Idempotent Line Model: Case Studies

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Abstract: **This paper presents a number of test cases using the new full frequency dependent idempotent line model (idLine) developed at the University of British Columbia. These cases include highly asymmetrical configurations of multicircuit overhead lines where the traditional constant transformation matrix frequency-dependent line model in the EMTP (fdLine) does not give accurate results. The accuracy of the idLine and fdLine time-domain line models is verified using the frequency-domain FDTP program.**

Keywords: Frequency dependent transmission line modelling, idempotent decomposition, EMTP modelling, time domain transients simulation.

I. INTRODUCTION

Traditional transmission line modelling in transients analysis of power systems makes use of modal decomposition (eigenvalue/eigenvector analysis) ([1], [2]) to decouple the travelling waves into their natural propagation modes before transferring the model into the system's coupled phase coordinates.

The frequency dependent transmission line model of [3] *fdLine* has been used in the electromagnetic transients program EMTP for a number of years for accurate and reliable modelling of overhead transmission lines with frequency dependent parameters. A fundamental limitation of this model, however, is that it assumes that the transformation matrix that relates modal and phase quantities can be considered as real and constant within the frequency range of the study. This assumption, even though valid in many cases, can lead to inaccurate results in cases of strongly asymmetrical multicircuit line configurations.

A number of solutions have been proposed for the problem of frequency dependence of the transformation matrix: from direct synthesis of this matrix in the frequency domain ([4]) to avoiding the use of transformation matrices by working directly in phase coordinates ([5]). Both of these approaches, however, have drawbacks. Direct synthesis of the transformation matrix with stable rational functions is difficult because the eigenvectors that make up the columns of these matrices are not uniquely defined at each frequency point. Direct phase-domain modelling is also difficult because an N-phase network has only N propagation modes and N time delays, and the N^2 self and mutual elements of $[A_{phase}]$ are not independent but a combination of these basic travelling times and modes.

II. IDEMPOTENT LINE THEORY

The idempotent line model *idLine* introduced in [6], expresses the line propagation functions directly in phase coordinates (thus avoiding the use of transformation matrices) but, instead of synthesizing the elements of the propagation matrix directly, it expresses this matrix in terms of the N natural propagation modes of the line (thus avoiding mixed-up travelling times and modes).

The theory of the new idLine model is described in detail in [7] and will be presented in an IEEE paper under preparation. Only the basics of the model are briefly reviewed here as a background for the simulation studies presented.

A. Phase-Domain Propagation Matrix

Idempotent decomposition (spectral decomposition) is used for the synthesis of the phase-domain propagation matrix. This allows us to write the propagation matrix as:

$$\left[A_{phase}\right] = \sum_{i=1}^{N}\left[M_i\right]A_{mode\,i} \tag{1}$$

where the $[M_i]$'s are the idempotent matrix coefficients and the $A_{mode\,i}$'s are the (scalar) propagation functions for each independent mode i. As opposed to the elements of the transformation matrices, which depend on an arbitrary scale factor, the elements of the idempotent matrices are uniquely defined at each frequency point.

If the elements of the idempotent matrices are synthesized with rational functions using stable *common poles*, and the mode propagation functions are synthesized with rational functions with a single time delay per mode, the phase-domain propagation matrix can be written as:

$$\left[A_{phase}\right]=\sum_{i=1}^{N}\left(\sum_{j=1}^{nPoles_i}\left[K_j^{(i)}\right]\frac{1}{s+p_j^{(i)}}\right)e^{-j\omega\tau_i} \qquad (2)$$

where superscript (i) indicates mode i. $[K_j^{(i)}]$ is a real constant matrix and $p_j^{(i)}$ and τ_i are real numbers.

The synthesis of all elements of each idempotent matrix with a common set of poles results in savings in simulation time in the order of N^2 (N = number of phases) as compared to fitting each element with a different set of poles. This also guarantees the absolute numerical stability of the solution since the common poles are simply factored out of the matrix.

B. Characteristic Admittance Matrix

Similarly to the case of the idempotent matrix, the characteristic admittance matrix can be also synthesized in terms of a common set of poles. This results in a very compact realization:

$$\left[Y_c\right]=\left[Y_{c0}\right]+\sum_{j=1}^{nPolesYc}\left[Y_{cj}\right]\frac{1}{s+p_j} \qquad (3)$$

where $[Y_{c0}]$ and $[Y_{cj}]$ are real, constant matrices, and p_j is real and constant.

C. Discretized equivalent circuit

With the synthesis of the phase-domain propagation function and characteristic admittance function as indicated in (2) and (3), time domain convolution integrals can be avoided and replaced by simple relationships between present and past values of the variables. This allows the derivation of the equivalent circuit in Fig. 1 in which the two line ends are decoupled and related only through current sources that depend on past values of the variables at the opposite line end.

III. CASE STUDIES

A number of simulations are next presented where the traditional constant-transformation matrix model (fdLine) does not give good results, while the idLine model gives results that are in close agreement with those obtained with the FDTP frequency domain program. (The FDTP program, developed by L. M. Wedepohl, solves the network directly in the frequency domain and, therefore, the solution is "exact" for frequency dependent lines.)

Five multicircuit line configurations are considered in this study (Cases A to E) and three tests were performed for each configuration: a steady state simulation, an open circuit energization, and a short circuit energization.

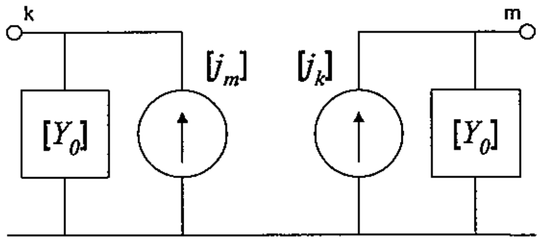

Fig. 1. Transmission Line equivalent circuit.

In the steady state tests, one circuit of the transmission line (phases 1, 2, and 3) was connected to balanced 60 Hz sinusoidal sources at both sending and receiving ends with RMS values of 500 kV L-L at the sending end and 485 kV L-L at the receiving end. The receiving-end sources lagged the sending-end sources by 15°.

For the energization tests, one circuit of the transmission line (phases 1, 2, and 3) was energized with a set of balanced, sinusoidal sources of peak value 1.0. The receiving end was either open (open circuit energization) or connected to ground (closed circuit energization), depending on the test. The other line circuits were connected to ground through 10 Ω resistances.

Case A. Two Horizontal Single-Circuit Transmission Lines

The first case analyzed is that of two horizontal single-circuit transmission lines. The physical configuration of the lines is shown in Fig. 2.

Case B. Double-Circuit Tower Configuration

This configuration corresponds to the common case of two lines on the same tower. The conductors configuration

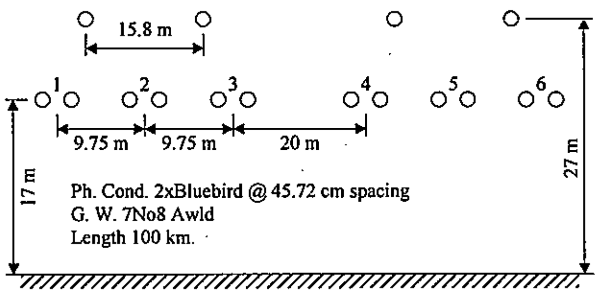

Fig. 2. Two Horizontal Single-Circuit Transmission Line.

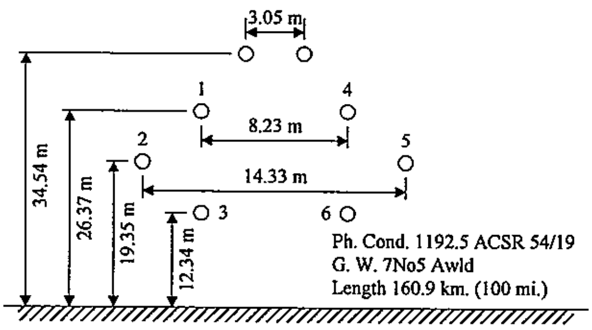

Fig. 3. Double-Circuit Tower configuration.

and line dimensions are shown in Fig. 3.

Case C. Two Delta-Configuration Parallel Circuits

This case corresponds to two single circuit parallel lines with conductors arranged in a triangular (delta) scheme. The configuration can be seen in Fig. 4.

Case D. Multi-Circuit Multi-Voltage Tower

In this case a 9-phase transmission line (1x500 kV and 2x230 kV circuits) is analyzed. The configuration and distances are shown in Fig. 5.

Case E. Quadruple-Circuit Parallel-Lines Configuration

The physical configuration for this 12-phase case is shown in Fig. 6

IV. SIMULATION RESULTS

Due to space limitations, only the results for the phase with the most noticeable difference between the new idLine model and the traditional fdLine model are shown.

In the case of open-circuit energization tests, all the plots correspond to voltages in the receiving end of the line for the phase showing the maximum difference between the idLine and the fdLine models. For the closed-circuit energization tests, the current in the receiving end for the phase with most noticeable difference is shown. The results obtained with the FDTP (frequency domain) program are plotted where available.

A. Steady State Tests

For the steady state tests, the maximum percentage

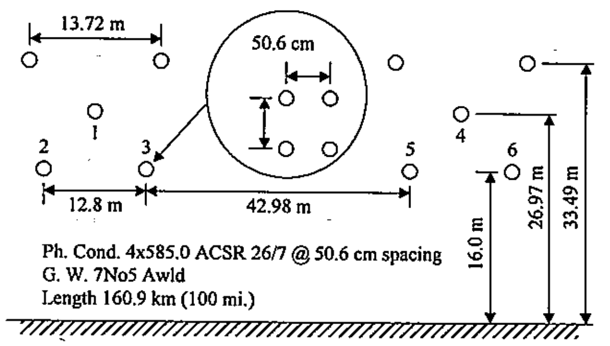

Fig. 4. Two Delta-Configuration Parallel Circuits.

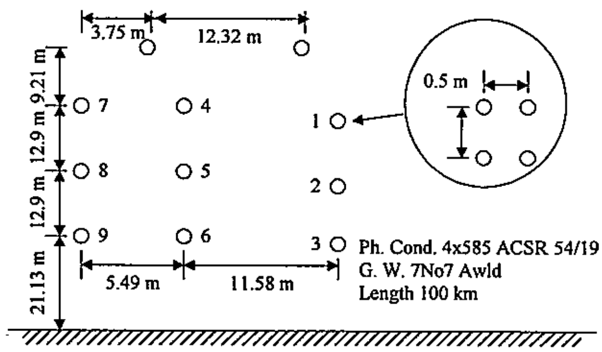

Fig. 5. Multi-Circuit Multi-Voltage Tower.

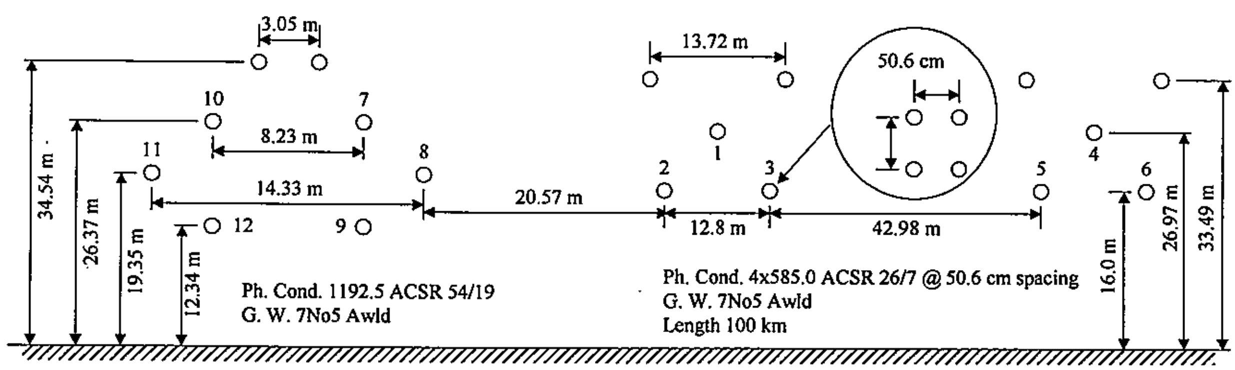

Fig. 6. Quadruple-Circuit Parallel-Lines Configuration.

differences between the magnitudes of the line currents at the sending end with respect to the exact solution are shown. The exact solution was obtained using a π equivalent circuit calculated at 60 Hz.

The steady state results are summarized in Table 1. The value of the currents in this table is given in per unit of the largest current in the system. Currents of less than 1% in magnitude were not included in this comparison. The results for the fdLine model correspond to the usual case where the constant transformation matrix is calculated for a frequency in the order of the kHz (at 1200 Hz in this case).

B. Energization Tests

The energization tests were performed with a time step of 5 μs and a total simulation time of 5.12 ms. There is one plot for each test in each configuration. The figures in the paper where those plots are presented are summarized in Table 2.

Table 1. Steady State Test Results. Line currents with maximum error. Magnitudes in per unit of largest current.

	Ph.	Exact	idLine	% Error	fdLine	% Error
A	5	0.024037 /-67.523	0.023619 /-67.528	-1.74%	0.027173 /-63.860	13.05%
B	6	0.085657 /-44.364	0.079422 /-42.752	-8.12%	0.102382 /-52.372	20.70%
C	6	0.012439 /-50.826	0.014739 /-28.998	18.50%	0.032581 /-52.411	161.9%
D	9	0.038582 /-42.612	0.040203 /-44.818	4.20%	0.060133 /-60.309	55.86%
E	6	0.015223 /-73.989	0.017646 /-74.501	15.91%	0.081033 /-83.218	432.31%

Table 2. Energization Test Plots.

Case	Test	Figure
A	Open circuit	Fig. 7
	Closed circuit	Fig. 8
B	Open circuit	Fig. 9
	Closed circuit	Fig. 10
C	Open circuit	Fig. 11
	Closed circuit	Fig. 12
D	Open circuit	Fig. 13
	Closed circuit	Fig. 14
E	Open circuit	Fig. 15
	Closed circuit	Fig. 16

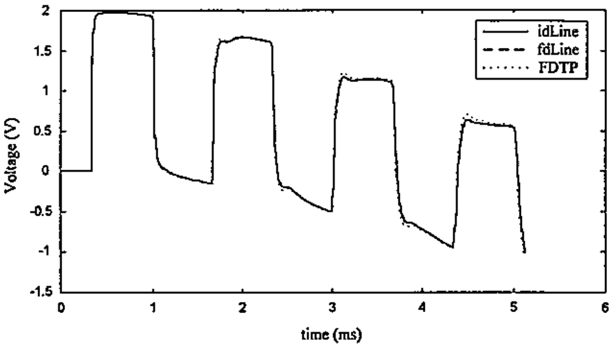

Fig. 7. Case A. Voltage at receiving end. Phase2.

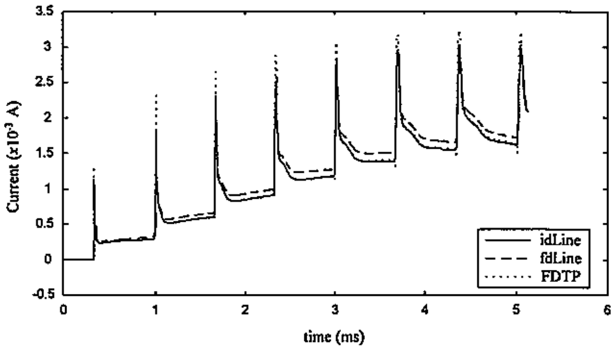

Fig. 8. Case A. Current at receiving end. Phase 4.

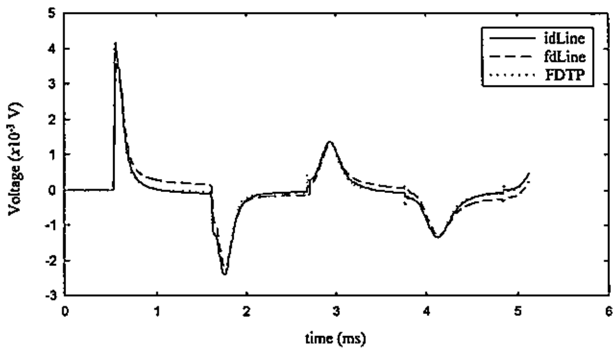

Fig. 9. Case B. Voltage at receiving end. Phase 6.

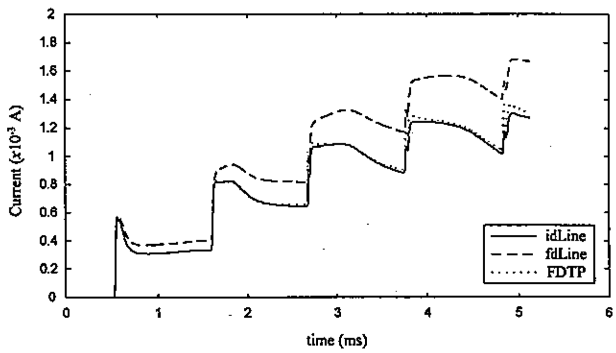

Fig. 10. Case B. Current at receiving end. Phase 6.

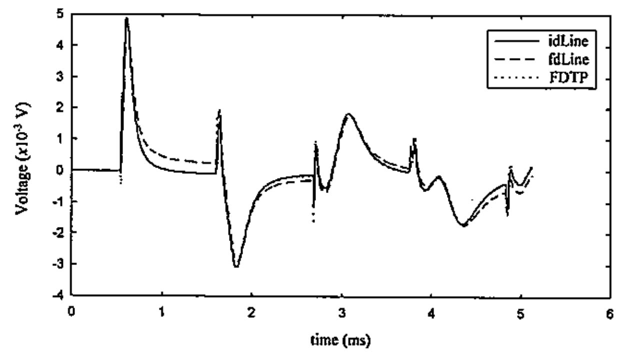

Fig. 11. Case C. Voltage at receiving end. Phase 5.

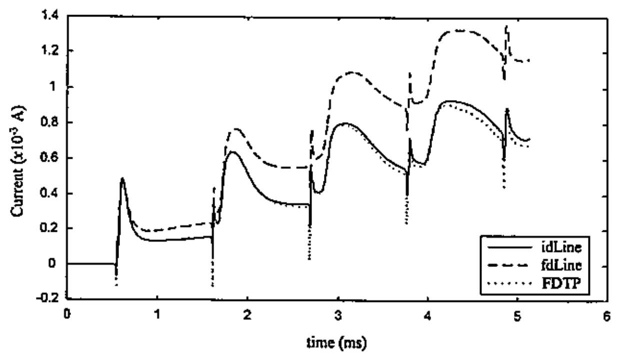

Fig. 12. Case C. Current at receiving end. Phase 5.

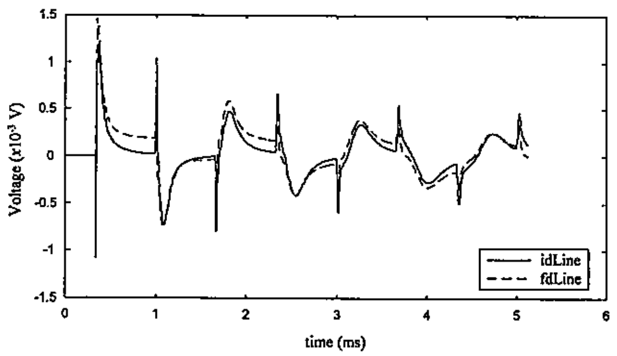

Fig. 13. Case D. Voltage at receiving end. Phase 7.

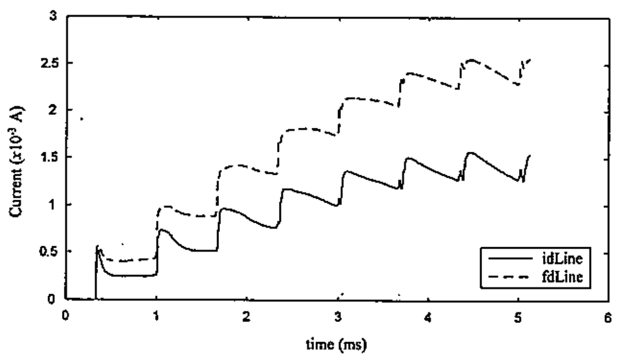

Fig. 14. Case D. Current at receiving end. Phase 9.

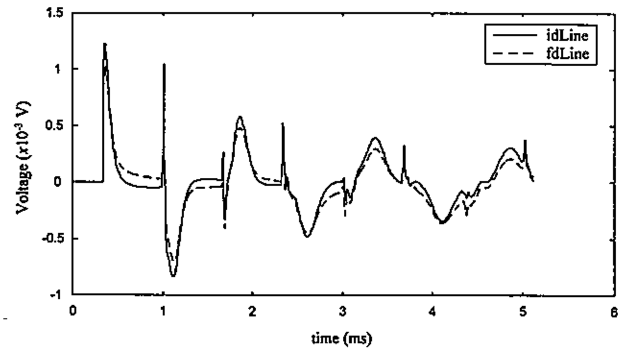

Fig. 15. Case E. Voltage at receiving end. Phase 7.

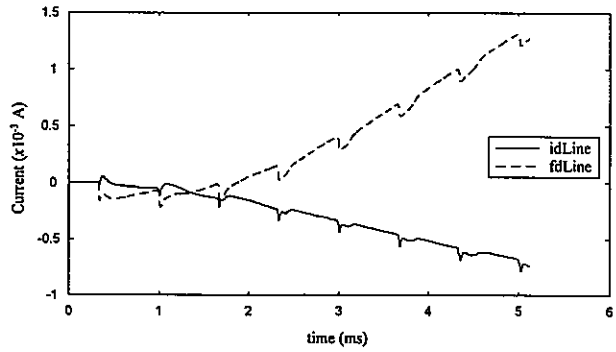

Fig. 16. Case E. Current at receiving end. Phase 7.

V. ANALYSIS OF THE RESULTS

A. Steady State Tests

From the results in Table 1, is it observed that the idLine model is more accurate than fdLine, especially when the configuration of the line is more unbalanced, or when the number of phases is larger. In interpreting these results, it is important to note, however, that the largest errors appear for the smallest currents. (Currents with magnitude less than 1% are ignored in the comparison.)

The error range for the fdLine model is between 13.05% to 432.31% depending on the line configuration, while the error range for the idLine model is between 1.74% to 18.50%.

It is important to point out that the model parameters used for the steady state simulations for the idLine and fdLine models are the same used for the transient tests.

B. Energization Tests

The new idLine model gives very good results compared with those obtained with the FDTP frequency dependent program. In the case of open circuit energizations, the fdLine also gave good results. However, for closed circuit energizations, the fdLine model results diverged from the results obtained with the FDTP program and idLine model. This is due to the fact that closed or short circuit tests strongly involve both the propagation matrix and the characteristic admittance part of the model, whereas open circuit tests tend to emphasize mostly the effect of the propagation matrix functions.

C. Solution Times.

Roughly, the number of operations required by a full frequency-dependent phase-domain transmission line model versus a constant transformation matrix frequency-dependent modal domain model is in the order of N^2 (N = number of phases). The technique of using common poles to fit the idempotent matrices and the characteristic admittance matrix used in idLine, however, considerably reduces the number of time domain convolutions. For the test cases presented in this paper, the timings ration between idLine and fdLine was of slightly less than $2N$. (It should be noted that the code of idLine is still at the development stage and, therefore, not as efficient as the code of fdLine.)

VI. CONCLUSIONS

The new idLine model gave better overall results for the studied cases when compared with those given by the fdLine model.

The idLine model gave very good results for both open and closed circuit energizations. The fdLine model, although being fairly accurate for open circuit energizations, gave some wrong results in the case of closed circuit energizations. This fact becomes more noticeable as the complexity of the system increases.

VII. ACKNOWLEDGMENT

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