Combined Iteration Algorithm for Nonlinear Elements in Electromagnetic Transient Simulation

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Abstract - **This paper presents a combined method of Modified Predictor-Corrector Iteration and Newton Raphson Iteration to extend Electromagnetic Transient Simulation to include nonlinear elements in the electrical network solution. The resultant non-linear models are efficient, stable, and more accurate than those using the basic nodal-conductance approach. The algorithm has been tested using large time steps, and with simultaneous abrupt changes of the characteristic of two or more nonlinear elements. The paper also discusses methods for testing convergence, and compares this method to results obtained with interpolation and to the basic nodal conductance matrix solution.**

Keywords : nonlinear circuit, transient calculation, Newton-Raphson iteration, modified predictor-corrector iteration, combined iteration method, interpolation.

I. INTRODUCTION

The nodal-conductance approach (NCA) is the basis of many Electro-Magnetic Transients Programs such as EMTP, ATP [1] and PSCAD/EMTDC [2]. Changing either the conductance or current injection of a component can be used to approximate non-linear elements. However, unstable or inaccurate solutions can result when applied to complicated systems, such as those utilizing HVDC (high voltage direct current), FACTS (flexible ac transmission system) or power electronic models (whenever a large number of nonlinear elements are to be represented). The inaccuracies occur because switching is limited to occur only on regularly spaced time step points, or because the branch impedance or current injections are based on circuit quantities from the last time step. Linear interpolation [3,4] can represent non-linear devices using piecewise linear approximations, and results in stable and accurately calculated results for any number of non-linear devices. However, it may be difficult to express all kinds of non-linear devices in this way. Therefore it is important to find a more general, accurate and stable representation for nonlinear elements.

Previous work in this area includes a Predictor-Corrector Iterative (PCI) method [5]. This method required the formulation of the conductance matrix G (of G*V = J) in all time steps and iterations, which results in long simulation times. It has been also found that the above method can become unstable on some complex electric circuits as HVDC and FACTS systems.

This paper presents a nonlinear element represented using a parallel connection of a piecewise linear conductance and a nonlinear current source using the combined iteration method of Modified Predictor Corrector Iteration (MPCI) algorithm and Newton Raphson Iteration (NRI) algorithm stably, accurately and fast. Complex nonlinear elements (like primary arc models) whose present state cannot be accurately decided from the previous state are represented in this paper.

This paper demonstrates the proposed algorithm with various devices and example systems, and compares the results to results obtained with linear interpolation, basic NCA solution and measured results.

II. FORMULATION OF NONLINEAR CIRCUIT

A. Presentation of a nonlinear element

PCI [5] requires to calculate the conductance matrix **G** (of **GV** = **J**) in all time steps and iterations, because a nonlinear element is presented as only a nonlinear conductance. This results in huge calculation times, and the results are unstable on some complex electric circuits such as HVDC and FACTS systems. But the method proposed in this paper need not change the conductance matrix in most time step loops because a nonlinear element is presented as the parallel connection of a piecewise linear conductance and a nonlinear current source. Therefore this proposed method is faster than other iteration methods.

Fig.1 and Fig.2 shows the representation of a non-linear device as the parallel connection of a piecewise linear conductance and a nonlinear current source. This can be modelled as shown in equation (1).

$$i_1 = G \times v_1 - J'_{non} \quad \left(J'_{non} = J + J_{non}\right) \tag{1}$$

where J'_{non}: a nonlinear current source, G: a piecewise linear conductance. When a target circuit includes some nonlinear elements, an iteration procedure is required to fi-

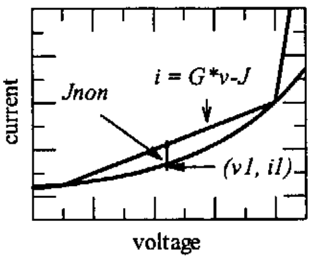


Fig.1 Example case

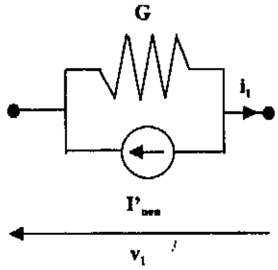


Fig.2 Equivalent circuit

nd the solution of the following nodal-conductance equation:

$$\mathbf{G}(t)\mathbf{v}(t) = \mathbf{J}(t) + \mathbf{J}_{\mathbf{non}}(t, \mathbf{v}(t)) \tag{2}$$

where J(t) : linear current injection vector. In this paper, the combined iteration method presented after this section is proposed to get the solution of equation (2).

B. Optimum ordering of nodes

When all elements in the circuit are linear, those elements are described by the trapezoidal rule of integration [1], and the equations of such circuits include only linear elements and can be expressed in the following matrix equation:

$$\mathbf{G}\,\mathbf{v}(t) = \mathbf{J}(t) \tag{3}$$

where **G** : linear nodal-conductance matrix, v(t) : node voltage vector, and **J**(t) : linear current injection vector. In this case, **G** is a constant matrix, and **J**(t) is time varying. The triangular factorization of **G** is performed only once before advancing to the time step loop, and v(t) is calculated by backward substitution. At the end of each time step **J**(t) is renewed to calculate v(t+Δt) which is the node voltage vector at the next time step.

When the circuit includes some nonlinear elements, which are expressed as nonlinear conductances, **G** can depend on many factors, such as the instantaneous voltage solution **v**(t) [5]. That is to say, the retriangulation of **G** is required whenever the factors change at a time step or an iterative step. In this proposed method (MPCI) the retriangulation of **G** is not required at each time step and each iterative step, because the nonlinear elements are expressed as a piecewise linear conductance and a variable current injection. The method of the triangular factorization of **G** is illustrated in Fig.3.

The ordering proceeds in the following order (as shown in Fig.3): linear nodes without switches, linear nodes with switches, nonlinear nodes without switches, nonlinear nodes with switches. When one or more nonlinear elements operate at that time step, the portion of the matrix from the smallest node number involved in the nonlinear elements to the end must be retriangulated. This ordering method is closely related to the proposed method in [6].

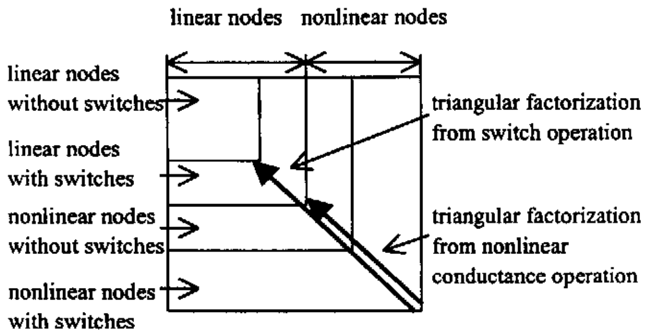


Fig.3 Optimum ordering of nodes in **G**

III. THE ITERATION PROCESS

A. Modified Predictor-Corrector Iterative Method (MPCI)

One of the proposed methods in this paper is MPCI method. This method is closely related to PCI method in [5]. The MPCI method doesn't require a lot of reconstitution of **G** at each time step loop and each iterative step, because it represents nonlinear elements differently than the PCI method. The details of MPCI is illustrated in this section.

The solution of the following equation $\mathbf{v}^{(0)}(t)$ gives the first estimation of the iteration (prediction):

$$\mathbf{G}(t)\mathbf{v}^{(0)}(t) = \mathbf{J}(t) + \mathbf{J}(t, \mathbf{v}(t-\Delta t)) \tag{4}$$

It should be noted that $\mathbf{v}^{(0)}(t)$ is different from the solution at the previous time step, because **J**(t) (linear current vector) has been already updated in (4). The improved solutions are repeatedly obtained by the following iteration scheme (correction):

$$\mathbf{G}(t)\mathbf{v}^{(k)}(t) = \mathbf{J}(t) + \mathbf{J}(t, \mathbf{v}^{(k-1)}(t)) \tag{5}$$

where k = 1, 2, ... is the number of iterations. When the maximum difference of an improved solution from the previous iteration step becomes smaller than a user specified error constant ε namely,

$$\max\left|\Delta v_i^{(k-1)}\right| = \max\left|v_i^{(k)} - v_i^{(k-1)}\right| < \varepsilon \quad (i : node\ index) \tag{6}$$

the $\mathbf{v}^{(k)}(t)$ is regarded as the final solution, and we now proceed to the next time step. If the maximum difference doesn't become smaller than ε within 2 or ^ times iterations, we proceed to iteration with NRI method.

B. Newton Raphson Iterative Method (NRI)

The multidimensional root finding method by NRI Method is discussed in [7]. NRI gives us a very efficient means of converging to a root, if a sufficiently good initial can be guessed. If it fails to converge, it indicates that the roots of the solution do not exist nearby.

A typical problem gives N functional relations to be zeroed, which involves variables x_i, i = 1, 2, ...,N:

$$F_i(x_1, x_2, \ldots, x_N) = 0 \qquad i = 1, 2, \ldots, N. \tag{7}$$

And each of the functions F_i in the equation (7) can be expanded in Taylor series

$$F_i(\mathbf{x} + \delta\mathbf{x}) = F_i(\mathbf{x}) + \sum_{j=1}^{N} \frac{\partial F_i}{\partial x_j}\delta x_j + O(\delta\mathbf{x}^2) \tag{8}$$

where **x** : the entire vector values x_i, **F** : the entire vector of functions F_i. The matrix of partial derivatives appearing in equation (8) is the Jacobian matrix **Jc**:

$$Jcij = \frac{\partial F_i}{\partial x_j} \tag{9}$$

In matrix notation equation (8) is

$$\mathbf{F}(\mathbf{x}+\delta\mathbf{x}) = \mathbf{F}(\mathbf{x}) + \mathbf{Jc}\cdot\delta\mathbf{x} + O(\delta\mathbf{x}^2) \tag{10}$$

By neglecting term of order $\delta\mathbf{x}^2$ and higher and by setting $F(\mathbf{x}+\delta\mathbf{x}) = 0$, a set of linear equations for the correction $\delta\mathbf{x}$ that move each function closer to zero is derived simultaneously, namely

$$\mathbf{Jc}\cdot\delta\mathbf{x} = -\mathbf{F} \tag{11}$$

The matrix equation (11) in electrical circuits can be solved efficiently by LU decomposition. The corrections are then added to the solution vector,

$$\mathbf{x}_{new} = \mathbf{x}_{old} + \delta\mathbf{x} \tag{12}$$

and the process is iterated to convergence. When the maximum difference of an improved solution from previous iteration step (the maximum of δx_i) become smaller than a user specified error constant ε, namely,

$$\max\left|\delta x_i^k\right| < \varepsilon \quad (i: node\ index) \tag{13}$$

the $\mathbf{x}_{new}$ is regarded as the solution, and we now proceed to the next time step.

The construction of the Jacobian matrix (including some nonlinear elements) is discussed below. The example case which includes a nonlinear element between node i and j is illustrated. As in equation (2), the circuit vector **F** in (11) is expressed as follows ((14) and (15)):

$$\begin{aligned} G_{ii} &= G_{Lii} + G_N \quad G_{ij} = G_{Lij} + G_N \\ G_{ji} &= G_{Lji} + G_N \quad G_{jj} = G_{Ljj} + G_N \\ J_i &= J_{Li} + J_N \quad J_j = J_{Lj} + J_N \end{aligned} \tag{14}$$

$$\mathbf{F} = \begin{bmatrix} G_{11} & \cdots & G_{1i} G_{1J} \\ \vdots & \ddots & \vdots \ \vdots \\ G_{i1} & \cdots & G_{ii} G_{ij} \\ G_{J1} & \cdots & G_{ji} G_{jj} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_i \\ V_j \end{bmatrix} - \begin{bmatrix} J_1 \\ \vdots \\ J_i \\ J_j \end{bmatrix} \tag{15}$$

where G_L: the conductance of linear elements, G_N: the conductance of a nonlinear element, J_L: the current source of linear elements, J_N: the current source of a nonlinear element. Therefore, the Jacobian matrix is constructed as the following differential equation (16).

$$\frac{\partial \mathbf{F}}{\partial \mathbf{V}} = \begin{bmatrix} G_{11} & \cdots & G_{1i} & G_{1j} \\ \vdots & \ddots & \vdots & \vdots \\ G_{i1} & \cdots & \left(G_{ii} - \frac{dJ_i}{dV_i}\right) & \left(G_{ij} - \frac{dJ_i}{dV_j}\right) \\ G_{i1} & \cdots & \left(G_{ji} - \frac{dJ_j}{dV_i}\right) & \left(G_{jj} - \frac{dJ_j}{dV_j}\right) \end{bmatrix} \tag{16}$$

Equation (17) below is derived from equation (14) (see appendix 1 for details).

$$\begin{aligned} G_{ii} - \frac{dJ_i}{dV_i} = G_{Lii} + \frac{dI}{dV_i}, \quad G_{ij} - \frac{dJ_i}{dV_j} = G_{Lij} + \frac{dI}{dV_j}, \\ G_{ji} - \frac{dJ_j}{dV_i} = G_{Lji} - \frac{dI}{dV_i}, \quad G_{jj} - \frac{dJ_j}{dV_j} = G_{Ljj} - \frac{dI}{dV_j} \end{aligned} \tag{17}$$

The nonlinear current from node i to j is expressed as:

$$I = f(V_i - V_j), \quad V = V_i - V_j \tag{18}$$

and

$$\frac{dI}{dV_i} = \frac{df}{dV}, \quad \frac{dI}{dV_j} = -\frac{df}{dV} \tag{19}$$

Using equations (18) and (19), we can rewrite equations (16) and (17) as follows:

$$\begin{aligned} G_{Lii} + \frac{dI}{dV_i} = G_{Lii} + \frac{df}{dV}, \quad G_{Lij} + \frac{dI}{dV_j} = G_{Lij} - \frac{df}{dV}, \\ G_{Lji} - \frac{dI}{dV_i} = G_{Lji} - \frac{df}{dV}, \quad G_{Ljj} - \frac{dI}{dV_j} = G_{Ljj} + \frac{df}{dV} \end{aligned} \tag{20}$$

$$\frac{\partial \mathbf{F}}{\partial \mathbf{V}} = \begin{bmatrix} G_{11} & \cdots & G_{1i} & G_{1j} \\ \vdots & \ddots & \vdots & \vdots \\ G_{i1} & \cdots & \left(G_{Lii} + \frac{df}{dV}\right) & \left(G_{Lij} - \frac{df}{dV}\right) \\ G_{i1} & \cdots & \left(G_{Lji} - \frac{df}{dV}\right) & \left(G_{Ljj} + \frac{df}{dV}\right) \end{bmatrix} \tag{21}$$

In (21), the Jacobian matrix can be efficiently constructed by the differential function df/dv, which can be calculated analytically or numerically. If the function f can be expressed analytically like the arrester model and diode model, the differential function df/dv can often be calculated by analytical differentiation of f. If the function f cannot be expressed analytically, the method to differentiate the function f can be calculated numerically. The arc model, which can't be expressed analytically, is illustrated as an example case for numerical differentiation in the following section.

IV. COMPARISON OF MPCI, NRI AND COMBINED METHODS

In Table 1, the number of iterations required for convergence are compared for the MPCI, NRI, and combined algorithm. The MPCI method converges very slowly in many example cases, but is very stable and reaches the approximate solution quickly. Hence this method is not suitable for complex circuits like FACTS and HVDC systems. The NRI method converges very quickly when the solution is close to the root, but can be very unstable elsewhere. NRI therefore requires a good method to reach the approximate solution. Therefore, the third method, which uses MPCI method at the initial 2 or 3 iterations, then the NRI method is superior. Such a combined iteration method has been used in this paper. This method is stable, fast and accurate. It should be noted that the proposed scheme does not impose restrictions on the number and configuration of nonlinear nodes. Also, because the proposed method does not modify the basic equation of the nodal-conductance approach, it can be implemented in existing EMTP-type programs [1-3].

Table.1 The average number of iterations
for the circuits in Fig. 4, 7, and 10.

	MPCI	NRI	MPCI+NRI
Arrester	76.0	2.64	4.82
Diode	5.17	×	4.00
Arc	19.4	×	5.17
Features	Stable Slow	Unstable Fast	Stable Fast

× : non-convergency

V. EXAMPLES

A. Arrester model in an Oscillatory Circuit

An arrester circuit illustrated in Fig.4 is analyzed using the proposed method. The arrester in the circuit is modeled

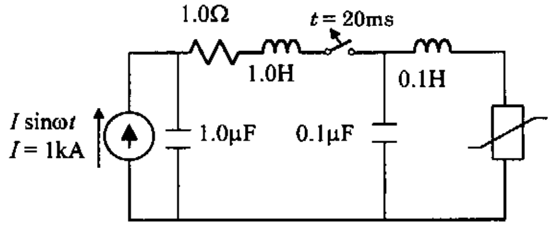


Fig.4 Oscillatory arrester circuit.

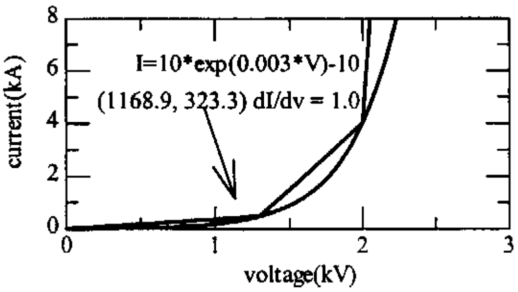


Fig.5 v-i characteristic of arrester-type model

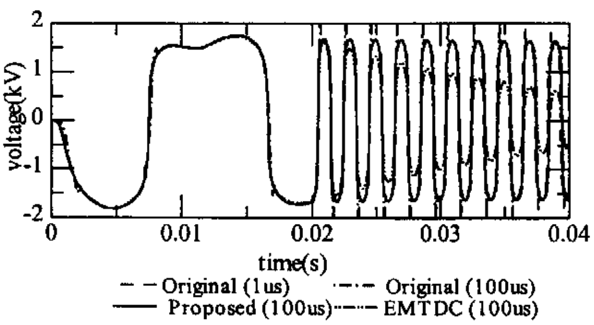


Fig.6 Calculated result

as a parallel connection of a piecewise linear resistance and a nonlinear current source. The $v - i$ characteristic is approximated by v(i) = 333.3 × ln (0.1 × I + 1.0) or i(v) = 10.0×{exp(0.003×v) - 1.0} as defined in Fig.5. The input function for iterations is sometimes voltage or current in different areas of the characteristic, depending on the slope.

Fig. 6 shows the circuit voltage for 4 different solution methods: NCA original method (Δt =1μs), NCA original method (Δt =100μs), EMTDC interpolated solution (Δt =100μs), and the proposed method (Δt =100μs) . The calculated result of the original method with Δt = 1μs compares closely with the proposed method which is solved using a much larger time step.

B. Diode-Bridge Rectifier Circuit

The diode-bridge rectifier circuit illustrated in Fig.7 is analyzed. The new method models each diode in the circuit as a parallel connection of a piece wise linear resistance and a nonlinear injective current source. The diode $v - i$ characteristic is approximated by v(i) = 81.65×10^{-3} ln (51.03 × 10^6 × I + 1.0), or i(v) = 1.95×10^{-8}{exp(12.25×v) - 1.0} as defined in Fig. 8.

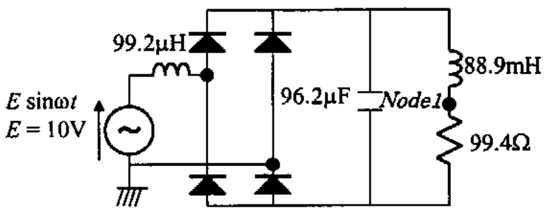


Fig.7 Diode-bridge rectifier circuit

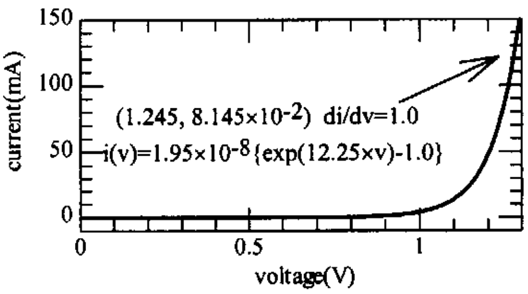
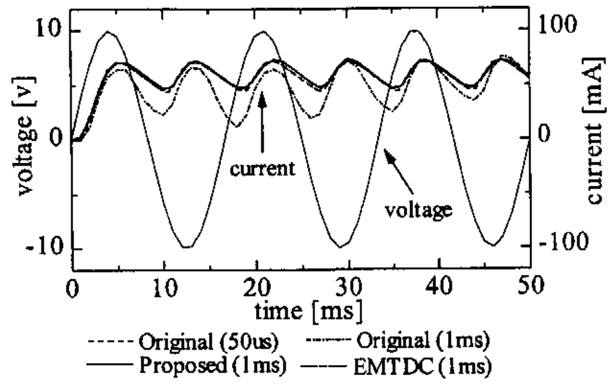
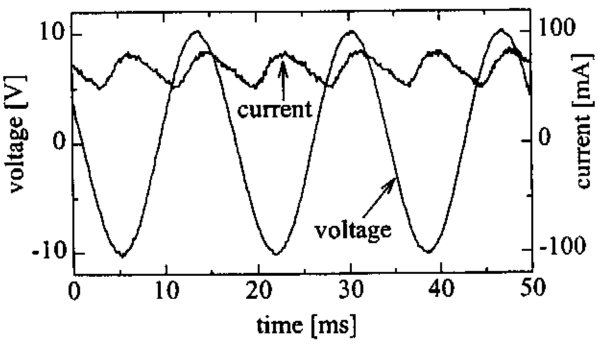


Fig.8 Voltage-current (v - i) characteristic of diode

Fig. 9 shows the circuit voltage and current for 4 different solution methods: NCA original method (Δt 50μs), NCA original method (Δt =1 ms), EMTDC interpolated solution (Δt =1 ms), and the proposed method (Δt =1 ms) . Fig. 9 shows the actual voltage and current measured from a physical test circuit. The calculated results of the original method with Δt = 50μs, the EMTDC solution solved with a 1 ms time step, and the new proposed method also with a 1 ms time step compare closely and match the measured waveforms.



(a) Calculated result (startup)



(b) Measured result (steady state)

Fig.9 Measured and calculated waveforms of input voltage and output current

C. Primary arc model

Earlier work has shown that a primary arc can be represented by the following equations [8,9,10] :

$$\frac{dg}{dt} = \frac{1}{T}(G - g) \tag{22}$$

$$G = \frac{|i|}{Vl} \tag{23}$$

The conductance g at time t tends toward the static conductance G which is determined by the present current i. The speed dg/dt with which g approaches G is determined by the arc time constant T. Various experimental studies have confirmed that the voltage drop along the main arc column is substantially independent of the current, and the value of stationary arc voltage per length is nearly constant in the arc cycle. It was shown that the average constant arc voltage gradient is about 15 V/cm over the range of current 1.4kA to 24kA in spite of some variation. For the primary arc, *T* and *l* (the length of arc) are considered as constant, and T can be given as following equation:

$$T = \frac{\alpha l}{I} \tag{24}$$

where the coefficient α is about 2.85×10^{-5} for the primary arc, I is the peak primary arc current in the first cycle (which is determined by using a small resistance instead of the primary arc model).

We are confronted by two problems in this primary arc model. The first is that the time varying arc conductance g can not be decided from previous information like v(t-Δt) or i(t-Δt), because of the strong non-linear nature of the arc. The second problem is that the primary arc model can not be expressed as piece wise linear approximation like arrester and diode model. Therefore the method to calculate the primary arc model by use of the combined iteration method is proposed. The following equation can be derived from (22) ~ (24) by the trapezoidal rule and MPCI rule at the first step.

$$g_{(0)}^{(0)}(t) = g(t-\Delta t)\frac{2T-\Delta t}{2T+\Delta t} + \frac{2\Delta t}{2T+\Delta t}\frac{|i(t+\Delta t)|}{Vl} \tag{25}$$

After the second step, the time varying arc conductance g is renewed by following equation.

$$g_{(k)}^{(n)}(t) = g(t-\Delta t)\frac{2T-\Delta t}{2T+\Delta t} + \frac{2\Delta t}{2T+\Delta t}\frac{\left|g_{(k)}^{(n-1)}(t)\, v(t)\right|}{Vl} \tag{26}$$

Where n : iteration times in arc model, k : iteration times in the main program (combined iteration method). When the maximum difference of an improved solution from previous iteration step (the maximum of $|g^n-g^{n-1}|$) becomes smaller than a user specified error constant ε, g^n is regarded as the solution of (22), and we now proceed to the combined iterative method in the main circuit which has already proposed in the previous section. However, the function f can't be expressed analytically like for the arrester and diode model. It is illustrated that the differential function df/dv can be calculated numerically. From (26), $I_{(m)}$,$V_{(m)}$ are derived from the solution of $g_{(m)}$ when k = m, and $I_{(m+1)}$,$V_{(m+1)}$ are derived from the solution of $g_{(m+1)}$ when k = m+1. Therefore df/dv is numerically calculated as $(I_{(m+1)} - I_{(m)})/(V_{(m+1)} - V_{(m)})$. Numerical differentiation is normally avoided because of stability concerns, but was found to be stable and efficient for this application.

A primary arc model test circuit is illustrated in Fig.10 and is analyzed by using the proposed method. The primary arc in the circuit is modeled as a parallel connection of a piece wise linear resistance and a nonlinear current source. The simulated arc characteristic is shown in Fig.11. Voltage waveforms are shown in Fig. 12 for the new proposed method (Δt =200μs) and the original NCA method

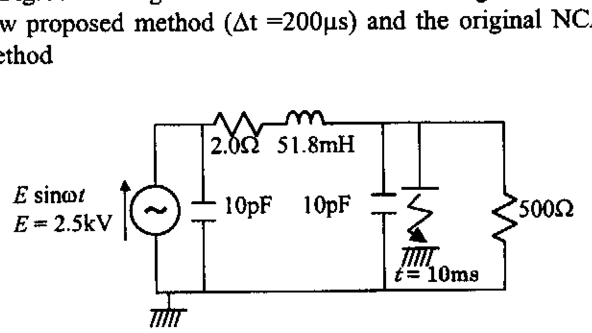


Fig.10 Primary arc model test circuit

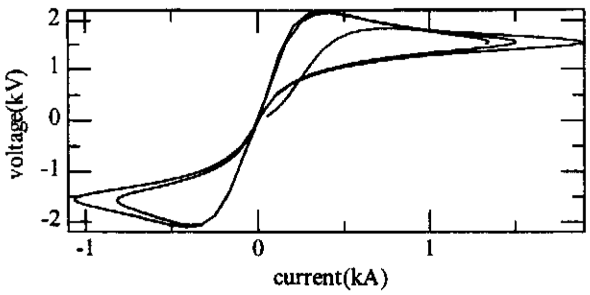


Fig.11 Primary arc characteristic

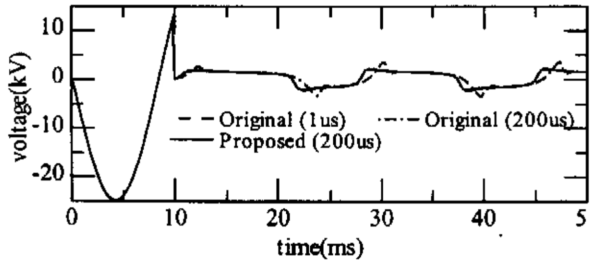


Fig.12 Calculated result

(1μs, 200μs). The calculated result by the proposed method is quite similar to the calculated result of original NCA method ($\Delta t = 1\mu s$).

VI. CONCLUSIONS

An extension to the NCA solution method has been presented to allow an arbitrary number and configuration of nonlinear elements in a subnetwork by use of a combined iteration method. This method has been applied to arresters, diodes, and primary arc characteristics. The calculated results agree well with actual measurements (where available), and with the small time step or interpolated solution methods. The proposed scheme has been shown to be accurate and stable even for a large time step, and can survive simultaneous and abrupt changes due to nonlinear elements.

This solution method has been made efficient by employing 2 different iteration techniques (MPCI and NRI). It is also made efficient by using a piece-wise linear conductance characteristic in parallel with a current injection updated during iterations. For complex non-linear characteristics, it has been shown that the Jacobian required for NRI iterations can be calculated numerically.

In the future, it is possible that this proposed method can be applied to other EMTP-type programs because it is an extension to the basic NCA method.

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VIII APPENDIX

1, Derivation of (17)

(17) is derived from Fig.1, (14) as following equation :

$$\begin{aligned} G_{ii} - \frac{dJ_i}{dV_i} &= G_{Lii} + G_N - \frac{d\left[J_{Li} + G_N(V_i - V_j) - J - I\right]}{dV_i} \\ &= G_{Lii} + G_N - G_N + \frac{dI}{dV_i} \qquad \text{(A.1)} \\ &= G_{Lii} + \frac{dI}{dV_i} \end{aligned}$$

Other equations in (17) are derived in a similar fashion.