Time domain cable modeling with frequency dependent parameters

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Abstract - **The aim of this paper is to propose a complementary method to improve the accuracy of cable parameters representation into time domain programs in which the effects of frequency upon resistances and inductances are required. A technique which will be able to represent a set of actual impedance measurements by a comprehensive multi-branches equivalent electrical circuit is presented. By using this approach it will be possible to feed time domain programs with more appropriate cable modeling.**

Keywords: Transient Analysis, Skin Effect, Modeling.

I- INTRODUCTION

When long cables are modelled for different computational studies, the inclusion of frequency dependent parameters may become a matter of great concern. Reference [1] describes the importance of considering this dependence when sub sea electrical installation with adjustable speed motor are fed via a long cable of several tenths of kilometres. According to the results given in that paper, significant errors may happen for the system frequency response and exaggerated resonant conditions are found when the cable parameters are kept constant. In addition, overvoltage calculation often requires better components representation due to the increased damping of transients for higher frequencies.

The skin effect is the main reason why the resistance dependency can be calculated by using frequency domain techniques, but this approach is not an easy way to include frequency dependent parameters into time domain programs. Reference [2] proposes an alternative methodology with no need of so many input data. The idea is to use a simple network with a proper R-L circuit combination to provide a first order approximation solution. A pair of resistance/inductance for two frequencies are used to calculate the equivalent circuit. Although the results are quite encouraging, the main limitation is that only two frequencies are precisely modelled in the equivalent circuit.

According to the proposed method, an exact agreement is only possible when an infinite number of branches is used. Although [2] has developed and tested a simpler equivalent circuit, the accuracy of the method increases with the number of branches. Computational studies carried out for different practical cables have shown that the first order approximation has presented discrepancies between theoretical results and real measurements. Therefore, the aim of this paper is to improve the accuracy of cable parameters representation into time domain programs. The approach here proposed consists of more than two R-L branches to provide the equivalent circuit and to verify the model computational performance and improvement in relation to the first order approximation.

II- THE METHOD

In accordance with [2] an exact agreement between real frequency dependent parameters and computational results can be obtained by an equivalent network with infinite number of branches. The method is well described in the above reference, which gives the circuit shown in Fig. 1 as the final proposal. The resistances and inductances are taken as constant values.

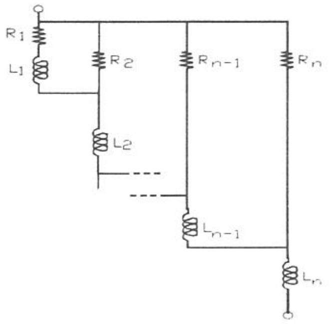

Fig. 1 - Equivalent circuit for frequency dependent parameters representation with infinite branches.

It should be noted that by rising frequency, the resistance increases the inductance decreases. Therefore, there is no use in adopting frequency dependent resistance and, at the same time, keeping the inductance as a constant value.

As an infinite number of branches would result in a non practical way of dealing with the problem, then a simpler representation must be adopted. The approach should be based on a given set of data derived from cable real measurements. The equivalent circuit given in [2] uses a first order approximation in which two branches are considered. This is illustrated in Fig. 2.

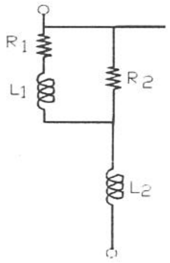

Fig. 2 - Equivalent circuit with two branches

By knowing two sets of cable parameters, i.e., a pair of resistance and inductance for two different frequencies, it is possible to evaluate the simplified equivalent circuit parameters (R_1, L_1, R_2, L_2) as follows:

$$L_2 = L_{eq(\omega 1)} - \frac{\Delta L . \left(\frac{\Delta R^2}{\Delta L^2} + \omega_2^2 \right)}{\omega_2^2 - \omega_1^2} \tag{1}$$

$$R_2 = \frac{\Delta R}{\Delta L} \left(L_{eq(\omega 2)} - L_2 \right) + R_{eq(\omega 2)} \tag{2}$$

$$L_1 = \frac{R_2^2 . \left(\omega_2^2 - \omega_1^2 \right)}{\Delta L . \left(\frac{\Delta R^2}{\Delta L^2} + \omega_1^2 \right) . \left(\frac{\Delta R^2}{\Delta L^2} + \omega_2^2 \right)} \tag{3}$$

$$R_1 = R_2 - L_1 . \frac{\Delta R}{\Delta L} \tag{4}$$

where:

$$\Delta L = L_{eq(\omega 1)} - L_{eq(\omega 2)} . \tag{5}$$

$$\Delta R = R_{eq(\omega 2)} - R_{eq(\omega 1)} . \tag{6}$$

and:

$L_{eq(\omega 1)}$ - cable inductance at the frequency ω_1;

$L_{eq(\omega 2)}$ - cable inductance at the frequency ω_2;

$R_{eq(\omega 1)}$ - cable resistance at the frequency ω_1;

$R_{eq(\omega 2)}$ - cable resistance at the frequency ω_2;

L_1 - 1st branch equivalent inductance;

L_2 - 2nd branch equivalent inductance;

R_1 -1st branch equivalent resistance;

R_2 -2nd branch equivalent resistance.

If the cable data provides more than two sets of parameter results, then a preciser equivalent circuit can be reached. In fact, if "n" pairs of resistance/inductance are known, then an equivalent circuit with up to "n" branches can be obtained.

Under the above condition, the following steps should be used:

Step 1: From a data consisting of "n" pairs of resistance/inductance obtained by measurements, two pairs of resistance/inductance, related to the highest available frequencies (ω_{n-1} and ω_n) shall be taken. Then, the inductance/resistance pair corresponding to the highest order branch "n" available can be evaluated through (1) and (2).

Step 2: Next, a parallel equivalent impedance for the remaining branches not yet evaluated must be found. This is illustrated in Fig. 3.

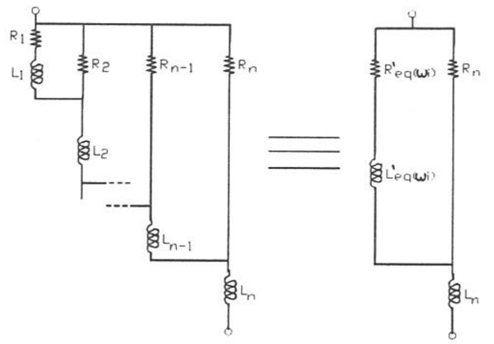

Fig. 3 - Cable equivalent circuit with "n" branches.

In Fig. 3:

$R'_{eq(\omega i)}$, $L'_{eq(\omega i)}$: resistance/inductance pair for the equivalent circuit, except the branch already evaluated in step "1".

R_n and L_n: resistance/inductance pair calculated in the previous step.

The pair $L'_{eq(\omega i)}$, $R'_{eq(\omega i)}$ can be evaluated through (8) and (9), respectively, to all remaining frequencies ω_{n-1}, ω_{n-2}, ω_{n-3},,...,ω_1.

$$L'_{eq(\omega i)} = \frac{R_n^2 \cdot \left(L_{eq(\omega i)} - L_n\right)}{\left(R_n - R_{eq(\omega 1)}\right)^2 + \omega_i^2 \cdot \left(L_{eq(\omega i)} - L_n\right)^2} \qquad (8)$$

$$R'_{eq(\omega i)} = \frac{R_n \cdot \left[R_{eq(\omega i)} \cdot R_n - R_{eq(\omega i)}^2 - \omega_i^2 \cdot \left(L_{eq(\omega i)} - L_n\right)^2\right]}{\left(R_n - R_{eq(\omega 1)}\right)^2 + \omega_i^2 \cdot \left(L_{eq(\omega i)} - L_n\right)^2} \qquad (9)$$

Where:

$R_{eq(\omega i)}$, $L_{eq(\omega i)}$: resistance/inductance pair related to the frequency ω_i as given by the supplied data.

This step can be referred to as a "sequential circuit order reduction".

Step 3: As already done in step "1", the inductance/resistance pair corresponding to the "n-1" branch can now be calculated from the four latest variables found ($L'_{eq(\omega_{n-1})}$, $R'_{eq(\omega_{n-1})}$ and $L'_{eq(\omega_{n-2})}$, $R'_{eq(\omega_{n-2})}$), and by using equations similar to (1) and (2), replacing ω_1 by ω_{n-2} and ω_2 by ω_{n-1}.

The step 2 and 3 should be repeated to all remaining frequencies ω_{n-2}, ω_{n-3},,...,ω_1 (that is, until the last branch -R_1 and L_1 - is reached). This final branch must be evaluated through (3) and (4).

III. COMPUTATIONAL RESULTS

With the above methodology, a computer program was elaborated to evaluate the equivalent circuit parameters for a general "n" branches circuit. In order to verify the approach performance, a specific cable was used. This is a 85mm^2 cable for which resistance and inductance measurements for different frequencies were carried out and the data supplied by the manufacturer. The results are given in Table 1.

Table 1 - Frequency dependent parameters for a typical 85mm^2 cable - Data supplied by the manufacturer.

Frequency[Hz]	Resistance[Ω/m]	Inductance[H/m]
60	0.0013170	0.0000044
180	0.0027195	0.0000024
300	0.0033938	0.0000017
420	0.0038317	0.0000015
540	0.0041841	0.0000013
660	0.0044985	0.0000012
900	0.0050793	0.0000011
1260	0.0058977	0.0000010
1980	0.0074289	0.0000009
2580	0.0085915	0.0000008
3300	0.0098469	0.0000007
6300	0.0139485	0.0000006

The computational studies here described were carried out under two different number of branch conditions. The first is related to a three branches equivalent circuit and the second, to a greater number of branches (12 branches). The results are fully described bellow.

Three branches equivalent circuit

In this case, a three branches equivalent circuit, derived from Fig. 1 will be determined. The three frequencies considered for calculations are:

$\omega_1 = 60$ Hz; $\omega_2 = 1980$ Hz and $\omega_3 = 6300$ Hz.

The higher order branch (n = 3) is firstly determined. This is done by feeding the corresponding R and L data associated to ω_2 and ω_3 (in Table 1) into (1) and (2). The following results will be achieved.

$$L_3 = 0.4668\ \mu H/m \qquad R_3 = 16.80\ m\Omega/m$$

Next, a circuit order reduction must be carried out by using (8) and (9). It must be emphasized that, by solving two sets of data, one associated to 1980 Hz and the other to 60 Hz, then two pairs of equivalent resistance/inductance are obtained. The results are:

$$L'_{eq1980} = 1.044\ \mu H/m \qquad R'_{eq1980} = 5.85\ m\Omega/m$$

$$L'_{eq60} = 4.598\ \mu H/m \qquad R'_{eq60} = 1.26\ m\Omega/m$$

Finally, the parameters of the 2nd branch can be found through (1) e (2) and the parameters of the 1st branch through (3) and (4). This yield to:

$$L_2 = 1{,}003\ \mu H/m \qquad R_2 = 5{,}90\ m\Omega/m$$

$$L_1 = 5{,}347\ \mu H/m \qquad R_1 = 1{,}02\ m\Omega/m$$

By calculating the three branches equivalent circuit resistances and inductances, and by comparing the results to the measured values, it is possible to evaluate the percentual errors as given in Fig. 4.

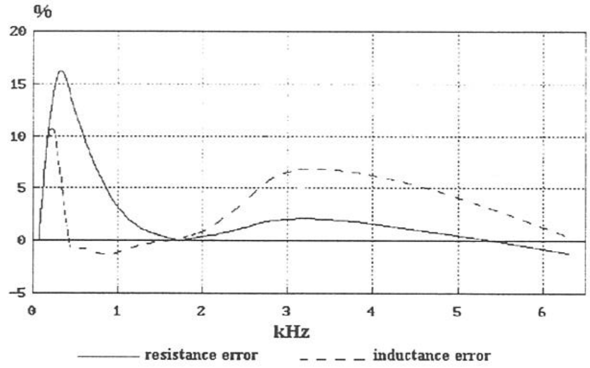

Fig. 4 - Percentual error as a function of the frequency for the three branches representation.

According to Fig. 4, the errors are rather small for the three frequencies adopted to evaluate the model parameters (60, 1980 e 6300 Hz). Actually, there are no errors for the two first frequencies because the final model was better adjusted to these two frequencies. Therefore, if there are some frequencies in what no error in modeling is desirable, then the data associated to three frequencies should be chosen to evaluate the model parameters.

Twelve branches equivalent circuit

The errors shown in Fig. 4 are due to the small amount of branches chosen for modeling the cable. The higher the number of branches the lower will be the discrepancies. For instance, Table 2 shows the results of the equivalent circuit parameters, related to the measurements show in Table 1, when 12 branches are used. The parameters were calculated in accordance to the steps already established.

Table 2 - 12 branches equivalent circuit parameters, for the 85mm^2 cable.

Branch	Resistance (Ω/m)	Inductance (H/m)
1	0.0003375952	0.0000002096
2	-0.0005483031	0.0000124569
3	0.0093395592	0.0000002672
4	-0.0102932403	0.0000189145
5	-0.0046000079	0.0000003093
6	0.0049181707	0.0000108380
7	0.0222065003	0.0000028476
8	-0.0253968107	0.0000031153
9	-0.0000791173	0.0000000001
10	0.0000782952	0.0000008106
11	0.0247657697	0.0000002199
12	0.0215682855	0.0000004142

The proposed method calculates the resistance and inductance of the branches in such a way that the final equivalent impedance approaches the data values. Negative values for these parameters in some branches may result in the calculations, but they have no physical meaning, being only part of the mathematical representation. It must be stressed however, that the final equivalent impedance always results in positive values for resistance and inductance, which approach the measured data.

Fig. 5 presents a comparison between the twelve branches equivalent circuit resistance behavior with the frequency and the measured values. The corresponding result for the equivalent circuit inductance is given in Fig. 6. These figures also illustrate the results found with the first order approximation, when the frequencies of 60 Hz and 6300 Hz are chosen as references. The original data of Table 1 is also presented in these two figures. It can be seen that the results provided by the approach proposed in this paper are in close agreement with those of Table 2.

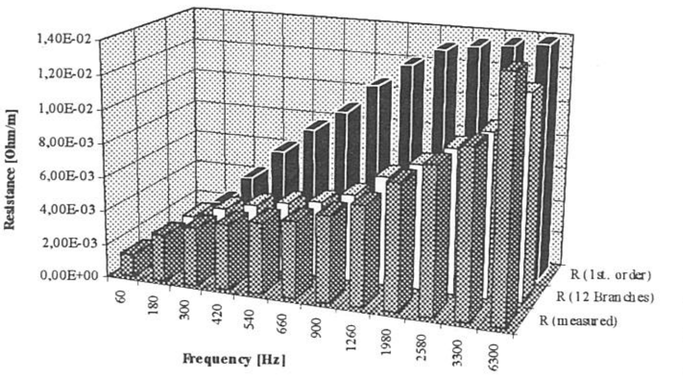

Fig. 5 - Resistance as function of frequency

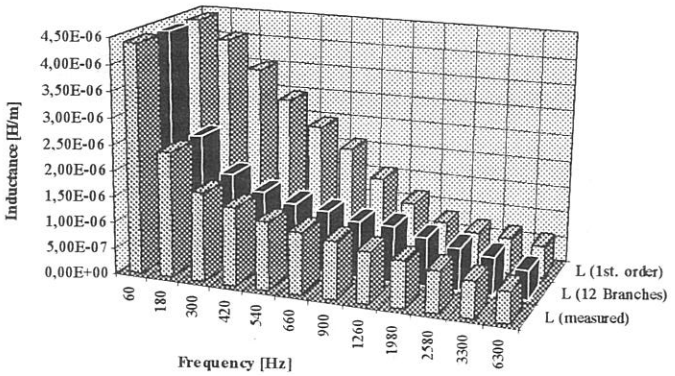

Fig. 6 - Inductance as a function of frequency

The inductance and resistance percentual deviations between Table 2 measurement data and the 12 branches equivalent circuit calculated are shown in Fig. 7. It can be noted that there are no diversions for the 60 Hz and 180 Hz frequencies. The most significant errors refer to the highest frequency considered (6300 Hz).

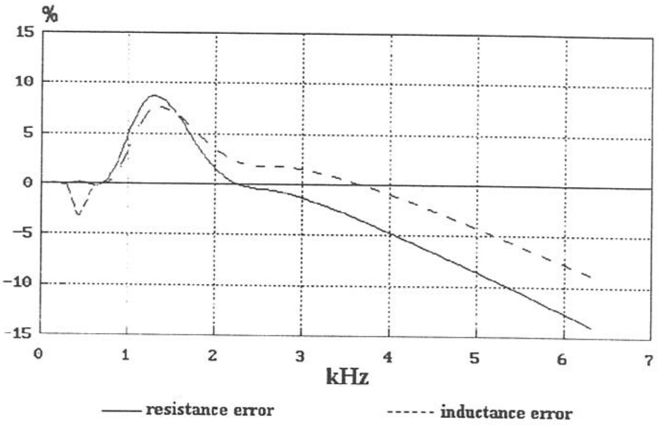

Fig. 7 - Percentual error as a function of the frequency for a 12 branches model.

IV. CONCLUSIONS

Time domain digital programs such as the ATP, usually requires the inclusion of the frequency dependency of both underground and overhead cable resistances and inductances. So far, this has been done mostly by using the Bessel functions. The approach proposed in this paper provides an improvement to the first order approximation method and allows for the determination of a "n" branches equivalent electrical circuit to represent the frequency dependency for both the resistive and the inductive cable parameters with the frequency. This model results in lower deviation over a wide range of the desirable frequency spectrum and offers a flexibility to choose the number of branches with respect to the amount of error allowed.

VI- REFERENCES

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[2] Henriksen, T.: "First order approximation to the frequency dependency of parameters in EMTP". LEC. 21st EMTP User Group Meeting - (1993) Portugal.