The Analysis of transient phenomena using the Wavelet theory

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Abstract — **The use of the Wavelet theory as a decomposition and composition tool in signal processing applications has proved to be efficient. However, the decomposition of a transient signal in Power Systems will only be useful if, due this decomposition, a safe decision can be made. The objective of this work is to evaluate the performance of the Wavelet theory in the detection and localization of transient phenomena.**

Keywords : Wavelets, Transients, Power Quality, Multiresolution Wavelet Method.

I. INTRODUCTION

In an ideal Power System the voltage and current signals must present purely sinusoidal wave forms. However, modern electrical systems are characterized by presenting distorted wave forms of the voltage and/or current. Traditionally, these distortions are studied by Fourier theory. Fourier theory is a powerful mathematical tool but, can only be used with success if the distortions are stationary [1]. Most physical phenomena nevertheless, create non-stationary distortions, where periodicity does not exist.

Recently, a new technique was suggested to deal with non-stationary distortions in Power Systems: Wavelet theory [1]. In this work, the author says that this new theory can in the near future become an efficient and powerful tool for identification and analysis of several types of non-stationary distortions, such as atmospheric surges, capacitor bank switching, etc.. Currently, the literature indicates that applications of Wavelet theory to Power Systems are still in an investigative phase. This work evaluates the performance of Wavelet theory, particularly the method proposed by [2], for the detection and localization of transients.

II. WAVELET THEORY

The fundamental idea in Wavelet theory is the scaling operation. Scaling renders possible the compression and dilation of a function called the mother wavelet. Informally, a mother wavelet $\psi(t)$ is a function that oscillates, has finite energy and zero mean value. The scaling mother wavelet, when translated in the time, creates daughter wavelets.

Mathematically, Wavelet theory results in an operation that decomposes a function $x(t)$ into a wavelet set. The Continuous Wavelet Transform (CWT) is then:

$$W_{\psi}(a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{+\infty} x(t)\, \psi^{*}\left(\frac{t-b}{a}\right) dt \qquad (1)$$

where, a (scale) and b (translation) belong to the Real set; $a \neq 0$ and "*" denotes the complex conjugate. The scale a corresponds to the inverse of the frequency.

The Continuous Wavelet Transform has great theoretical interest, especially for the development and comprehension of its mathematical properties. But its discretization is necessary for practical applications.

In the Discrete Wavelet Transform, only the parameters of the transform are discretizated. One typical discretization is: $a = a_o^m$ and $b = n a_o^m b_o$, with m and n belonging to the Integer set, $a_o > 1$ and $b_o \neq 0$ [3].

The discretization process leads to the Discrete Wavelet Transform (DWT):

$$W_{\psi}(m,n) = \frac{1}{\sqrt{a_o^m}} \int_{-\infty}^{+\infty} x(t)\, \psi\left(\frac{t - n a_o^m b_o}{a_o^m}\right) dt \qquad (2).$$

For Time Discrete Systems, it has the Time Discrete Wavelet Series (TDWS) given by:

$$W_{\psi}(m,n) = \frac{1}{\sqrt{a_o^m}} \sum_{k=-\infty}^{\infty} x(k)\, \psi\left(\frac{k - n b_o a_o^m}{a_o^m}\right) \qquad (3).$$

If the purpose of the discretization process is to eliminate the redundancy of the continuous form and to ensure inversion, then the choice of a_o and b_o must be made so that the daughter wavelets form an orthonormal basis [4]. This condition is satisfied, for example if $a_o = 2$ and $b_o = 2$ [3].

III. MULTIRESOLUTION ANALYSIS

The Wavelet method decomposes a signal at different scales or resolutions. This multiresolution decomposition is the essence of Wavelet theory.

In Multiresolution Analysis [5], the mother wavelet function $\psi(t)$ is defined via a function $\phi(t)$, called the scaling function. The daughter wavelets form an orthonormal basis only if an appropriate choice of the function $\phi(t)$ is made. According to [6], this choice is made so that there exists a squared summable sequence $\{c_n\}$ such that:

$$\phi(t) = 2 \sum_{n=-\infty}^{+\infty} c_n \phi(2t - n) \quad (4).$$

Equation (4) is the fundamental equation of the Multiresolution Analysis. Using this equation, the mother wavelet function is defined as follows:

$$\psi(t) = 2 \sum_{n=-\infty}^{\infty} d_n \phi(2t - n) \quad (5)$$

where the sequence $\{d_n\}$ is squared summable.

The structure of Multiresolution Analysis with the application of a recurrent algorithm was utilized by [7] to construct orthonormal wavelet sets. This work uses the DAUB4 wavelet, which is the simplest and most localized member of these sets. This gives the desired time localization.

IV. THE MULTIRESOLUTION WAVELET METHOD

Using the structure of Multiresolution Analysis, the Time Discrete Wavelet Series can be calculated. In this case, it was decided to call it the Multiresolution Wavelet Method (MWM) [8].

Schematically, it has the follow structure (Fig. 1 and 2):

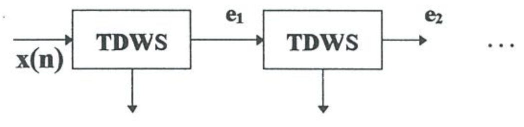


Fig. 1. MWM structure

Due to the dizimation process of the method (Fig. 2), if the signal $x(n)$ has N samples, the signals e_1 and w_1 will have only $N/2$ samples each in the same interval of time, and similarly for other stages.

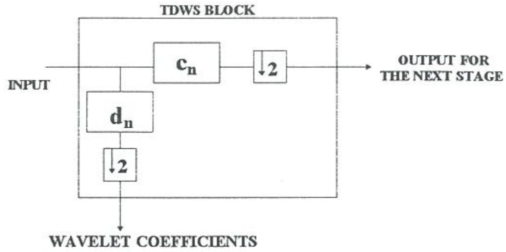


Fig. 2. One stage of the MWM

In the context of Signal Processing, the sequences $\{c_n\}$ and $\{d_n\}$ are considered as being the two conjugate quadrature filter channels (CQF). The filter c_n is a low-pass filter and the filter d_n is a high-pass filter.

The resulting signal from c_n (e_n) is a smoothed version of the original signal, and the resulting signal from d_n (w_n) is a detailed version of the original signal, which corresponds to the wavelet coefficients.

V. COMPUTATIONAL IMPLEMENTATION OF MULTIRESOLUTION WAVELET METHOD

For computer implementation of MWM, the wavelet subroutines WT1 and DAUB4 were used [9]. These subroutines, when implemented, provide the wavelet coefficients, or the inverse operation, of the original signal. The DAUB4 subroutine executes the DAUB4 wavelet, which according to [2] is the most adequate for the detection and localization of disturbances. The scaling used was the dilation of the mother wavelet.

VI. VALUATION OF THE PERFORMANCE OF THE MULTIRESOLUTON WAVELET METHOD

To evaluate the performance of the MWM, vectors were used to represent the electrical signals, which had size N=1024. It was adopted: θ, initial phase of the signal and δ, relative position of the disturbance in the signal.

A. Performance of the MWM in the detection and localization of disturbances

In order to perform this evaluation, the signals showed in the Figs. 3a and 4a were simulated. The MWM decomposes these signals into their detailed and smooth representation. The detailed representation (output of the filter d_n) contains the higher frequency components of the signal because d_n is a high-pass filter. So the disturbances present in the signals are detected in this representation.

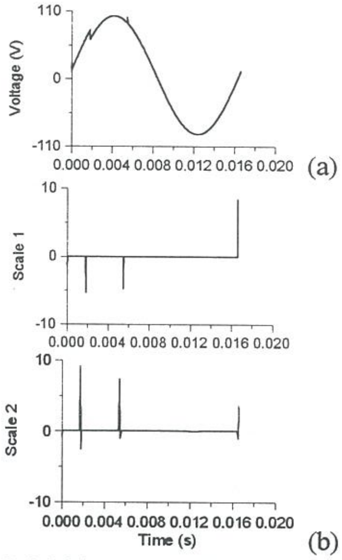


Fig. 3. Multiresolution Wavelet Method.
(a) voltage signal (b) scales 1 and 2

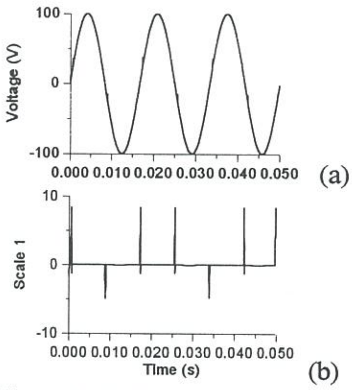


Fig. 4. Multiresolution Wavelet Method.
(a) voltage signal (b) scale 1

Each spike in the detailed representation (wavelet coefficients) corresponds to the disturbances present in the signal. The spike at the end of the signal is "seen" by the MWM as a transient [10].

Each stage of the MWM corresponds to a scale. Fig. 3b shows scales 1 and 2, and Fig. 4b shows scale 1. The fast and short transient disturbances are detected at scale 1, because the DAUB4 wavelet is most localized in time. The slow and long transient disturbances will be detected at higher scales, because the DAUB4 wavelet becomes less localized in time.

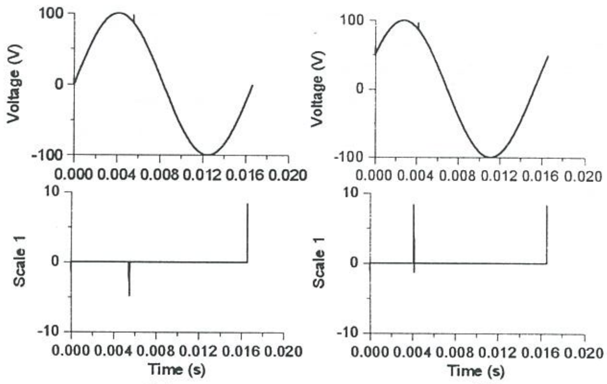
With regard to Fig. 3b, it can be observed that the slowest disturbance has the greatest value at second scale. However, it is not possible to say "absolutely" what the type of disturbance is.

So, the Figs. 3b and 4b show the excellent performance of the MWM in the detection and location of fast disturbances present in the signal. According to [8], the MWM is a powerful tool for the detection and location of fast and short transient disturbances.

B. Performance of the MWM with the variation of θ

To evaluate the performance of the MWM with the variation of θ, a base signal (sinusoidal of 60 Hz and 110V of amplitude) was used, to which was applied a spike of 10V of amplitude (Figs. 5a and 5b). In these figures, only the first scale is shown.

These figures show clearly the sensitivity of the MWM to the "windowing" applied to the signal: the MWM presents wavelet coefficients with different values to the same disturbance. This fact is a problem of the step of the method ([11] and [12]). This phenomenon of the method is named sensitivity to translations, that is, the wavelet coefficients of two signals may be quite different if two signals just differ by a time shift. This is one drawback of the method, but it can be by-passed.



(a) (b)

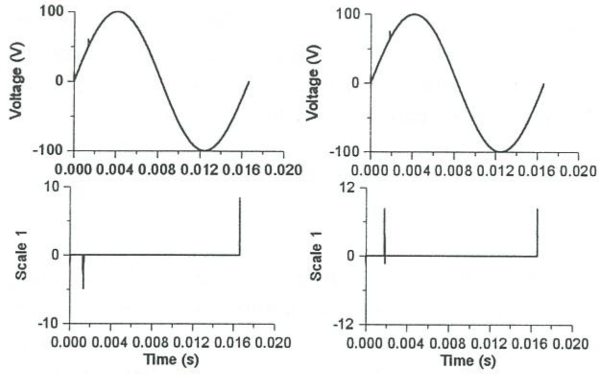
Fig. 5. Multiresolution Wavelet Method.
(a) $\theta=0^\circ$, $\delta=120^\circ$ (b) $\theta=30^\circ$, $\delta=120^\circ$

C. Performance of the MWM with the variation of δ

Here, the same signals as item B were used, but now the spike was translated in the signal (Figs. 6a and 6b).

As it was shown in the previous case, the MWM is sensitive to the location of the disturbance in the signal: the MWM presents wavelet coefficients with different values to the same disturbance.

Therefore, except for this inconvenience, Figs. 6a and 6b show the detection and localization in the time of the disturbances present in the signal.



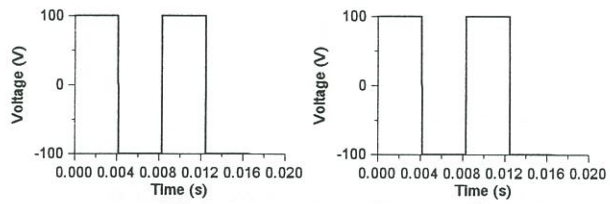
(a) (b)

Fig. 6. Multiresolution Wavelet Method.
(a) θ=0°, δ=30° (b) θ=0°, δ=40°

D. Performance of the MWM in the reconstruction of signals

To evaluate the performance of the MWM in the reconstruction of signals, the squared wave function was used. The Fig. 7a illustrates the original function and the Fig. 7b illustrates the reconstructed function by MWM. The mean-squared error between the original function and the reconstructed one by MWM was calculated. The value obtained was equal to $7.8\text{x}10^{-5}$.

In this way, the efficiency of the MWM in the reconstruction of signals was proved. The MWM can thus be used in the reconstruction of stationary power system disturbances too.



(a) (b)

Fig. 7. (a) Squared Wave Function.
(b) Reconstructed function by MWM.

VII. APPLICATION OF THE MULTIRESOLUTION WAVELET METHOD TO REAL MEASUREMENTS

The measurements utilized in this section were supplied by Companhia Energética de São Paulo (CESP), one of the electrical supply companies of Sao Paulo State. To apply the MWM to the measurements, vectors with 1024 samples were used. According to [2], the squared wavelet coefficients were used, in order to reduce the effect of the present noise in the real electrical signals.

A . Fusion time of the fuses (Fig. 8)

This test determines the fusion time of the fuses of a monopolar automatic switch. The Fig. 9a represents the voltage in the switch during the test and the Fig. 9b, illustrates its wavelet coefficients up tol fourth scale.

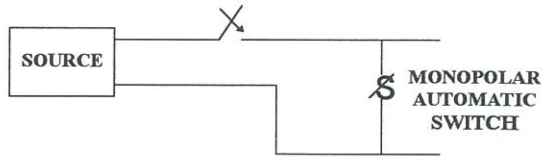
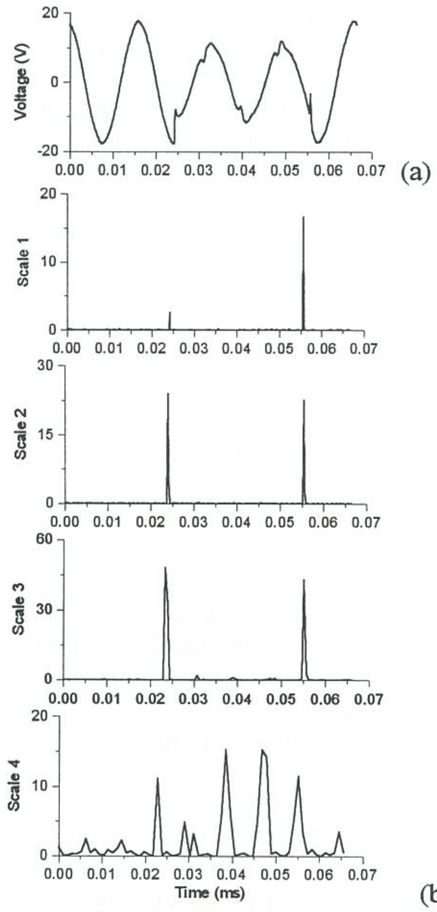


Fig. 8. Fusion time of the fusibles



(b)

Fig. 9. (a) Voltage in the switch of the Fig. 8
(b) Multiresolution Wavelet Method

The efficiency of the MWM is observed in the detection of the beginning (around 0.025 ms) and end (around 0.055 ms) of the fusion (Fig. 9b). The fast variations (beginning and end of the fusion process) are detected in the first and second scales and the slow variations, in the third and forth scales.

Note that the wavelet coefficients relating to the beginning and end of the fusion (fast transient disturbance) persist at the same temporal location over scales 1, 2, and 3, according to [2].

B. Transient in a voltage signal (Fig. 10a)

The Fig. 10a shows a voltage signal with a fast transient around 0.027 ms, which can be easily seen in the first scale and its propagation until the third scale. Another transient around 0.03 ms can be seen in first scale but, with smaller intensity. The slowest variations are only detected after the third scale.

Note that the MWM detects the fast disturbances at scales 1 and 2 and the slow disturbances, at scales 3 and 4.

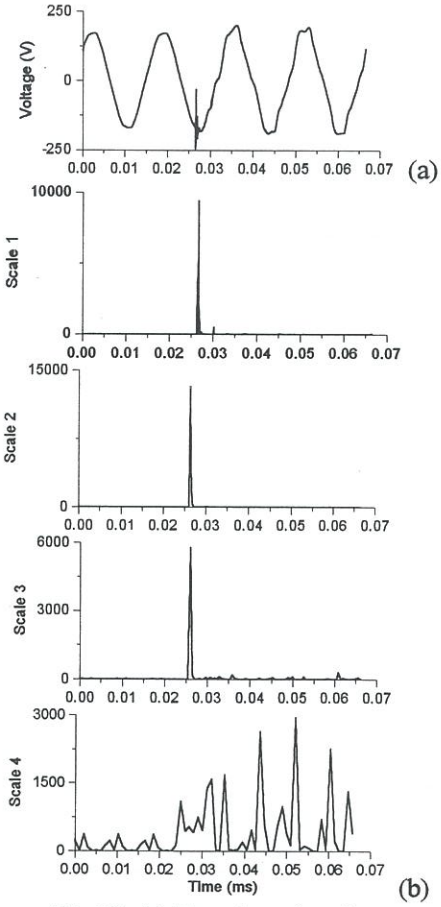


Fig.10. (a) Transient signal
(b) Multiresolution Wavelet Method

C. Reenergization of one 13,8 kV circuit to supply ground return monophasic loads (Fig. 11)

The reenergization voltage with its wavelet coefficients are shown in the Figs. 12a and 12b.

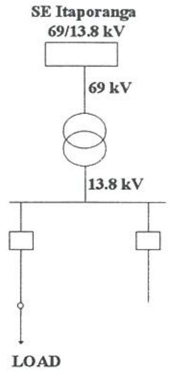


Fig. 11. Reenergization of circuit

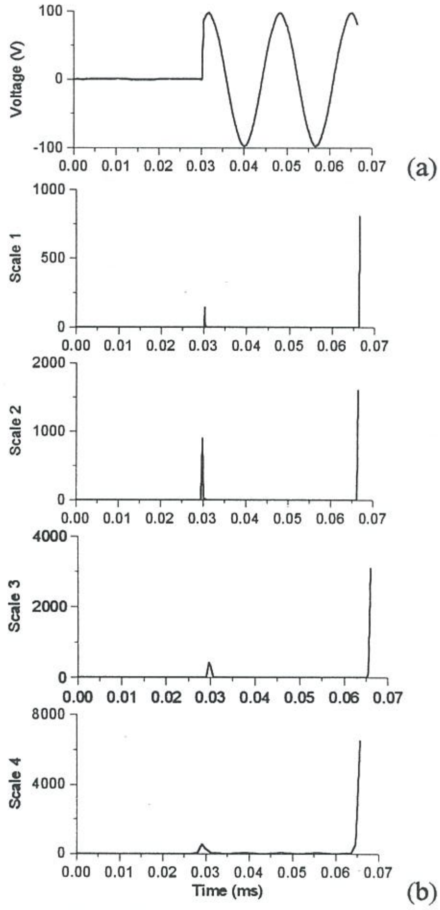


Fig. 12. (a) Reenergization voltage
(b) Multiresolution Wavelet Method

The reenergization time can be seen in the first scale, around 0.03ms. As the signal doesn't show any other disturbances, only the coefficients relating to the reenergization time can be seen in the figures previously cited.

Note that the wavelet coefficients of the reenergization instant persist at the same temporal location over all scales. Observe that the end of the signal is seen as transient.

VIII. CONCLUSIONS

Since the proposal of utilization of the Wavelet Theory in Power Systems is still recent, we tried to initiate the exploration of its potential in this work. In this way, based on the results shown in this work, we can conclude:

1. The MWM is a useful tool to detect the occurrence of abrupt variations in wave forms in the time domain.

2. The efficiency of the MWM in the analysis and synthesis of signals was confirmed.

3. The MWM is sensitive to "windowing" made in the signal and sensitive to location of the disturbance in the signal. Therefore, in order to use the MWM to disturbances classification, modifications in this method should be made.

4. The meaning of the magnitude of the scales is still not clear. The B and C cases of item VI, show the variation of the values of the wavelet coefficients for the same signal with the same disturbance.

In Power Systems, some disturbances and/or transients must be detected through signal decomposition. Disturbance detection is important for edequate the system operation. For example, of protection, active filters, etc.. So, decomposition cannot be allowed to lead to false diagnoses. If this happens, serious damage can occur to the system. So a clear conception of the scale magnitudes is needed.

IX. ACKNOWLEDGEMENTS

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X. REFERENCES

[1] P.F. Ribeiro, "Wavelet Transform: an advanced tool for analyzing non-stationary harmonic distortions in Power Systems", *Proceeding of IEEE International Conference on Harmonics in Power Systems*, pp. 365-369, September 1994, Bologna.

[2] S. Santoso et al., "Power Quality Assessment Via Wavelet Transform Analysis", *IEEE PES Summer Meeting*, July 1995, Portland.

[3] I. Daubechies, *Ten Lectures on Wavelets*, SIAM, 1992, Philadelphia.

[4] M. Vetterli and C. Herley, "Wavelets and Filter Banks: Theory and Design", *IEEE Trans. on Signal Processing*, vol. 40, September 1992.

[5] S. Mallat, "A theory for Multiresolution Signal Decomposition: The Wavelet Representation", *IEEE Trans. on Pattern Anal. and Mach. Intell.*, vol. 11, pp. 674-693, July 1989.

[6] B. Porat, *Digital Processing of Random Signals - Theory and Methods*, Prentice-Hall,1994.

[7] I. Daubechies, "Orthonormal Bases of Compactly Supported Wavelets", *Communications in Pure Applied Math*, pp. 909-996, 1988.

[8] N.S.D.Brito, "Teoria Wavelet e sua aplicação em Sistemas de Energia Elétrica", M. Sc. Thesis, 1996, UNICAMP, Campinas.

[9] W.H. Press et al., *Numerical Recipes in FORTRAN*, Cambridge University Press, 1992.

[10] C.N. Rasmussen, "Implementing the Wavelet Transform in MATLAB", Internal Report, 1996, UNICAMP, Campinas.

[11] S. Mallat, "Zero-Crossings of a Wavelet Transform", *IEEE Trans. on Information Theory*, vol. 37, July 1991.

[12] J. Liang and T.W. Parks, "A Translation-Invariant Wavelet Representation Algorithm with Applications", *IEEE Trans. on Signal Processing*, vol. 44, February 1996.

XI. BIOGRAPHIES

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