specific cases, a complex interaction can occurs between winding and capacitors, which is known as ferroresonance. Much has been studied about problems related to this practice, and many complete and consequently complexes models were developed. This situation can be readily analysed by power system Engineers if the proposed modelling is considered. To do so, the voltage source must have its internal resistance considered and a power corrective capacitor is parallel connected to the transformer. A new state-variable arises, that is capacitor voltage. Consequently, another equation is added to the analysis, that is the equation of displacement current in capacitor. For the same transformer used before, by using proposed modelling with same pure sinusoidal voltage source, Fig, 4 shows the behaviour of capacitor voltage along time, in the steady-state. It can be depicted the presence of harmonics as a consequence of interaction between capacitor and winding. Also, it is possible to notice that overvoltage occurs, since maximum voltage exceeds 311 V, that is the maximum voltage of the source. Internal impedance of voltage source has the value of 0.2 Ohms, the value of capacitor is 4.6 μF and $\Phi = 0$.

Fig. 4 - Voltage capacitor for the case of ferroresonance.



Also, for purposes of ferroresonance analysis, it is suggested that proposed modelling should be used for the cases when voltage source has its effective value over the normal one. This is to evaluate the influence of overvoltages in the capacitor lifetime, since its current strongly grows. Also interesting variables for analysis are current on the voltage source, capacitor current and total power factor, for the steady-state. As this situation has non-linear elements, some interesting results are obtained. For example, there is a considerable difference between values of corrective capacitance obtained from the theory of traditional power circuits and that from simulation, or practice. This situation is constantly faced by power system Engineers.

Transient analysis is also important, and can be done with this modelling by using different values of Φ.

D. *Additional Remarks*

Even though this proposed modelling has been presented though the analysis of power system problems, some extended analysis are allowed by this modelling. For example, it is possible to analyse the behaviour of magnetic flux linkage as voltage steps are applied in view of obtaining experimental dc magnetisation curve. After all, this modelling presents one of the intrinsic characteristics of hysteresis, which is the memory effect. In this model it is represented by the initial value of λ.

V. CONCLUSIONS

A hysteresis modelling is proposed for simplifying the analysis of most frequent problems related to power transformers, that are faced by power system Engineers. Presented results give an understanding of how voltage harmonics influence on magnetic losses, also how ferroresonance gives origin to overvoltages or how inrush current are dependent on the initial conditions. All of presented results are based on Euler's numerical method for solution of differential equations. All calculations and graphics were developed in spreadsheets, that are common to any Engineering office. Therefore, any power system Engineer will be able to reproduce these results and thus he/she will be able to simulate particular cases.

VI. REFERENCES

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