Transmission Lines: Fitting Technique Optimization

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Abstract - **In the EMTP, the transmission line model proposed by J. Martí is largely used. Multiphase lines are first decoupled through modal transformation matrices. For each line mode, the characteristic admittance $Y_c(\omega)$ and the propagation function $A(\omega)$ are approximated by rational functions using the accurate asymptotic fitting technique. In this paper, an optimization procedure to fit rational functions to $Y_c(\omega)$ and $A(\omega)$, is presented. It is shown that the number of poles and zeroes of the fitted functions can be considerably smaller than those obtained by the asymptotic procedure, over the entire frequency range and preserving good accuracy. Time domain simulations are performed for three-phase overhead lines. A considerable reduction in computer time is achieved.**

Keywords: EMTP, Line modelling, Optimization.

I. INTRODUCTION

Frequency-dependent transmission line modelling is very important for transient studies [1,2]. J. Martí's work [2] led to a solution method which is largely used today. For each line mode one needs to know the propagation function $A(\omega)$ and the line characteristic impedance $Z_c(\omega)$:

$$Z_c(\omega) = \sqrt{\frac{R'(\omega) + j\omega L'(\omega)}{G'(\omega) + j\omega C'}} = \sqrt{\frac{Z'(\omega)}{Y'(\omega)}},$$

and

$$A(\omega) = e^{-\gamma(\omega) d}.$$

where:

$Z'(\omega) = R'(\omega) + j\omega L'(\omega), \quad Y'(\omega) = G'(\omega) + j\omega C'$

$\gamma(\omega) = \sqrt{Z'_{mod}(\omega).Y'_{mod}(\omega)}$ = propagation constant

$R'(\omega)$ = series resistance; $L'(\omega)$ = series inductance;

$G'(\omega)$ = shunt conductance; C' = shunt capacitance

d = line length; τ = travel time.

(primed quantities are in per unit length)

With J. Martí's formulation, $A(\omega)$ can be easily interpreted. Consider the open-ended single-phase transmission line of Fig. 1, in which the sending end is connected to a voltage source V_1. $A(\omega)$ is the voltage ratio:

$$A(\omega) = \frac{V_0(\omega)}{V_i(\omega)} = e^{-\gamma(\omega) d}.$$

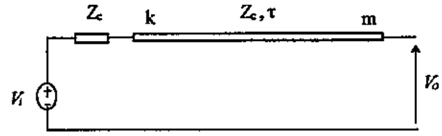


Fig. 1: Single-phase transmission line.

The transmission line equivalent circuit, in the frequency domain, is shown in Fig. 2, where

$$B_k(\omega) = V_k(\omega) - Z_c(\omega).J_{km}(\omega) = [V_m(\omega) + Z_c(\omega).J_{mk}(\omega)].A(\omega);$$

$$B_m(\omega) = V_m(\omega) - Z_c(\omega).J_{mk}(\omega) = [V_k(\omega) + Z_c(\omega).J_{km}(\omega)].A(\omega).$$

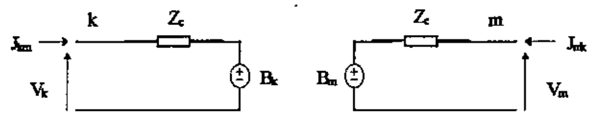


Fig. 2. Transmission line equivalent circuit [2].

II. TIME-DOMAIN MODEL

For time-domain simulations, $Z_c(\omega)$ is approximated by $Z_{eq}(\omega)$, produced by a series of RC parallel blocks (Foster I realization - Fig. 3).

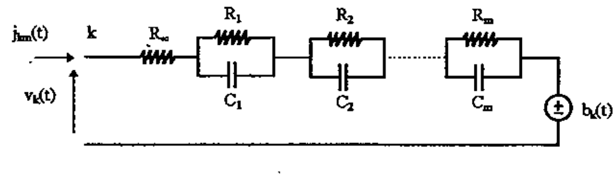


Fig. 3. Equivalent circuit for time-domain simulation.

The RC parameters are calculated according to the following steps:

1. in the complex plane ($s = \sigma + j\omega$), $Z_{eq}(\omega)$ is written in rational function form:

$$Z_{eq}(s) = \frac{N(s)}{D(s)} = H \cdot \frac{(s+z_1).(s+z_2)......(s+z_n)}{(s+p_1).(s+p_2)......(s+p_m)}, \text{ with } n=m; \quad (1)$$

2. Writing $Z_{eq}(s)$ in partial fraction form,

$$Z_{eq}(s) = k_\infty + \frac{k_1}{(s+p_1)} + \frac{k_2}{(s+p_2)} + \dots + \frac{k_m}{(s+p_m)}, \quad (2)$$

the RC parameters are obtained: $R_\infty = k_\infty$; $R_i = k_i/p_i$; $C_i = 1/k_i$, for $i = 1,2...,n$.

In Fig. 2,

$B_k(\omega) = V_k(\omega) - Z_c.J_{km}(\omega) = \left[V_m(\omega) + Z_c.J_{mk}(\omega)\right].A(\omega)$.

Then, from the circuit in Fig. 3:

$$b_k(t) = \int_{\tau}^{+\infty} f_m(t).a(t).dt \qquad (3)$$

where: $F_m(\omega) = V_m(\omega) + Z_c(\omega).\ J_{mk}(\omega)$.

$f_m(t)$ = inverse transform of $F_m(\omega)$;

$a(t)$ = inverse transform of $A(\omega)$.

$a(t)$ is obtained from $A(\omega)$ in the following way:

1. $A(\omega)$ is approximated by $A_{eq}(\omega)$ which, in the complex plane ($s = \sigma + j\omega$), is written in the form:

$$A_{eq}(s) = P(s).e^{-s.\tau_{min}},$$

where τ_{min} is the travel time of the fastest waves, and

$$P(s) = \frac{N(s)}{D(s)} = H.\frac{(s+z_1).(s+z_2)......(s+z_n)}{(s+p_1).(s+p_2)......(s+p_m)}, \quad \text{with } n<m \qquad (4)$$

2. $A_{eq}(s)$ can be written in the form,

$$A_{eq}(s) = \left\{\frac{k_1}{s+p_1} + \frac{k_2}{s+p_2} + \cdots + \frac{k_m}{s+p_m}\right\}.e^{-s.\tau_{min}}, \qquad (5)$$

and in the time domain,

$$a_{eq}(t) = \{k_1 e^{-p_1(t-\tau_{min})} + k_2 e^{-p_2(t-\tau_{min})} + \ldots + k_m e^{-p_m(t-\tau_{min})}\}.u(t-\tau_{min}). \qquad (6)$$

$a(t)$ is replaced by $a_{eq}(t)$ and equation (3) is evaluated with recursive convolution at each time step.

III. THE ASYMPTOTIC FITTING TECHNIQUE

The success of the described approach is the quality of the rational function approximations for $Z_c(\omega)$ e $A(\omega)$. J. Marti uses the asymptotic fitting technique based on Bode's procedure for approximating the magnitude of the functions. Since the rational functions have no poles or zeroes in the right hand side of the complex plane, the corresponding phase functions are uniquely determined from the magnitude functions. The basic principle is to approximate the given curve by straight-line segments which are either horizontal or have a slope which is a multiple of 20 decibels/decade. The points where the slopes change define the poles or zeroes of the rational function. The number of poles and zeroes are not determined a priori. The approximation is done in a step by step fashion starting from DC to the highest frequency.

The asymptotic fitting technique is accurate, but it can lead to a large number of poles and zeroes of the approximated functions [3,4]. A smaller number of zeroes and poles may be found if a reduction is made in either the frequency range or accuracy [5]. In the next section, an optimization procedure to fit rational functions to $Y_c(\omega)=[Z_c(\omega)]^{-1}$ and $A(\omega)$, is presented. It is shown that the number of poles and zeroes of the fitted functions can be considerably smaller than those obtained by using the asymptotic procedure, over the entire frequency range, without sacrificing accuracy.

IV. THE OPTIMIZED FITTING TECHNIQUE

$Y_{eq}(s)$ and $P(s)$ in (4) are determined from the magnitude of the phase functions $Y_c(\omega)$ and $A(\omega)$. For $s=j\omega$, $[Y_c(\omega)]^2$ and $[P(\omega)]^2$ can be written in the forms

$$|Y_c(\omega)|^2 = H^2.\frac{(A_1.\omega^2+1).(A_2.\omega^2+1)......(A_n.\omega^2+1)}{(B_1.\omega^2+1).(B_2.\omega^2+1)......(B_n.\omega^2+1)} \qquad (7a)$$

and

$$|P(\omega)|^2 = H^2.\frac{(A_1.\omega^2+1)}{(B_1.\omega^2+1)}......\frac{(A_n.\omega^2+1)}{(B_n.\omega^2+1)}.\frac{1}{(B_{n+1}.\omega^2+1)}.\frac{1}{(B_m.\omega^2+1)} \qquad (7b)$$

for $n<m$.

The parameters H, A_i and B_i are found iteratively using the nonlinear least square optimization procedure due to Levenberg-Marquardt [6,7,8], in which the function to be approximated (equation 7) and its Jacobian are known in analytical form. Once H, A_i and B_i are determined, the poles and zeroes of $Y_{eq}(s)$ and $P(s)$ are obtained by:

$$z_i = 1/\sqrt{A_i} \quad \text{and} \quad p_i = 1/\sqrt{B_i}.$$

Computational Procedure

1. Characteristic Admittance

a) The user defines the number of poles n equal to the number of zeroes. The frequency axis is divided in n equally spaced intervals in logarithmic scale, starting with the interval between ω_{min} and ω_1 and ending with the interval between ω_n and $\omega_{máx}$(Fig. 4). To start the iterative process, the initial guesses for A_i and B_i are taken in such way that the poles p_i are at the centre of each interval and the zeroes at z_i=0.8 p_i. The initial guess for H is $H = Y_c(\omega_{min})$.

b) find the error function χ^2 between the known $[Z_c(\omega)]^2$ and the estimated $[Z_{eq}(\omega)]^2$ curves, for the whole frequency range;

c) use the Levenberg-Marquardt method starting the iterative process. A_i, B_i and H, are obtained when χ^2 is minimum.

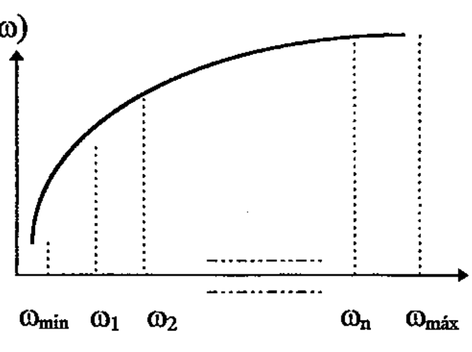


Fig. 4. Estimation of zeroes and poles of $Y_{eq}(s)$.

2. Propagation Function $A(\omega)$

In computing the characteristic admittance $Z_{eq}(s)$, no convergence problems were found. However, they may occur when computing $P(s)$. To avoid them, the fitting process is made in one or more steps:

a) from FDDATA™ [9] output files, one know the phase angle of $A(\omega)$, or $P(\omega)$. If the phase angle of $P(\omega)$ is less then 90^0 degrees, the fitting is made in a way the number of poles exceeds the number of zeroes by one. The iterative process is started and the parameters A_i, B_i and H are found;

b) the new pole is added at $p_{i+1}=1.2p_i$ and the iterative process is re-started taken the parameters obtained in the previous step as initial guesses.

V. CALCULATED RESULTS

Consider the three-phase untransposed overhead transmission line with four bundle conductors and an earth wire of Fig. 5 [10]. The line data are shown in Table 1.

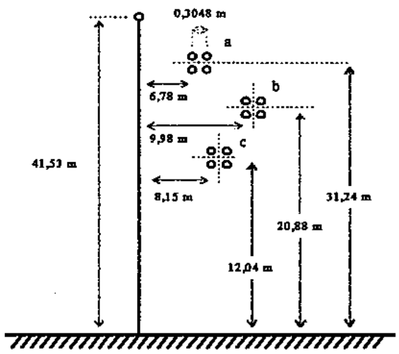


Fig. 5. Untransposed three-phase transmission line.

Table 1 - Basic transmission line data of Fig. 5.

Number of circuits	1
Number of conductors per phase	4
Number of earth wires	1
Conductor resistivity	32 nΩ.m
Earth-wire resistivity	26,9 nΩ.m
Conductor strand and earth-wire diameter	28,6 mm
Earth resistivity	20,0 Ω.m
Line length	160,0 km

From FDDATA™, the zero, positive and negative modes of $Y_c(\omega)$ were computed for a frequency range from 10^{-2} Hz to 10^6 Hz. The zero mode of $A(\omega)$ was computed from 10^{-2} Hz to 10^5 Hz and the positive and negative modes from 10^{-2} Hz to 10^6 Hz. The asymptotic (FDDATA™) and the optimized fitting procedures were used to find the rational function approximations for $Y_c(\omega)$ and $A(\omega)$. The fitting results for the zero mode of $Y_c(\omega)$ are shown in Fig. 7 to 10. In the whole frequency range, the accuracy or the function obtained by the optimized procedure is better than the larger order function obtained by the asymptotic method. For the zero mode of $A(\omega)$, the number of poles exceeds the number of zeroes by two (Fig. 11 to 14). This may be the cause of higher errors in magnitude and phase of both fitted curves. The maximum errors for the magnitude of the fitted functions are summarized in Table 2. The optimized fitting procedure produces lower order rational functions.

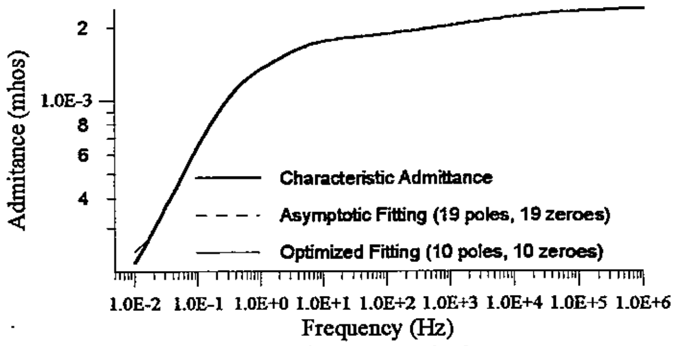


Fig. 7 - Admittance magnitude.

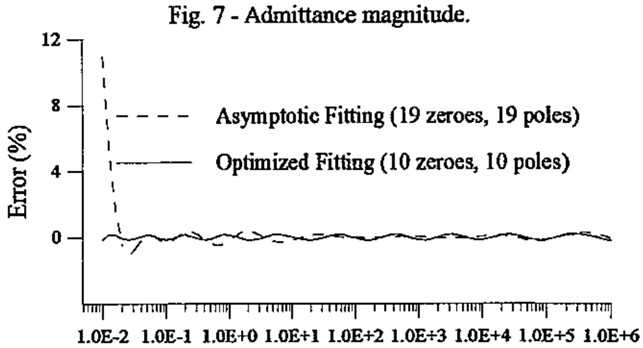


Fig. 8 - Magnitude error curves for $Y_{ceq}(\omega)$.

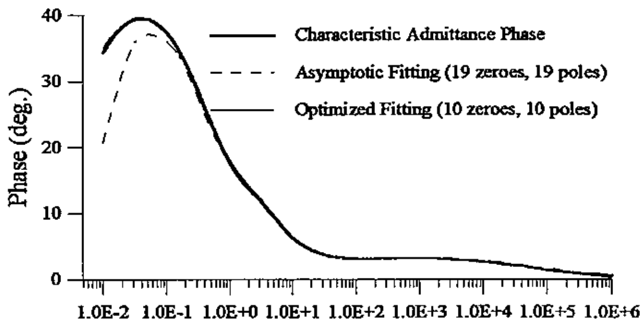


Fig. 9 - Admittance Phase.

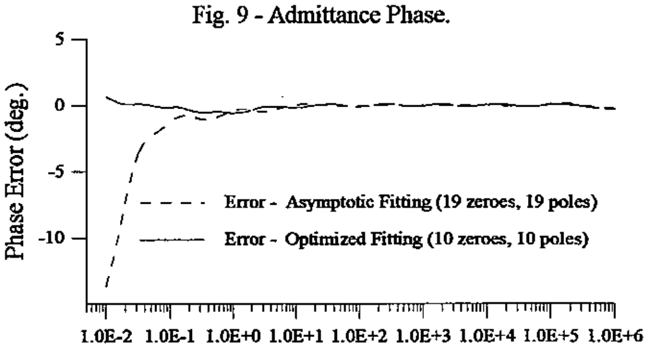


Fig. 10 - Phase Error Curves for $Y_{ceq}(\omega)$.

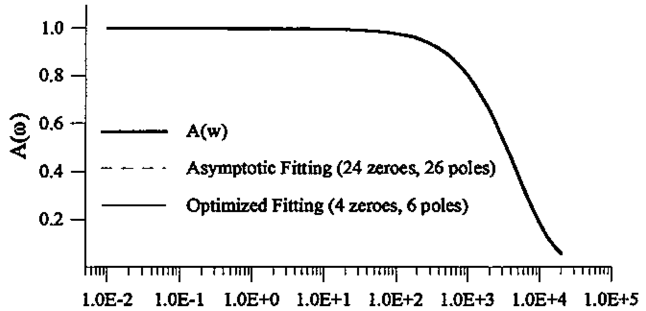


Fig. 11: $A(\omega)$ - Zero mode frequency response.

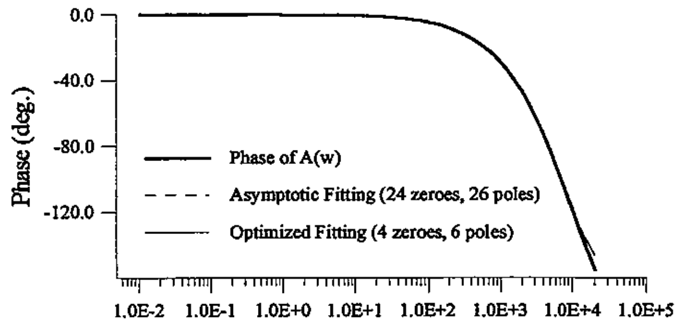


Fig. 13: $A(\omega)$-Zero mode phase.

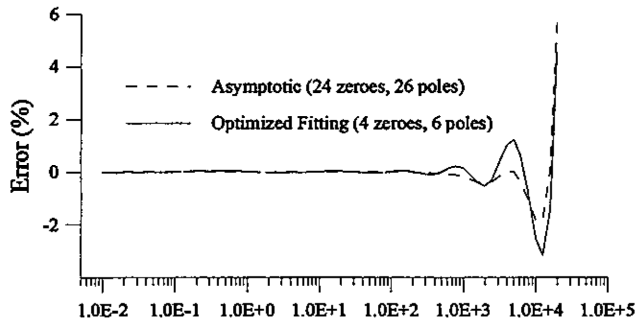


Fig. 12: $A(\omega)$-Zero mode error (%).

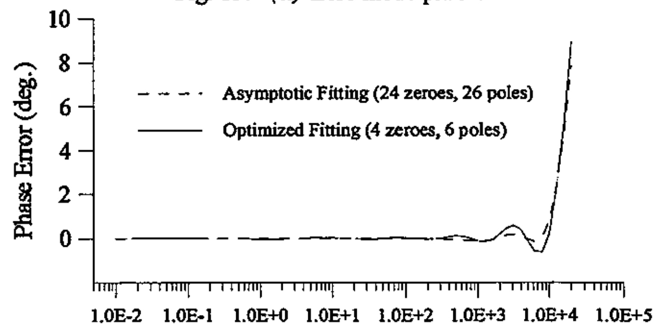


Fig. 14: $A(\omega)$-Zero mode phase error.

Table 2 - The fitting procedures.

		Asymptotic [2]			Optimized Fitting		
	Mode	number of zeroes	number of poles	maximum error(%)	number of zeroes	number of poles	maximum error(%)
$Y_c(\omega)$	zero	19	19	10.960	10	10	0.236
	positive	13	13	10.170	08	08	0.178
	negative	14	14	10.670	09	09	0.073
$A(\omega)$	zero	24	26	5.675	04	06	4.567
	positive	20	21	8.949	05	06	0.136
	negative	24	25	3.569	05	06	0.530

VI. DIGITAL SIMULATIONS

MICROTRAN® [9] was used to carry out time domain simulations for the open-ended line of Fig. 5, using data obtained with the asymptotic and the optimized fitting procedures. A unit step voltage was applied to each phase at the sending end. A time step $\Delta t = 1\mu s$ and the modal transformation matrix, calculated *1,2 kHz* from FDDATA™, were used. The simulated results are shown in Fig. 15.

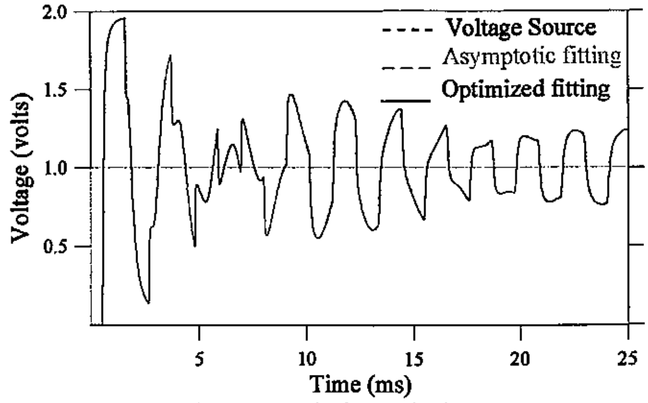
The simulation times for the frequency dependent models with the two model parameters are compared to the constant parameter model, taken as the reference model, produced by MTLINE™[9]. The results are shown in tables 3 and 4. For the model obtained with the asymptotic fitting procedure, the simulation takes 82,75% more time. However, for the model in which the parameters were obtained by the optimized fitting procedure, the increment in time would be only 58,62%.

Table 3 - Simulation time per time step

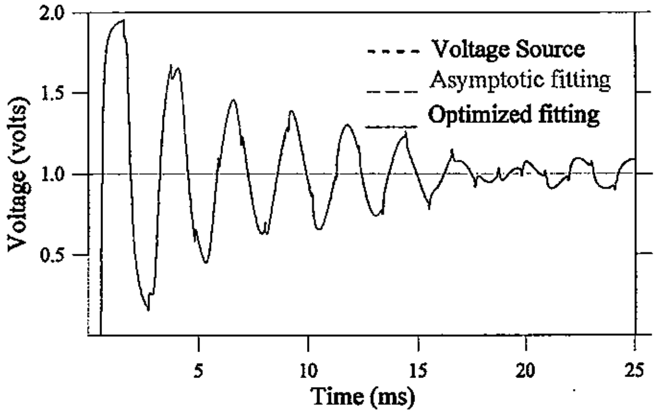
Simulation time / Number of time steps		
Constant parameter model	Model obtained from asymptotic fitting	Model obtained from optimized fitting
29,0 / 25000 = 1,16 ms	53,0 / 25000 = 2,13 ms	46,0 / 25000 = 1,83 ms

Table 4 - Total processing time.

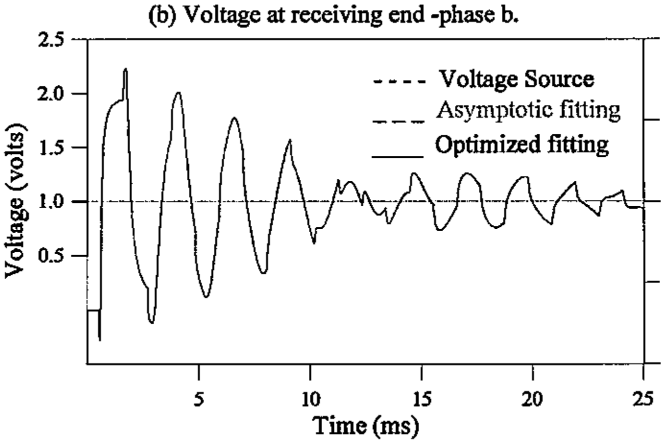
Total processing time		
Constant parameter model	Model obtained from asymptotic fitting	Model obtained from optimized fitting
29,0 s	53,0 s	46,0 s



(a) Voltage at receiveing end -phase a.



(b) Voltage at receiving end -phase b.



(c) Voltage at receiving end -phase c.

Fig. 15: Energization of a three-phase line.

VII. CONCLUSIONS

An optimization procedure to fit rational functions to $Z_c(\omega)$ and $A(\omega)$, was presented. Simulations were performed for a three-phase overhead transmission line. It was shown that the number of poles and zeroes of the fitted function can be considerably smaller than those obtained by using the asymptotic procedure, over the entire frequency range and preserving good accuracy.

Low order models may be important for real time simulation investigations and for studies in which transmission lines need to be subdivided into several sections, such as corona studies [11].

VIII. ACKNOWLEDGEMENTS

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