Computation of Switching Transients using Low-Order, Multi-Port Network Equivalents

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Abstract — **Network equivalents can be used to represent parts of a power system to reduce the complexity and the computation time during the simulation of electromagnetic transients. It is computationally efficient to represent in detail only those components, which are of primary interest for the transient phenomena investigated. The remainder of the system can be modeled using low-order network equivalents without significant loss of accuracy. Using frequency domain approach, the network equivalent is modelled by developed rational admittance branches in a well-known transients program.**

Keywords : Network equivalents, Electromagnetic Transients, System Identification, Modelling, EMTP.

I. INTRODUCTION

Power system electromagnetic transients are simulated most efficiently using time domain methods implemented in digital computer programs [1,2]. Detailed modelling of system components, modelling the entire system of a large-scale power network may be computationally demanding, especially for parametric or statistical studies, AC/DC system's transients analysis, etc. Additionally, the management and validation of increased system data become difficult for the system engineer. It is therefore a common practice to represent in detail only those components that are of primary interest. Without significant loss of accuracy, the rest of the system can be modelled using low-order equivalents.

Various researches have recognized the need for accurate network equivalents valid over a wide frequency range for many years [3-6]. Based mainly on the frequency domain approach, the objective was to produce equivalents using techniques of network synthesis like ladder R,L,C networks of particular structure, that might introduce an extra error due to their mismatching frequency curves in certain frequency intervals. Recently, a hybrid EMTP has been created which preserves the advantages of the standard EMTP, while adding the direct frequency domain modelling of some peripheral components [7]. Abur and Singh [8] proposed a time domain model to represent multi-port external systems in form of a discrete-time Norton equivalent. This method requires the complete representation of the system in the EMTP to obtain the equivalent, which is generally of high order.

This paper deals primarily with the aspects and strategies to create low-order multi-port network equivalents of passive three-phase balanced power systems based on the core idea and techniques developed in [9]. The frequency response of port admittances of a network are approximated by reduced-order rational functions using a nonconstraint system identification technique with freely selectable error criteria [10,11]. A developed electrical branch of rational admittance type enables direct representation of the network equivalent in the ATP-EMTP [2].

II. PROPERTIES OF MULTI-PORT NETWORKS

Network equivalents obtained by the approximation of frequency and/or time domain responses of passive systems must satisfy the realizability conditions in the well-known field of network analysis and synthesis [12,13]. When the realizability conditions are not satisfied due to insufficient fitting of system parameters, the equivalent model obtained may act like a active circuit, hence it will be "unstable" in a wide sense.

A passive multi-port is represented by its reduced nodal admittance matrix [Y(jω)] seen from the terminals at each angular frequency:

$$[I(\omega_i)]=[Y(\omega_i)][U(\omega_i)] \quad for\ \omega_{min}\le\omega_i\le\omega_{max} \tag{1}$$

with

$$[Y(\omega_i)]=\begin{bmatrix} y_{11}(\omega_i) & y_{12}(\omega_i) & \cdots & y_{1N}(\omega_i) \\ y_{21}(\omega_i) & y_{22}(\omega_i) & \cdots & y_{2N}(\omega_i) \\ \vdots & & \ddots & \vdots \\ y_{N1}(\omega_i) & y_{N2}(\omega_i) & \cdots & y_{NN}(\omega_i) \end{bmatrix}$$

The internal busses are eliminated after the matrix [Y] has been assembled. The resultant matrix [Y(jω)] is symmetrical with N(N+1)/2 distinct elements. As described in [9,14] the frequency response of distinct elements of the matrix [Y(jω)] is computed over a given frequency range taking into account the frequency dependence of system components. Frequency dependent admittance submatrices of overhead lines and cables are calculated from the geometrical and electrical data using the supporting routines LINE CONSTANTS and CABLE CONSTANTS [2]. The three-phase transformers are represented by linear RLC circuits including equivalent terminal capacitances. The frequency dependence of short-circuit impedance is considered using empirical f

Each of the matrix elements $y_{jk}(s)$ in (1) can be expressed in form of a rational function, which should satisfy the following realizability conditions:

1. All the coefficients of the numerator and denominator polynomials of $y_{jk}(s)$ are real.
2. None of the elements of [Y(s)] has poles in the open right-hand plane (rhp) (Re s > 0).
3. All the poles on the imaginary axis (Re s = 0) are simple and the matrix of residues at those poles must be positive semidefinite.
4. The real part [G(jω)] of the matrix [Y(jω)] must be nonnegative for all ω; in other words, the matrix [G(jω)] must be positive semidefinite. The symmetrical real part matrix is positive semidefinite if and only if its eigenvalues are all positive at each frequency [15]. Pivoting is necessary, if the matrix [G(jω)] is of rank r (r < N) [15].

In general, the 3. condition does not apply to real power systems, because there always is some damping at each system resonance. Consequently, the system has no poles on the jω-axis. The real part condition 4 is difficult to satisfy during the approximation process for multi-port networks, because all the matrix elements of [Y] cannot be fitted simultaneously by applying the condition 4. The diagonal elements of the matrix [Y] are driving-point (dp) functions, conditions of which can be satisfied during approximation process. The off-diagonal elements are not driving-point functions indicating that they may possess zeros in the open rhp.

III. APPROXIMATION OF NETWORK ADMITTANCES

A π-type equivalent of the multi-port network is preferred for the low-order modeling in the transients program ATP-EMTP [2] as shown in Fig. 1.

The relation among π-circuit elements of Fig. 1 and elements of the [Y] matrix (small letters) is given as follows:

$$Y_{j0} = \sum_{k=1}^{N} y_{jk} \tag{2}$$

$$Y_{jk} = -y_{jk} \quad (j \neq k) \tag{3}$$

At first instance, it may seem reasonable to approximate the frequency response of all elements of [Y] by rational functions. The π-equivalent elements can be determined using (2) and (3). However, in electrical power systems it is generally valid at low frequencies (in the range of system frequency) that $Y_{j0} \ll Y_{jk}$. In this case the determination of Y_{j0} would be inaccurate at low frequencies, because Y_{j0} will be calculated by subtracting of almost two equal quantities; diagonal element y_{kk} and sum of off-diagonal elements y_{jk} ($j \neq k$) belonging to the same row of [Y]. The addition in (2) should be interpreted as subtraction at low frequencies because of the phase shift of approx. 180° between the diagonal element and off-diagonal elements belonging to the same row of the matrix [Y].

In order to avoid this drawback explained in the previous paragraph, the following strategy has been developed. In addition to the frequency response curves of all distinct elements of the [Y] matrix, the frequency response of shunt elements Y_{j0} (j=1,..,N) of the π-circuit is calculated over a desired frequency range using (2). Note that only the diagonal elements of [Y] matrix are positive real functions. Total N(N+1)/2 frequency response curves should be matched by rational functions utilizing common poles. Remaining one transfer admittance Y_{jk} ($j \neq k$) belonging to each row of [Y] matrix can be determined using (2).

The frequency response of admittances available at discrete frequencies in polar form, $Y(\omega_i) = |Y(\omega_i)| \cdot \exp(j\theta_i)$, are approximated by rational functions of the following general form using a nonconstraint optimization program, which allows to define various error criteria for both magnitude and phase in desired frequency intervals:

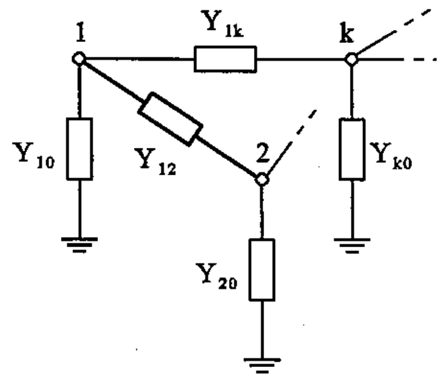

Fig. 1 Equivalent π-representation

$$Y_M(s) = Y'_M(s) \cdot Y''_M(s) \quad (4)$$

with

$$Y''_M(s) = K'' \frac{\prod_{i=1}^{J''} (s-\sigma_{n_i}'') \prod_{i=1}^{L''} (s-\delta_{n_i}'' \pm j\omega_{n_i}'')}{s^m \prod_{i=1}^{M''} (s-\sigma_{p_i}'') \prod_{i=1}^{N''} (s-\delta_{p_i}'' \pm j\omega_{p_i}'')} e^{-sT''} \quad (5)$$

The rational function $Y_M(s)$ of the model system consists of two identical parts; $Y'_M(s)$ is the "fixed" part containing all known parameters (known poles, zeros, gain and dead time) and the "free" part $Y''_M(s)$ includes unknown parameters to be determined by the optimization. During the approximation procedure, parameters can be exchanged between two parts. The unknown "free" parameters of (5) are collected to form a parameter vector [r]:

$$[r] = [K'' \; T'' \ldots \ldots \sigma_{n_i}'' \ldots \delta_{n_i}'' \ldots \omega_{n_i}'' \ldots \sigma_{p_i}'' \ldots \delta_{p_i}'' \ldots \omega_{p_i}'' \ldots]^t \quad (6)$$

The number of free parameters depends on the order of the rational function selected by the user.

The continuous performance function

$$a^*([r]) = a([r]) + \frac{1}{\rho} \ln \left\{ \sum_{i=1}^{N_G} \exp \left[\rho \left(\frac{G_i([r])}{C_i} - a([r]) \right) \right] \right\} \quad (7)$$

with

$$a([r]) = \max_{1 \le i \le N_G} \{G_i([r])/C_i\}$$

G_i: Error (quality) criterion (see point 3 below)
C_i: Upper limit for each error criterion
N_G: Number of error criteria applied
ρ: sufficiently large real number

is minimized using one of the available various direct search algorithms. Note that the optimization technique used is of unconstrained type, since the error criteria are incorporated into the performance function, that will be minimized without any external constraints.

The advantages of the used approximation technique [10,11] can be summarized as follows:

1. Common poles of network admittances are determined only once and can be passed to the remaining admittances, which will be fitted next. Hence the rational functions created are of low order.

2. As given in (5) the poles and zeros of the rational functions are unknown parameters, which can be of real and/or of complex conjugate type. The stability of the model can be tested (realizability condition 3) easily by checking directly pole and zero locations.

3. Maximum and average deviation from the original frequency curve can applied to magnitude and phase separately in desired frequency intervals as error criterion G. This feature enables to produce a model system, which is accurately approximated at steady-state frequency as well as over a desired high frequency range.

4. The rational functions obtained by the approximation program can be represented in EMTP directly by the developed electrical branch "KIZILCAY F-DEPENDENT" in s- or z-domain [2]:

$$Y(s) = K \frac{a_0 + a_1 s + \cdots + a_m s^m}{b_0 + b_1 s + \cdots + b_n s^n} \quad (8)$$

$$Y(z) = K \frac{A_0 + A_1 z^{-1} + \cdots + A_m z^{-m}}{B_0 + B_1 z^{-1} + \cdots + B_n z^{-n}} \quad (9)$$

The time delay function e^{-sT} in (5) might be incorporated into the existing rational admittance branch of EMTP [2] to take the travel time on lines between terminals into account.

User interaction during fitting procedure allows to control the quality of fitting, the order of the model system and selection of the dominant system resonances to produce physically meaningful low-order models.

The initial estimate of the frequency response, i.e. approximate corner frequencies of system resonances should be provided by the user. The fitting method is sensitive to initial estimates, specially if system resonances are very close to each other. Usually successive approximations are necessary to obtain the model system.

IV. PRACTICAL CONSIDERATIONS

From the practical point of view it is appropriate to create multi-port network equivalents with high number of internal busses and low number of external ports in order to keep the preparatory work to create network equivalents as low as possible. Presently available fitting methods are of SISO (single input single output) type and allow only to approximate one admittance function at a time. Therefore it is impossible to incorporate the 4th condition given in section II into the approximation routine. This important condition can be checked first after all admittances have been matched by

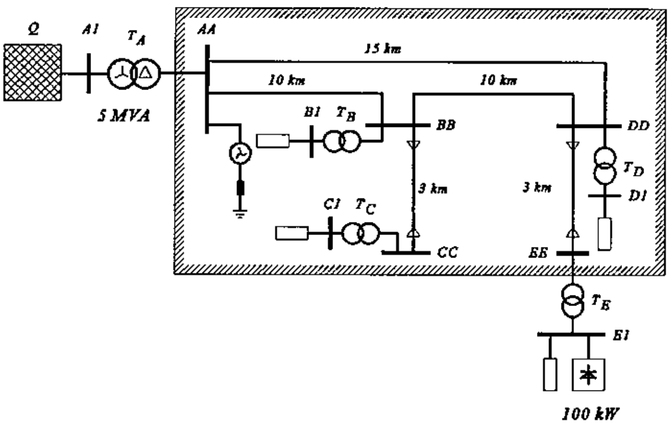

Fig. 2 20-kV distribution network

approximating rational functions. If it is not satisfied, it becomes a trial and error procedure to improve the performance of created network equivalent.

One of the significant simplifications is to assume a balanced power system, since the components of interest in a transients simulation will be normally modelled in detail. Additionally it can be justified that a large network modelled by a network equivalent behaves most likely almost symmetric among the phases. This assumption leads to two identical uncoupled networks (positive and negative sequence) and an independent third network (zero sequence), which would be coupled in the case of multi-circuit lines and uncoupled in the case of single-circuit lines. The nodal admittance matrix of a balanced network has to be calculated over a frequency range, and decomposed into positive and zero sequence networks using well-known transformation matrices of symmetrical components or Karrenbauer transformation. The use of sequence admittances reduces the number of approximations significantly and hence the preparatory work to create the network equivalent will become acceptable. The frequency range selected for the approximation determines the validity of the model for subsequent transient calculations. Taking into account the increasing order of the network equivalent over a wider frequency range, it is most suitable to simulate middle-frequency switching transients using network equivalents.

V. EXAMPLE

The balanced 20-kV distribution system shown in Fig. 2 is used to illustrate the validity of the network equivalent proposed in this paper. The shaded part of the network between three-phase busbars AA and EE is represented by a low-order, two-port, three-phase equivalent.

Since no zero sequence voltages and currents can be transferred practically from the LV to HV side of the transformer T_E with vector group Dyn5, it is sufficient to model only the positive sequence network of the shaded part. A 6-pulse bridge rectifier with a load of 100 kW is connected to the LV side of the transformer T_E. The harmonic propagation along this 20-kV network has been simulated using the low-order network equivalent and for comparison the same case was analyzed using full network representation, in which overhead lines were modelled using frequency dependent line model JMARTI SETUP [2]. Due to numerical

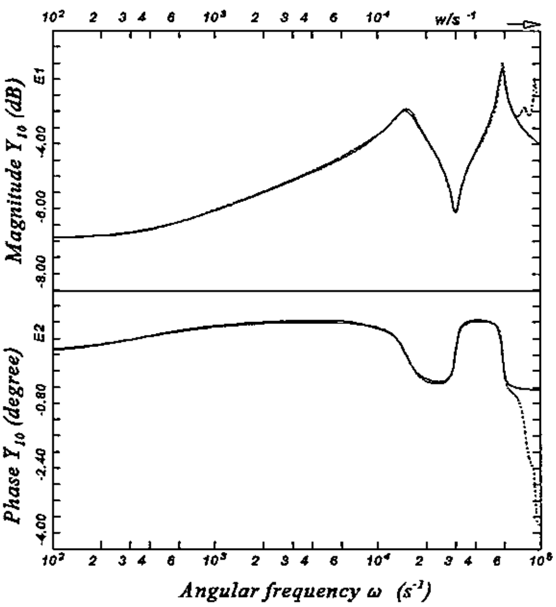

Fig. 3 Frequency response of the positive-sequence shunt admittance Y_{10}

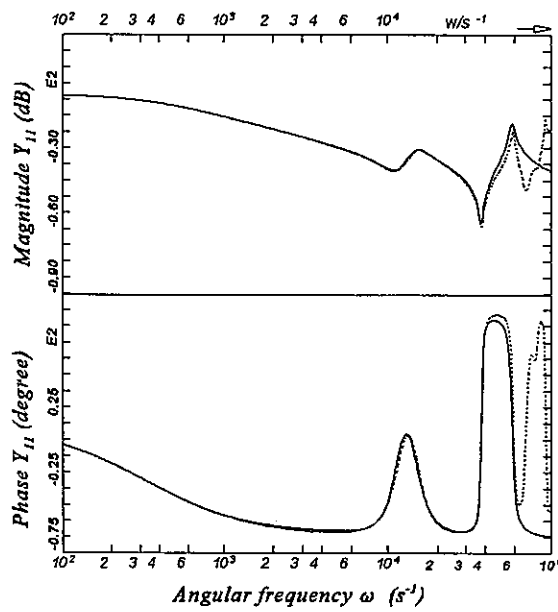

Fig. 4 Frequency response of element y_{11} of the positive-sequence nodal admittance matrix

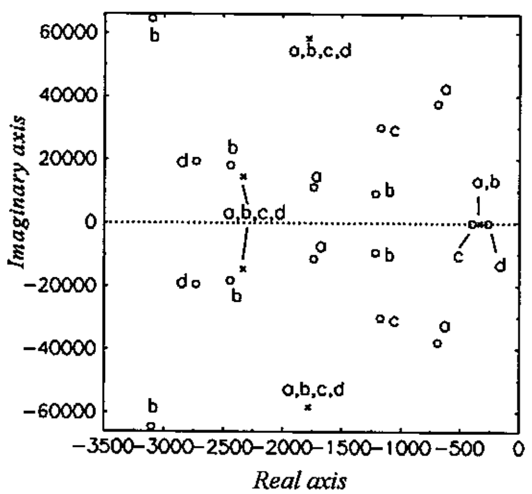

Fig. 5 Pole (x) and zero (o) locations of approximated admittances
a - y_{11} b - y_{22} c - Y_{10} d - Y_{20}

instability, cables were modelled using constant parameter distributed line model.

The diagonal elements $y_{11}(j\omega)$, $y_{22}(j\omega)$ of the [Y] matrix as well as the shunt admittances $Y_{10}(j\omega)$ and $Y_{20}(j\omega)$ of π-equivalent of the positive sequence system have been approximated by rational functions of the form (5) over the frequency range from 10 Hz up to 10 kHz. Node numbers 1 and 2 correspond to busbars AA and EE in Fig. 2, respectively. As an example, the original frequency responses and their approximations are shown for the shunt admittance $Y_{10}(j\omega)$ and corresponding diagonal element $y_{11}(j\omega)$ in Fig. 3 und Fig. 4, respectively. The pole and zero locations of the fitted four admittances are given in Fig. 5.

According to Fig. 5 the approximated rational admittance functions are identified by real and complex conjugate poles and zeros. The complex conjugate poles are common to all admittances leading to a network equivalent of lowest order. The maximum order of 6 was obtained for the diagonal element y_{22}.

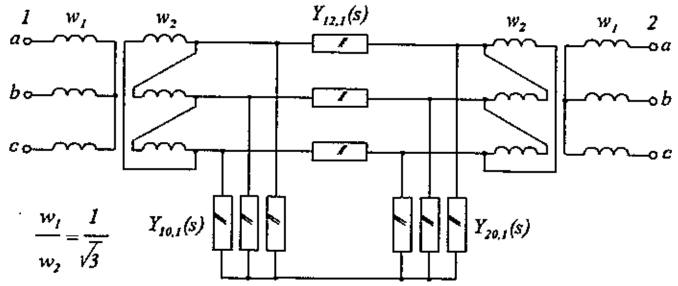

Fig. 6 Three-phase low-order equivalent of the positive-sequence system between two terminals

The reduced-order network equivalent between busbars AA and EE was represented by the three-phase model shown in Fig. 6. The ideal yd-transformers at the three-phase terminals 1 and 2 decouple positive- and zero-sequence voltages and currents. The admittances shown in Fig. 6 are represented by the rational admittance branch [9] and they constitute a three-phase π-equivalent of the positive-sequence system.

The transfer admittance $Y_{12}(s)$ in the model was determined by following rational function operations:

$$Y_{12}(s) = y_{11}(s) - Y_{10}(s) \tag{10}$$

$$Y_{12}(s) = y_{22}(s) - Y_{20}(s) \tag{11}$$

The best approximation, which satisfies with other admittance functions the real part conditions, was selected from above equations to model $Y_{12}(s)$.

The computed line currents (phase a) at busbars AA and EE using the low-order network equivalent and the full system are

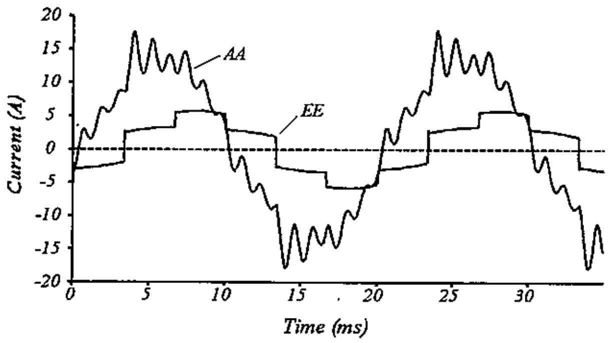

Fig. 7 Simulation using the network equivalent

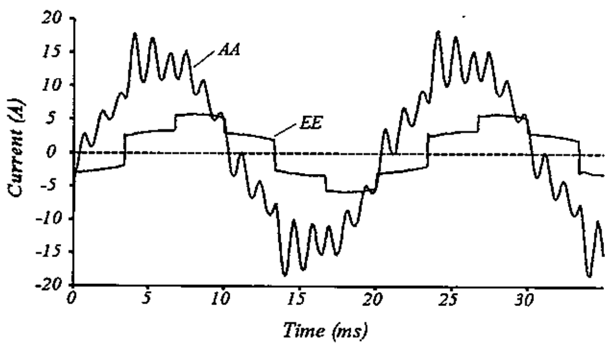

Fig. 8 Simulation using the full system

shown in Fig. 7 and Fig. 8, respectively. Both figures show good agreement between the results obtained using the full system and the low-order equivalent. The cpu time to perform the simulation using the low-order equivalent is less than the half of the cpu time required for the full system, when the same time step is used. The time step of the simulation using the low-order network equivalent is not restricted by travel times of lines and it can be increased in the case of computationally demanding transient studies.

VI. CONCLUSIONS

This paper decribes a method to produce low-order multi-port network equivalents, which are suitable to simulate switching transients in large power networks, which involve frequencies from a few Hertz up to 10 Khz. The network equivalent is based on frequency responses of admittance functions taking into consideration the frequency dependence of system components.

An unconstraint optimization method is used interactively to match calculated frequency curves by rational functions with freely selectable error criteria, which are applied separately to magnitude and phase in desired frequency intervals. Since the approximation program delivers poles and zeros of the model system, the realizability conditions can be checked easily. The order of the model system can be reduced significantly, when the common poles of rational admittance functions are taken into account.

The developed electrical branch in form of a rational admittance function represents in EMTP directly the fitted admittances. This avoids the tedious work of network synthesis to produce equivalents.

In order to keep the preparatory work as small as possible, a large part of a power network with many internal busses and small number of external ports, which determine the size of the reduced nodal admittance matrix [Y], should be modelled using network equivalents. In this respect, the assumption of a balanced network significantly reduces the number of elements to be approximated.

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