REAL TIME, OSCILLATION FREE NETWORK DIGITAL SIMULATION

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Abstract - **Real time power system digital simulation is a tool for the development and testing of measurement and protection equipment. This work introduces a new approach to simulate electrical networks, reducing the total amount of computation. Although based on the theory of digital multirate signal processing, the final result is a single rate state space digital filter structure. The new approach also eliminates numerical oscillations, common to switched power systems simulation. The method being oriented for digital machines with inner product facilities, as is the case of digital signal processors (DSP), this work focuses on the algorithm validation by off line software simulation, rather than real time implementation.**

Keywords: Real time simulation, transient simulation, trapezoidal integration, digital filter.

I. INTRODUCTION

Digital simulation of analog systems, such as electrical networks, is usually done by some sort of approximation procedure. The aim is to approach the analog system continuous-time operation using a discrete-time counterpart. Clearly, the time step used in the procedure will somehow limit the approximation quality. As a result, very small time steps are usually employed during simulation, which is a solution for the approximation quality, but a problem for real time simulation.

A very common digital approximation method is the Trapezoidal Integration procedure, used in EMTP [1]. This integration rule has a frequency domain replica known as Bilinear Transformation. From this transformation one realizes, as expected, that analog frequencies are nonlinearly mapped into digital frequencies, the degree of distortion increasing with frequency. Just a limited analog frequency bandwidth around zero is almost linearly mapped to its equivalent discrete counterpart (about 1/16 sampling frequency).

One way to keep close to the region of linear mapping is to reduce the integration time step. As it reduces, the upper digital frequency limit, $f_t/2$ – half of the sampling frequency – increases. A rule of thumb is to consider the frequency mapping linear up to $f_t/16$ [2].

This work introduces a new approach to the Bilinear Transformation (Trapezoidal Integration), extending the linear range of the mapping to almost 80% of $f_t/2$. Consequently, one may use larger time steps in the simulation, reducing the computational burden.

II. BILINEAR MULTIRATE TRANSFORMATION

The basic idea behind this approach is to extend for higher frequencies the Bilinear Transformation almost linear mapping, restricted to the low frequencies band. Usually a higher sampling rate achieves this result, at the cost of an also larger computational burden.

The method proposes an alternate signal processing mapping, the Bilinear Multirate Transformation. Contrary to the traditional Bilinear Transformation, this approach is developed based on a multirate digital filter type of platform, although requiring a single rate to operate. The following steps illustrate the method [3-4] (please refer to Fig.1):

- It supposes the input signal at a low sampling rate, typically by limiting the representative bandwidth to at most 80% of the Nyquist frequency, $f_t/2$.
- An Expander increases this low sampling rate. An expansion by L uniformly places (L-1) zero samples between every pair of true samples. Therefore it increases by L the original sampling rate.
- The cost to be paid by the Expander action is observed in the generated signal, distorted by the added zeros. It is shown that in the frequency domain that distorted signal has its scaled original spectrum superposed to (L-1) uniformly spaced shifted versions of it [4].
- The distorted signal is fed to a lowpass digital filter, rejecting all those (L-1) undesired images. This allows the original signal to appear at the digital filter output with L times the original sampling rate.
- Unfortunately, the digital filter delays the signal and one is not really able to recover the true signal, but just a delayed version of it.
- However, since the distorted signal, at the lowpass digital filter input, has (L-1) zeros after every true sample, one may advance the filter response by (L-1) samples, corresponding to the (L-1) zeros.

- A fifth order elliptic lowpass digital filter of almost exactly (L-1) samples of delay was used. The filter response is shown in Fig.2. Advancing its response by (L-1) sample intervals, the true signal at higher rate is obtained. This is represented in Fig. 1 by the $z^{+(L-1)}$ block.
- The lowpass filter high rate signal output is connected to the digital counterpart of the system to be simulated. This counterpart is generated from the analog system state equations, by the conventional Bilinear Transformation. Since the time step is very small (large sampling rates are being used at this part of the simulator network), the desired original signal bandwidth now lies at lower digital frequencies, preserving the mapping linearity. As such, at these frequencies the digital system response is very close to the analog model.
- At this point one notices the high rate input signal is bandlimited, due to the lowpass filtering. The rejection of the undesired distorted signal images led to a lowpass output with almost all energy limited to the lower 1/L of the full digital spectrum.
- A second multirate digital signal processing operation, Decimation, is performed at the digital system output. Only one sample is retained of every consecutive L samples. This way, the sampling rate is reduced to the original sampling rate. Due to the bandlimited signal situation, described in the previous item, theory shows there is no loss of information in this rate reduction [2].
- Since only one of L system output samples is preserved, there is no need to evaluate the other (L-1) samples. The state equations are used for this purpose.

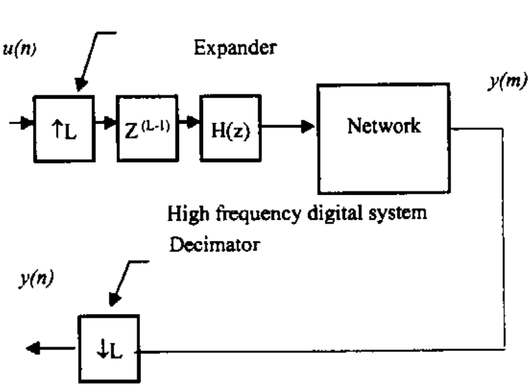

Fig.1 -Multirate Bilinear Transform Derivation

III. AVOIDING NUMERICAL OSCILLATIONS

The frequently found numerical oscillations in the simulation of switched power networks are shown to have origin in the Bilinear Transformation, or equivalently in the Trapezoidal Integration rule [5]. Eventual continuous time signals with a pole at infinity have their digital pole counterpart located at –1 of the z-plane unit circumference [2]. Positioning a section of the fifth order lowpass digital filter with a zero at –1 at the output of the simulated network cancels the effects of the critical pole, with no extra delay. This is shown in Fig. 3, where part of the lowpass interpolation filter, H(z), is shifted to the output of the system discrete counterpart, A. Note in Fig.3 there was no change in the overall gain from the expander output to the decimator input, since the order of a product does not alter its result. Breaking H(z) into $H'(z)(1+z^{-1})$ and placing the last factor subfilter at the simulator output cancels eventual signal pole at –1. This method, besides eliminating numerical oscillations in the simulator output, has shown to present better time response.

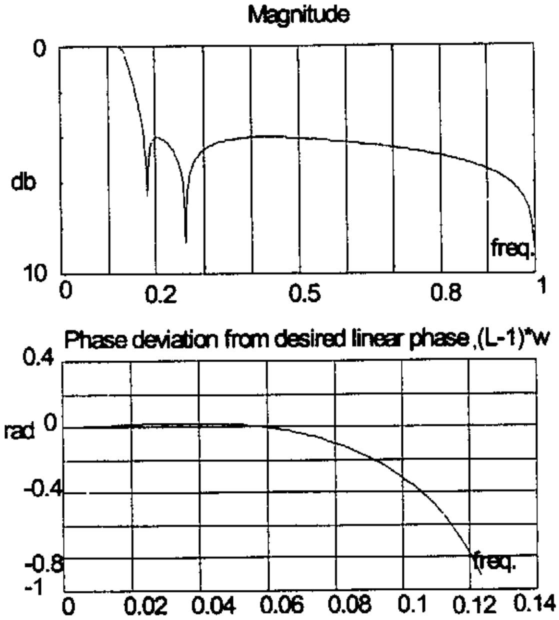

Fig. 2- 5th order elliptic interpolation filter

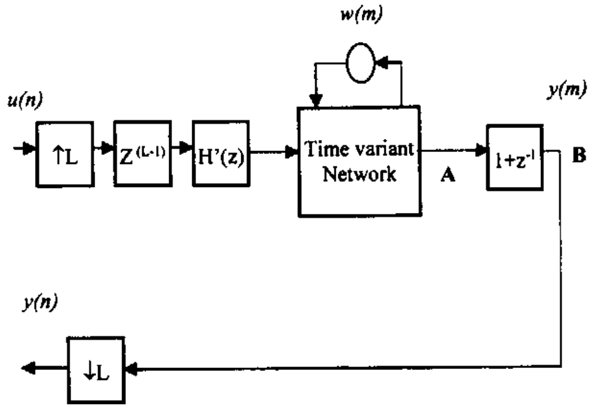

Fig. 3- Numerical oscillation elimination method

IV. RESULTS

The method was tested in comparison with the usual Bilinear Transformation, with respect to the computational burden for approximately the same simulation quality. This is achieved by increasing the usual Bilinear Transformation sampling rate with respect to the used in the Multirate Bilinear Transformation. Typically the required increase in sampling rate is of a factor four or five. This oversampling factor is labeled L`. The computational burden corresponds to the following parameters:

N (lowpass interpolation filter order),
p (number of network inputs),
q (analog network order)
L'(oversampling factor)

Fig.4 shows the curves relating number of multiplies versus network order (L'=4, N=5 and p=1). Note for networks with order three or more the Multirate method will have lower computational requirements.

Example 1: Comparative frequency responses of models derived by the Bilinear Multirate Transformation and by the Conventional Transform.

The network to be used is the cascade of pi sections, a common practice to approximate transmission lines. Fig.5 shows a network with equal inductors, 0.16 H, capacitors, 0.8784 μF, and resistors, 1.84 ohms.

The V_r/V_s magnitude frequency response (when a 600Ω load resistor is connected from terminal B to ground) is shown in Fig.6. Note that the Multirate Bilinear Method has similar response to the analog network model, while the Conventional Bilinear Transform presents larger errors.

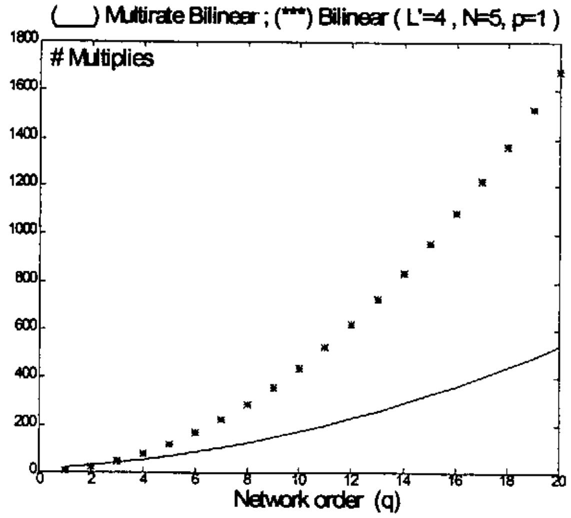

Fig.4 – Computational burden: comparison

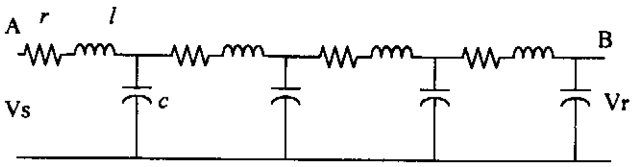

Fig.5. Four Sections Line Model

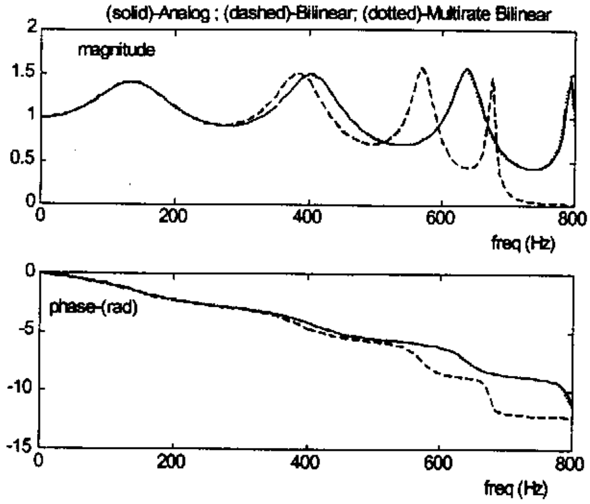

Fig.6. Vr/Vs frequency response

The time step used was T=333μs. However, to present similar performance, the Conventional Bilinear Transform has to use an oversampling factor of 5. The phase response has similar behavior.

Example 2: Comparative transient responses.

A simple power system network is shown in Fig. 7 with a single phase transmission line, connecting the source to an LC load (L=0.08 H and C=879.52 nF). The line parameters are frequency dependent, corresponding to mode 0 of the 160 Km three phase line described in Appendix A.

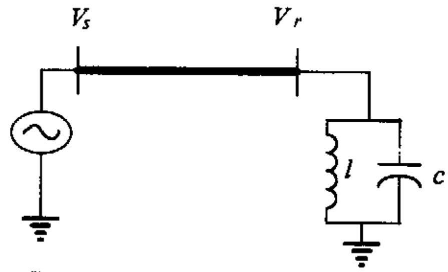

Fig.7- Single phase power network

Simulations were performed using a MATLAB based program and also the EMTP [5]. The first program implements the Multirate Bilinear Transform, while EMTP uses the Trapezoidal Integration Method (Bilinear Transform). The network is simulated using EMTP with time steps T, T/2 and T/8 (T=2.8e-4 s). The smaller time step is used to generate a reference response. The Multirate Bilinear simulator uses T as time step. The voltage $V_r(t)$ is shown in Fig. 8. Three curves are found in this figure: solid line represents the EMTP(T/8), dashed line stands for the EMTP(T), and dotted line represents the Multirate Bilinear Transform(T) output. Note that the Multirate Bilinear Transform produces an output closer to the EMTP(T/8) output than that of the EMTP(T).

The error plot, using EMTP(T/8) as a reference response, is also shown in this figure. One observes better Multirate Bilinear Transform performance than that of the

EMTP(T/2). This means it is possible to, at least, double the multirate time step, still preserving similar simulation qualities. One can consider the oversampling factor L' at least equal to two.

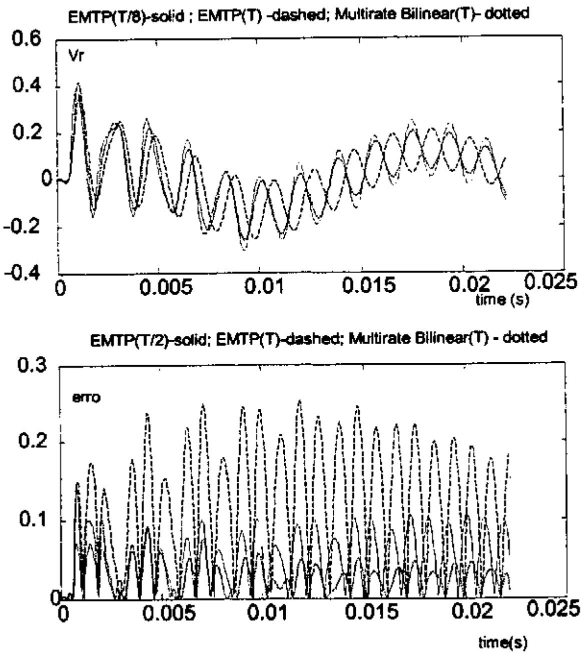

Fig.8 - Transient Time response

The comparative computational effort between Conventional and Multirate methods has to include transmission line simulation. It is possible to show that each mode in the frequency dependent line model uses $2(2Nf + Nz + 2)$ multiplies, where Nf and Nz stand for the order, respectively, of the impulse and of the characteristic impedance transfer function [7]. Typically Nf and Nz exceed 10 [7]. The computational effort for the Conventional Bilinear transform is given by $q(q+p)L'$ [3]. Table 1 resumes the computational burden associated to EMTP and to the Multirate Bilinear Method, using L'=2, N=5, p=1, q = 2 and $Nf = Nz = 10$

It is possible to notice the Multirate Bilinear Method has smaller error than EMTP(T/2) as well as a reduced computational burden. EMTP(T/2) requires 12 multiplies for the load and 128 for the line. The Multirate Bilinear Method requires 36 for the load and 64 for the line.

Table 1. Computational burden comparison

	EMTP(T/2) (Load + line)	Multirate Bilinear (T) (Load + line)
Multiplies Number	12+2*64 = 140	36+64 = 100

Example 3: Oscillation free simulation.

This illustrative case deals with the suppression of numerical oscillations, usually found in switched analog network computer simulations. The illustrative case relates to the network shown in Fig. 9. Note that, when the switch is open, the voltage on the switch terminal connected to the inductor will present numerical oscillations, in case the trapezoidal integration is used.

A common method to eliminate the numerical oscillations, found in literature, is the CDA (Critical Damping Adjustment) [6]. This method changes the numerical integration process when the switch status changes (substituting by Euler method) and returns to the trapezoidal integration approach, as soon as the oscillation is eliminated. This procedure is used here for comparative purposes. The multirate method is implemented as in Fig. 3, where the source w(m) represents the switch.

Simulation results are presented in Fig. 10. In this figure solid line represents the CDA(T/8) method, dashed line stands for the CDA(T) method, and dotted line represents the Multirate Bilinear Transform(T) method (the time step T is 0.5ms). In the figure, the impulse response caused by the opening of the switch, was removed for clarity

It may be observed that the Multirate approach result comes closer to that of the CDA(T/8), the reference.

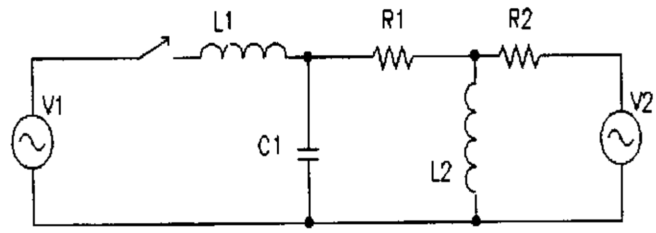

Fig.9- Network example

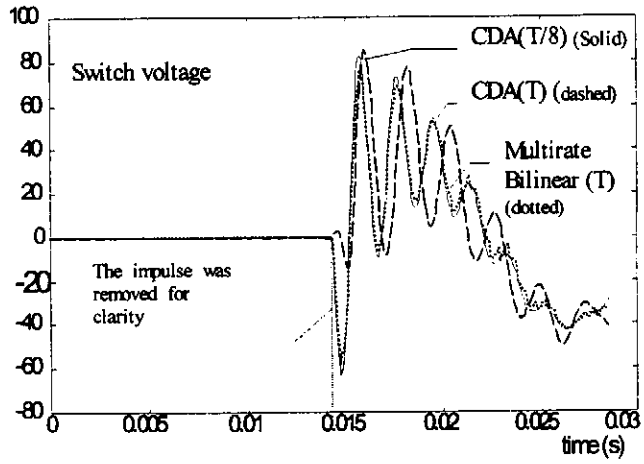

Fig.10- Transient simulation

V. CONCLUSION

A new approach was introduced for the real time digital simulation of electrical networks, leading to better quality in the simulation, as well as to reduced computational requirements. The software simulations focused on the algorithm validation, rather than on fast real time implementation. This allowed a comparative performance analysis, taking into account the number of arithmetic operations necessary for network simulation.

Comparisons always used as ideal responses those obtained applying EMTP at a high sampling rate. Multirate and Conventional Bilinear Transformation (Trapezoidal Integration) simulations used different sampling rates to obtain approximately close results. Multirate methods consistently could use larger time steps than the Conventional approach. The actual range of time steps range from two to four, representing the ratio between the Multirate time step to the Conventional Bilinear time step, both with similar results. This may represent a significant reduction in computational burden, mostly for large networks with several transmission lines.

The method yields a new state space representation in the discrete time domain, favoring the simulation by , but not requiring, inner product oriented digital machines.

Numerical oscillations caused by impulsive responses due to network switching are possible to be eliminated in the final simulation result. The impulsive term identifies a pole at infinity in the signal Laplace transform. This pole is mapped by the Trapezoidal Integration procedure into a discrete counterpart, located on the z-plane unit circumference at z = -1. This pole perfectly canceled by a lowpass filter section zero , properly located at the discrete system output. Ilustrative cases suport this result.

VI. REFERENCES

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APPENDIX A

Basic transmission line data

Phase conductors (tubular)
T/D = 0.231
DC Resistence = 0.0522 Ohms/Km
Outside diameter D = 3.18 cm

Sky wires (solid)
T/D = 0.5
DC Resistence = 0.36 Ohms/Km
Outside diameter D = 1.46 cm

Earth resistivity = 250 Ohms/m

400 kV Transmission line conductor spacings