# Analysis of Core Type Transformer Models Based on the Principle of Duality in Electromagnetic Transients Studies

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*Abstract* — **This paper presents some theoretical aspects of how to obtain equivalent electric circuits for transformers based on the principle of duality. Some results of simulations with these equivalent circuits obtained with the program ATP are included. Finally a comparison is made between the results obtained using BCTRAN and those from topology based models.**

**Keywords:** Electromagnetic transients, Duality models of power transformers.

## I. INTRODUCTION

In the existing power transformer matrix representation in EMTP and ATP programs the core modelling is defined simply on the basis of a specified excitation current (positive and zero sequence), permitting linear magnetizing branches to be incorporated in the matrix. Nonlinear inductances are connected separately at winding terminals to introduce core nonlinearities, however these are not identified physically with individual core legs, yokes, etc. On account of that the authors present in this paper some theoretical aspects of how to obtain equivalent electric circuits for a transformer based on the principle of duality. Models based on the duality existing between magnetic and electric circuits identify a means for correctly interfacing core flux with air flux, permitting core nonlinearities to be incorporated on a physical basis. Presented in this work are some topology based magnetic models for three-phase core type three-windings transformers. It's important to remark that they can be simulated in EMTP and ATP programs. Other authors presented an hybrid model using BCTRAN for the leakage inductanes and topology based models for the core structure [6].

## II. MODELLING BASED ON THE PRINCIPLE OF DUALITY [1] [2] [3 ][4] [5]

The first and most difficult step in developing an equivalent electric circuit is the reduction of the flux distribution in a transformer into a magnetic circuit of lumped magnetic reluctances. To do this it's necessary to assume some paths for the flux in the iron core and in the air. The second step is the derivation of an electric circuit that is the equivalent of the lumped magnetic circuit. When the magnetic circuit contains no cross-over branches the equivalent electric circuit may be derived directly from the magnetic circuit by applying the topological principle of duality.

It's important to note that the accuracy of the equivalent electric circuit is largely dependent on the manner in which the magnetic system is reduced to a magnetic circuit.

### A. Magnetic circuit

The development of a magnetic circuit for the coil with N turns and an iron core shown in Fig. 1 affords an example of the method where $v_a$ is the instantaneous voltage between the terminals of the coil and $i_a$ is the instantaneous current through the coil. Fig. 1 shows the mean flux paths assumed where $\phi_1$ is the flux confined to the iron core between points A and B, $\phi_2$ is the flux confined in an external air path and $\phi_3$ is the flux confined to the yokes and the other leg.

The application of Ampere's law in mean path (1) gives:

$$\oint \bar{H} d\bar{p} = N i_a = \mathscr{F}_a \qquad (01)$$

where $\bar{H}$ is the magnetic field intensity, $\bar{dp}$ is the differential of length and $\mathscr{F}_a$ is the magnetomotive force. This equation can be rewritten as:

$$\oint \frac{\phi}{\mu \cdot S} \cdot dp = \mathscr{F}_a \qquad (02)$$

where $\bar{B}$ is the flux density and $\mu$ is the absolute permeability of the iron, $\phi$ is the magnetic flux and S is the cross-sectional area.

Dividing path (1) in the iron core in two partial paths, each of which has a uniform flux, equation (02) becomes:

$$\phi_1 \cdot \Re_{AB} + \phi_3 \cdot \Re_{BCA} = \mathscr{F}_a \qquad (03)$$

$\Re_{AB}$, $\Re_{BCA}$ - lumped reluctances that represent the two sections of the iron core associated with path (1) and carry fluxes $\phi_1$ and $\phi_3$ respectively

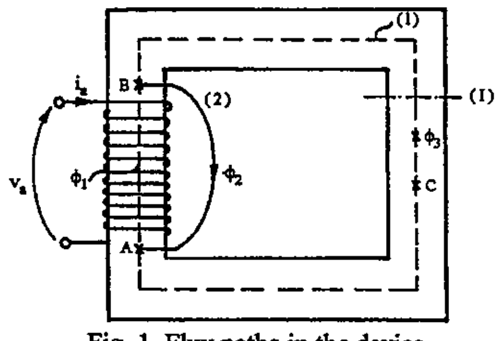


Fig. 1. Flux paths in the device

In terms of magnetomotive forces this equation can be rewritten as:

$$\mathcal{F}_1 + \mathcal{F}_3 = \mathcal{F}_a \qquad (04)$$

where $\mathcal{F}_1 = \Re_{AB} \cdot \phi_1 \quad \mathcal{F}_3 = \Re_{BCA} \cdot \phi_3$

The application of the same procedure for main path (2) gives:

$$\phi_1 \cdot \Re_{AB} + \phi_2 \cdot \Re_f = \mathcal{F}_a \qquad (05)$$

$$\mathcal{F}_1 + \mathcal{F}_2 = \mathcal{F}_a \qquad (06)$$

where $\mathcal{F}_2 = \Re_f \cdot \phi_2$ and $\Re_f$ is the lumped reluctance corresponding to an external air path and carries flux $\phi_2$.

Finally the relationship between fluxes $\phi_1$, $\phi_2$ and $\phi_3$ is:

$$\phi_1 = \phi_2 + \phi_3 \qquad (07)$$

From equations (03), (05) and (07) it's possible to synthesize the equivalent magnetic circuit of the magnetic system of Fig. 1 as shown in Fig. 2.

*B. Duality between magnetic and electric circuits*

Each flux of Fig. 1 will be associated with a fictitious coil of arbitrary number of turns N, for whichs Faraday-Lenz's law gives:

$$v_i = N \cdot \frac{d\phi_i}{dt} \qquad i = 1, 2, 3 \qquad (08)$$

Multiplying equation (07) by N and performed the differentiation results:

$$v_1 - v_2 - v_3 = 0 \qquad (09)$$

So a nodal flux equation in the magnetic circuit becomes in a mesh voltage equation in the electric circuit.

Equation (06) corresponds to the left mesh in the magnetic circuit of Fig. 3, and can be expressed as:

$$Ni_a = Ni_1 + Ni_2 \qquad (10)$$

or

$$i_a = i_1 + i_2 \qquad (11)$$

where $i_1$, $i_2$, $i_3$ are the currents through fictitious coils.

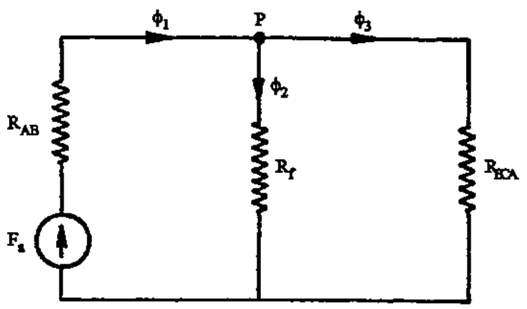


Fig. 2. Equivalent magnetic circuit of the device of Fig. 1

So a mesh magnetomotive force equation in the magnetic circuit becomes in a nodal current equation in the electric circuit. Magnetomotive force $\mathcal{F}_1$ can be written:

$$\Re_{AB} \cdot \phi_1 = Ni_1 \qquad (12)$$

Faraday-Lenz's law for this fictitious coil gives:

$$v_1 = N \frac{d\phi_1}{dt} \qquad (13)$$

Equations (12) and (13) lead to:

$$v_1 = \frac{N^2}{\Re_{AB}} \cdot \frac{di_1}{dt} \qquad (14)$$

The last equation shows that a lumped reluctance in the magnetic circuit corresponds a lumped inductance in the electric circuit. This establishes the duality between magnetic and electric circuits.

*C. Topological technique of duality*

This technique allows to draw the equivalent electric circuit from the equivalent magnetic circuit being not necessary to write the duals equations of the magnetic circuit's equations.

The topological technique by which the electric circuit may be obtained is shown in Fig. 3. A point is marked within each mesh of the magnetic circuit and an additional reference point is marked outside the circuit.These points will be the nodes of the dual network. Each pair of points are joined by solid lines, each one should pass through each element common to the two meshes of the magnetic circuits enclosing these two points. These solid lines will be the branches in the dual network. If a solid line passes through a reluctance then an inductance should be inserted between the corresponding nodes of the equivalent electric circuit.In the case of magnetomotive force a driving current should be inserted between them. Fig. 4 shows the graphically derived equivalent electric circuit, where $L_f$ is a linear inductance and $L_{AB}$, $L_{BCA}$ are non-linear inductances. It will be observed that the topological form of the system of solid lines is identical with the form of the equivalent electric circuit shown in Fig. 4.

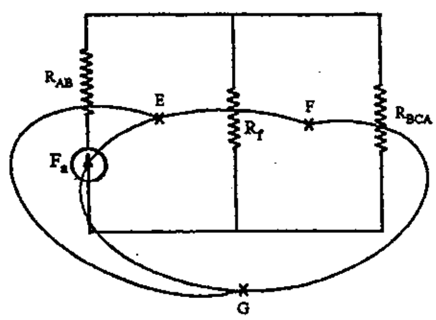


Fig. 3. Topological development of equivalent circuit.

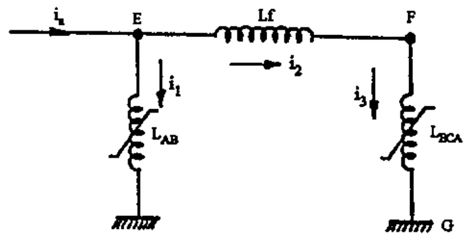


Fig. 4. Equivalent electric circuit of the magnetic circuit of Fig. 3

In the approach the authors choosed the arbitrary number of turns of the fictitious coils equal to N, now it will be assumed that the number of turns is equal to $N_1$ and it will be shown how this modifies the equivalent electric circuit of Fig. 4.

In terms of ampere-turns equation (06) can be expressed as:

$$N i_a = N_1 i_1' + N_1 i_2' \qquad (15)$$

or

$$i_a' = \frac{N}{N_1} \cdot i_a = i_1' + i_2' \qquad (16)$$

From this it can be concluded that before the equivalent electric circuit may be used it's necessary to add an ideal transformer having ratio N:$N_1$ to the terminals of the equivalent circuit as indicated in Fig. 5. In this new circuit all the elements have to be referred to an arbitrary number of turns $N_1$.

## III. CORE TYPE THREE PHASE THREE WINDINGS TRANSFORMER MODELS

In this section different models for a core type three phase three windings transformer will be obtained using the principle of duality. Fig. 6 shows the core type transformer with concentric windings considered in the present work, where:

$N_1, N_2, N_3$ - number of turns of the primary, secondary and tertiary windings
$i_{1a}, i_{1b}, i_{1c}$ - currents in the coils of the primary winding
$i_{2a}, i_{2b}, i_{2c}$ - currents in the coils of the secundary winding
$i_{3a}, i_{3b}, i_{3c}$ - currents in the coils of the tertiary winding

Fig. 7 shows the main mean flux paths assumed in the transformer of Fig. 6, where $\phi_1$, $\phi_2$, $\phi_3$ are the fluxes confined to the iron core between points A and B, C and D, F and G respectively, $\phi_{f1}$, $\phi_{f2}$, $\phi_{f3}$ are the leakages fluxes confined in an external air paths, $\phi_{ta}$, $\phi_{tb}$, $\phi_{tc}$ are the fluxes confined to the paths

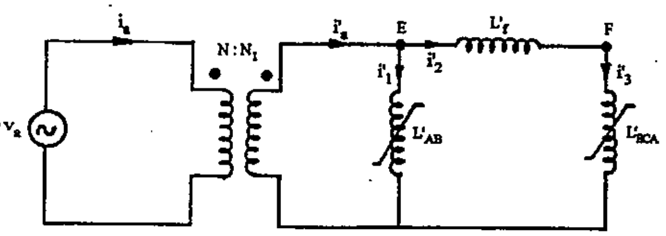


Fig. 5 - Equivalent electric circuit referred to $N_1$ turns

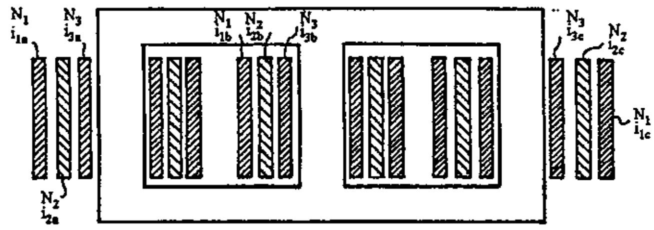

Fig. 6. Core type three phase transformer

from the top of the transformer through the air and tank to the bottom of the core and $\phi_{ab}$, $\phi_{bc}$ are the fluxes confined to the yokes.

The application of Ampere's law in path (1), (2) and (3) gives:

$$N_1 i_{1a} = \Re_{f1} \cdot \phi_{f1} \qquad (17)$$
$$N_2 i_{2a} = \Re_{f2} \cdot \phi_{f2} \qquad (18)$$
$$N_3 i_{3a} = \Re_{f3} \cdot \phi_{f3} \qquad (19)$$

$\Re_{f1}$, $\Re_{f2}$, $\Re_{f3}$ lumped reluctances corresponding to external air paths.

The application of Ampere's law in the path ABCDA and ABEA results:

$$-N_3 i_{3a} - N_2 i_{2a} + N_1 i_{1a} - N_1 i_{1b} + N_2 i_{2b} + N_3 i_{3b} = \\ = \Re_{AB} \cdot \phi_1 + (\Re_{BC} + \Re_{DA}) \cdot \phi_{ab} - \Re_{CD} \cdot \phi_2 \qquad (20)$$

$\Re_{AB}$, $\Re_{BC}$, $\Re_{DA}$, $\Re_{CD}$ lumped reluctances corresponding to iron core paths.

$$N_1 i_{1a} - N_2 i_{2a} - N_3 i_{3a} = \Re_{AB} \cdot \phi_1 + \Re_{ta} \cdot \phi_{ta} \qquad (21)$$

$\Re_{ta}$ lumped reluctance corresponding to the path through air and tank. Finally the last equations are:

$$\mathcal{F}_{ka} = N_k i_{ka} \qquad k = 1, 2, 3 \qquad (22)$$

$$\phi_1 = \phi_{ta} + \phi_{ab} \qquad (23)$$

Similar equations can be written for the other legs. From these equations it's possible to synthesize the equivalent magnetic circuit as shown in Fig. 8.

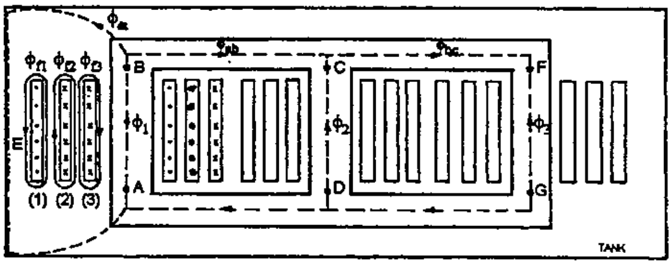


Fig. 7. Main flux paths assumed in the transformer

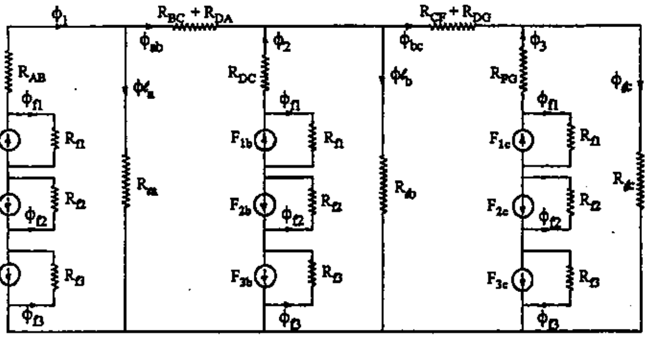

Fig. 8. Equivalent magnetic circuit of the magnetic system of Fig. 7

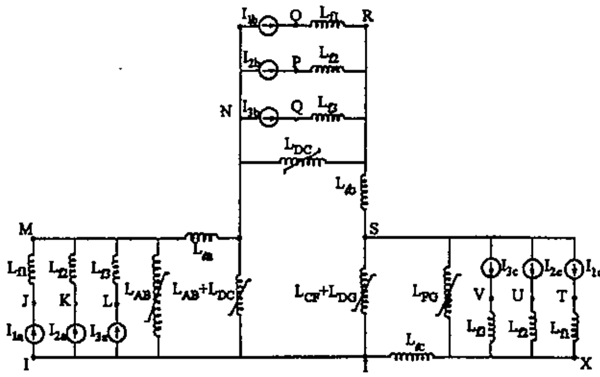

Fig. 10. Equivalent electric circuit (Model I) of the magnetic circuit of Fig. 8

The magnetic circuit of Fig. 8 is planar, then it's possible to apply the topological technique as indicated in Fig. 9 to obtain the equivalent electric circuit of Fig. 10, denominated Model I, where $L_{f1}$, $L_{f2}$, $L_{f3}$ are linear inductances, $L_{\ell a}$, $L_{\ell b}$, $L_{\ell c}$ are linear inductances, $L_{AB}$, $L_{DC}$, $L_{FG}$ are non-linear inductances and $L_{BC}$, $L_{DA}$, $L_{CF}$, $L_{DG}$ are non-linear inductances.

The ideal transformers have not been shown because of the added complexity.

The following approximation is made: the lumped reluctances $\Re_{\ell a}$, $\Re_{\ell b}$, $\Re_{\ell c}$ are equal to $\Re_{\ell}$ then a new equivalent electric circuit denominated Model II is obtained. In this model $L_{\ell a} = L_{\ell b} = L_{\ell c} = L_{\ell}$ being this the only difference with Model I.

In order to simplify Model II another approximation is made: in the magnetic circuit of Fig. 8 it can be observed that fluxes $\phi_{\ell a}$, $\phi_{\ell b}$, $\phi_{\ell c}$ are less than $\phi_1$, $\phi_{ab}$, $\phi_2$, $\phi_{bc}$, $\phi_3$ so the reluctances associated with the yokes are added to the reluctances of the legs in such a way to obtain three equal reluctances $\Re$ as shown in Fig. 11, which means we are using fictitious dimensions of the limbs.

The application of the topological technique in the circuit of Fig. 11 leads to the equivalent electric circuit of Fig. 12, denominated Model III.

Model III has been utilized in electromagnetic transients studies carried out with TNA (Transient Network Analyzer).

## IV. CHARACTERISTICS OF THE MODELS [5] [6]

In the next section are reported the main characteristics of the models derived above.

### *A. Model I*

The non-linear inductances can be calculated from the geometric data and from the material's characterisitc B x H of the iron core. The data of usual short-circuit and excitation tests given by the manufacturer don't give the necessary number of equations in order to obtain the linear inductances, so that some additional hypothesis are required.

### *B. Model II*

The non-linear inductances can be calculated from the geometric data and from the material's characteristics B x H of the iron core. From the data of positive-sequence short-circuit test can be calculated all leakage inductances and only $L_{\ell}$ is needed to complete the characterization of all air flux components. This requires just one zero-sequence test, so that the other tests are redundant.

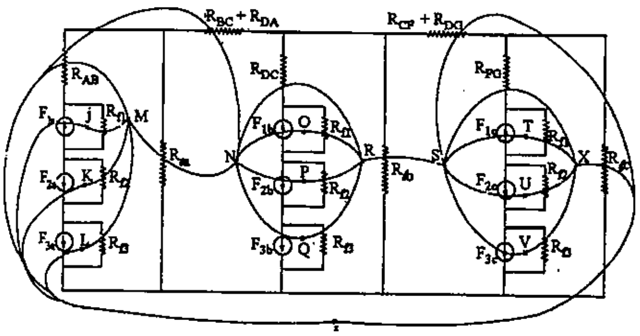

Fig. 9. Topological development of equivalent circuit

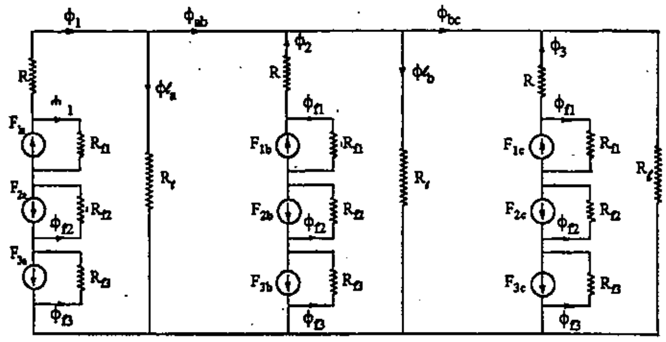

Fig. 11. Simplified equivalent magnetic circuit

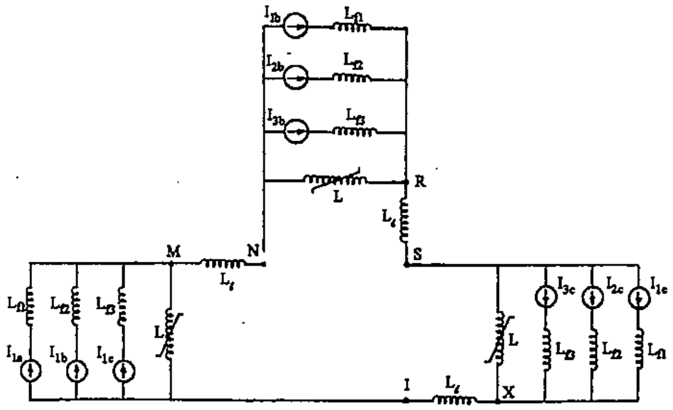

Fig. 12. Equivalent electric circuit (Model III) of the magnetic circuit of Fig. 11

### C. Model III

From the positive-sequence excitation test given by the manufacturer the characteristic Vrms x Irms is obtained which has to be converted to a λ x i curve, and this represents the non-linear inductance of the model. The leakage inductances and $L_\ell$ are calculated as indicated in Model II. Model III is appropriate in the case the user does not have geometric data. Individual leakage networks on each phase are efectively decoupled during short-circuit tests (positive-sequence and zero- sequence) in accordance with the flux distribution assumed in the derivation. For positive-sequence short-circuit tests the measured per unit impedance is the same regardless of which winding is excited, this is not valid for zero-sequence short-circuit tests since $L_\ell$ cannot be neglected.

### D. Common characteristics to all models

These electric equivalent circuits represent the magnetic system referred to an arbitrary number of turns N, before these may be used it's necessary to add ideal transformers having ratios $N_1$:N, $N_2$:N, $N_3$:N to their terminals and with them is taken into account the type of transformer's connection. In order to improve the accuracy of the electric models the following additions should be made: a) the resistances of the coils are placed outside the ideal transformers b) resistances are placed in parallel with non linear inductances in order to represent the iron core losses.

## V. CASE STUDIES

In this section some results of digital simulations using Models II, III and BCTRAN for a core type three phase two windings transformer are presented, they were obtained utilizing ATP program.

Finally a comparison is made between the results obtained with these models and those obtained from manufacturer tests.

Models II and III implementation was made in the following way: a) nonlinear inductances were represented through Type 98 element b) the branches containing the leakage inductances in series with driving currents were represented through two Saturable Transformer Components per phase. This model was used because it has an ideal transformer incorporated. c) the rest of the circuit's elements were represented using linear elements.

The transformer studied has the following characteristics: voltage ratings 60/15.75 kV, apparent power rating 7.5 MVA, type of connection Yd11 and the neutral of the primary winding is directly earthed. From the data of short-circuit and excitation tests, core's geometry, material's characteristic BxH and number of turns of the windings given by the manufacturer the authors calculated all models' elements.

Models II, III and BCTRAN were used to simulate positive-sequence excitation test conditions.The results of simulations as well as manufacturer data are presented in Table I.

Fig. 13 illustrates positive exciting currents computed using the above models.

The same models were used to simulate positive and zero-sequence short-circuit tests conditions.The results of simulations as well as manufacturer data are presented in Table II.

From the results presented in Table I it can be concluded that: a) each phase of Model II presented different peak and rms current values because the equivalent electric circuit is not symmetric. This electrical behavior agrees with other manufacturer's test data b) Model III presented equal values for all the phases, the same happened for BCTRAN and test data. It's important to remark that this two models don't consider the magnetic's asymmetry and the manufacturer only provided average values for currents d) Models III and BCTRAN presented a good agreement with test data and Model II gave an average difference of 14% in rms values. This difference is due to the following reasons: the manufacturer gave a typical material's characteristic BxH for one plate and he said that the limb's characteristic BxH would be different.

The shapes of the curves showed are similar and the results from short-circuit tests had good agreement.

## VI. CONCLUSIONS

In the present work, equivalent electric circuits were developed from the magnetic circuit by a topological duality technique.

Table I - Positive excitation test results

| | Phase A | | Phase B | | Phase C | |
|---|---|---|---|---|---|---|
| | $I_{peak}$ (A) | $I_{rms}$ (A) | $I_{peak}$ (A) | $I_{rms}$ (A) | $I_{peak}$ (A) | $I_{rms}$ (A) |
| Model II | .289 | .1574 | .194 | .1307 | .292 | .1775 |
| Model III | .333 | .1855 | .333 | .1855 | .333 | .1855 |
| BCTRAN | .304 | .1868 | .304 | .1868 | .304 | .1868 |
| Test Data | --- | .1804 | --- | .1804 | --- | .1804 |

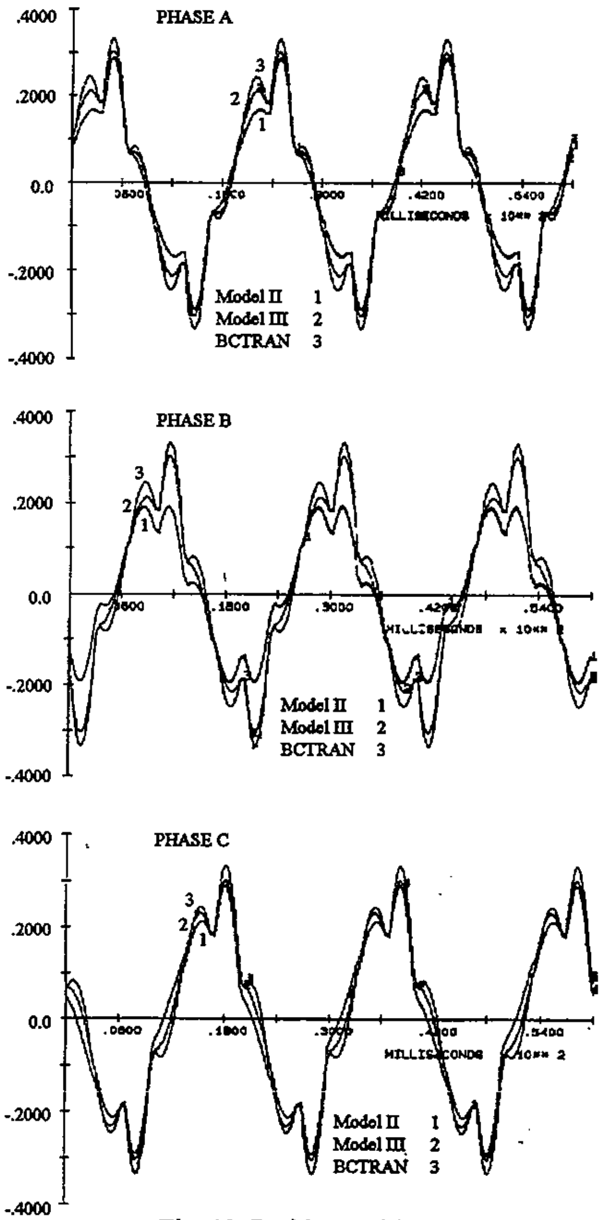


Fig. 13. Positive exciting current

The accuracy of these models is largely dependent on the manner in which the magnetic system is reduced to a magnetic circuit.

The models presented here were implemented using ATP and EMTP programs which are available in most electrical utilities.

Through the equivalent electric circuit presented here the model Saturable Transformer Component would be used to represent a three phase core type transformer without the addition of a fictitious extra delta-connected winding.

From Models II and III can be observed that zero-sequence short-circuit impedances are not reciprocal since $L_\ell$ cannot be neglected, in other words $Z_{PSO}$ is different from $Z_{SPO}$. Existing ATP and EMTP models don't always make this distinction, introducing errors.Existing ATP and EMTP models don't always make this distinction, introducing errors.

From this work the authors can conclude that the equivalent electric circuits derived become an alternative in relation to other models available.

Finally, the authors feel that it's necessary to conduct more research in this topic in order to obtain better models for the different types of transformer utilized in electric utilities.

Table II - Short-circuit tests results

| | $Z^{pos}$ (%) | $Z^{zero}$ (%) |
|---|---|---|
| Model II | 7.9696 | 7.3279 |
| Model III | 7.9696 | 7.3278 |
| BCTRAN | 7.9702 | 7.3301 |
| Test Data | 7.97 | 7.33 |

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