Quasi-Modes Multiphase Transmission Line Model

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Abstract **- This article presents a new model to represent transmission lines including the frequency dependence of longitudinal parameters. The model uses the natural modes, for ideally transposed lines, and "quasi-modes" for non-transposed lines, and is applied to lines that have a vertical symmetry plane. The line is represented through π-circuits, with one π-circuit for each mode. The transformation matrix is modeled using ideal transformers. The model is described for three phase lines, dc lines, double three phase lines and six phase lines. A 440 kV three phase transmission line illustrates it and it is made a comparison with a frequency dependent EMTP line model, the Semlyen one.**

Keywords : Transmission line model, frequency dependence, transformation matrix, EMTP.

I. INTRODUCTION

One of the main difficulties when dealing with transient simulation studies in a digital simulator program like EMTP [1] is the correct representation of transmission lines. The EMTP works in the time domain and the line is generally represented by its phase quantities. Nevertheless, the transmission line parameters, namely the longitudinal parameters, varies with distance and frequency.

The former is represented through the hyperbolic function in the distributed parameter model or through π-circuits. With the last the line is represented by cascading the π-circuit, and some care should be taken with the π length. The number of cascaded elements can become very large if the line is too long.

To model the frequency dependence it is more complex. First, the impedance matrix varies with frequency, what means that there is one full matrix for each frequency. The impedance matrix is a full one due to the phase (and ground wire) coupling. As a program like EMTP works in time domain the frequency dependence of an element is not a straight model.

It is proposed then to work not with phase but with modes and therefore deal with diagonal rather than full matrix. In mode domain there is no coupling and the impedance frequency dependence can be properly represented with synthetic circuits. Nevertheless, there is the transformation matrix, that makes the link between phase and mode domain, which also varies with frequency.

The present model is exemplified for lines with a vertical symmetry plane, but for ideally transposed line there is no need of this symmetry plane. The transformation matrix is real and frequency independent, which allows its representation through ideal transformer in EMTP.

The theory is described for a three phase line, dc, double three phase and six phase line. A 440 kV three phase transmission line illustrates the model and it is made a comparison with a frequency dependent EMTP line model, the Semlyen method incorporated in EMTP [2].

II - TRANSMISSION LINE ELECTRICAL PARAMETERS CALCULATION

Once the transmission line configuration is established, it is possible to calculate its electrical longitudinal and transversal parameters, using the formulae for the electromagnetic behavior in the frequency range of 10 Hz to 1 MHz.

These parameters are calculated in phase domain. For each frequency there is a full longitudinal and transversal impedance matrix. Nevertheless, it is difficult to work with these full matrices and it is proposed to transform them in mode matrices, where the main characteristics of the line are easier to deal with.

For calculating the electrical parameters in phase domain it was used classical Carson's formulae [3]. It must be emphasized that the proposed method allows to consider other consistent formulations of the line parameters, namely considering soil electric permitivity, soil conductivity and permitivity frequency dependence, non uniform soil, e.g. with layers of different electric parameters.

III - MODE DOMAIN

Once the electrical parameters (longitudinal and transversal impedance) have been properly calculated in phase domain, the line can be represented to start the desired simulations. It is proposed then to work with mode components.

The transformation matrix is unique for each impedance matrix, which means that for a defined line there is one impedance matrix for each frequency, apart consequences of multiple eigenvalues, as it is the case of ideally transposed lines, and there is only one transformation matrix associated to each frequency.

This seems to make the mode determination very complex. It is usual to make some simplifications like assuming one single transformation matrix calculated for a chosen frequency and use it for the entire frequency range. With the vertical symmetry plane restriction there is a real and frequency independent matrix that separates exactly two groups of modes. For a three phase line, in one group there is the exact mode and in the other the two other modes. A real and frequency independent transformation matrix separates this group in quasi modes, that may be treated as a good approximation of the exact modes. For an ideally transposed line the quasi modes are exact modes, till if the line has no symmetry plane.

The transformation matrix is real and constant, frequency independent. It is a consequence of the line geometry and has no simplifications implied. As the transformation is a real one, and is constant, it can be modeled in a program like EMTP by using ideal transformers.

For three phase lines, Clarke's transformation [4] is applied and for double three phase and six phase lines first a media/antimedia (m/a) transformation, which sums and makes differences, is applied and then Clarke again is used [5]. For dc lines only m/a transformation is necessary.

The model is presented for the three phase line and then it is explained for the others lines. After obtaining the transformation matrix the modal impedance is calculated and the frequency dependence can easily be seen. This dependence is then synthesized with series and parallel resistor and inductors, as is shown in [3].

IV - THREE PHASE TRANSMISSION LINE MODEL

Suppose a three phase transmission line with the ground wire already reduced, as shown in Fig. 1.

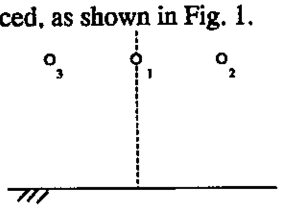

Figure 1 - Schematic representation of a single three phase line

In phase domain

$$-\frac{d|u|}{dx}=|Z|.|i| \quad (1) \qquad -\frac{d|i|}{dx}=|Y|.|u| \quad (2)$$

The impedance matrix, in phase component, is :

$$|Z_{ph}|=\begin{bmatrix} A & D & D \\ D & B & F \\ D & F & B \end{bmatrix} \quad (3)$$

Due to the vertical symmetry axis, Clarke's transformation can be applied and the currents in the conductors are divided as shown in Fig. 2, for each component :

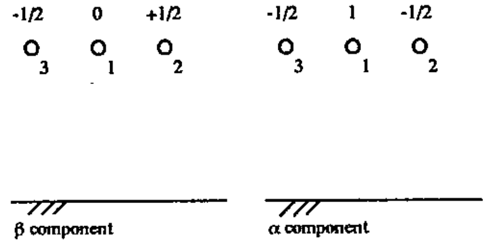

Figure 2 - Current in the conductors, for Clarke's components, in non rationalized form

The transformation matrix is

$$[T_{Cl}]=\begin{bmatrix} 2/\sqrt{6} & -1/\sqrt{6} & -1/\sqrt{6} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix} \quad (4)$$

Applying this matrix to current in phase there comes :

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix}=[T_{Cl}].\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (5)$$

and

$$i_{abc}=[T_{Cl}]^{-1}.i_{\alpha\beta 0} \quad (6)$$

Using these equations in (1)

$$-\frac{d}{dx}\left([T_{Cl}]^{-1}.u_{\alpha\beta 0}\right)=|Z_{ph}|.[T_{Cl}]^{-1}.i_{\alpha\beta 0} \quad (7)$$

$$-\frac{d}{dx}u_{\alpha\beta 0}=[T_{Cl}].|Z_{ph}|.[T_{Cl}]^{-1}.i_{\alpha\beta 0} \quad (8)$$

which makes

$$|Z_{\alpha\beta 0}|=[T_{Cl}].|Z_{ph}|.[T_{Cl}]^{-1} \quad (9)$$

and the same for the admittance matrix results in :

$$|Y_{\alpha\beta 0}|=[T_{Cl}].|Y_{ph}|.[T_{Cl}]^{-1} \quad (10)$$

Applying (9) in (3) there comes :

$$|Z_{\alpha\beta 0}|=\begin{bmatrix} z_\alpha & 0 & z_{\alpha 0} \\ 0 & z_\beta & 0 \\ z_{0\alpha} & 0 & z_0 \end{bmatrix} \quad (11)$$

where

$$z_\alpha=\frac{1}{3}(2A+B-4D+F) \quad (12)$$

$$z_\beta=B-F \quad (13)$$

$$z_{\alpha 0}=z_{0\alpha}=\frac{\sqrt{2}}{3}(A-B+D-F) \quad (14)$$

$$z_0=\frac{1}{3}(A+2B+4D+2F) \quad (15)$$

The β component is a real mode, as there is no coupling between it and the others. The same is not true for α and zero components. However, the mutual terms are formed by the difference of the impedance self terms and the difference of the impedance mutual terms.

For non transposed lines the self terms are almost the same, what is also true for the mutual terms, therefore it is a good assumption to discard this coupling term ($z_{\alpha 0}$) and treat α and zero components as real modes, or "quasi-modes".

If the line is transposed, then the self terms are equal and the mutual terms are also the same, which results in null coupling between α and zero components, that in this case are exact modes. For ideally transposed lines there are only two distinct modes, α equal β, and zero. Any linear combination of α-β modes are also modes, such as positive and negative components. The impedance matrix is :

$$\left|Z_{\alpha\beta 0}\right| = \begin{bmatrix} B-F & 0 & 0 \\ 0 & B-F & 0 \\ 0 & 0 & B+2F \end{bmatrix} \qquad (16)$$

The system represented in EMTP is generally in phase components, then there will be phase elements and mode elements, linked by the transformation matrix modeled through ideal transformers. The line is represented through cascade of π-circuits, one for each mode, as shown in Fig. 3.

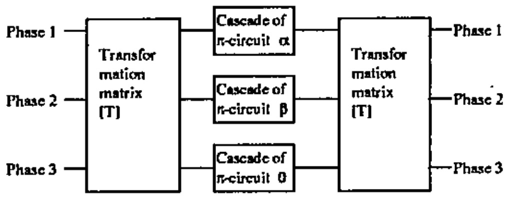

Figure 3 - Schematic representation of three phase line in EMTP

The transformation matrix is composed of ideal transformers that reproduce the relation between the phases and modes currents and voltages, as is presented in Figs.4-6.

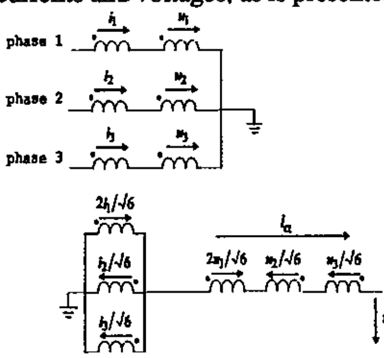

Figure 4 - Link between phases and mode α

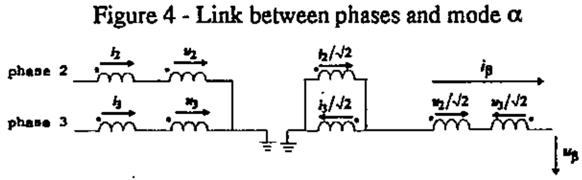

Figure 5 - Link between phases and mode β

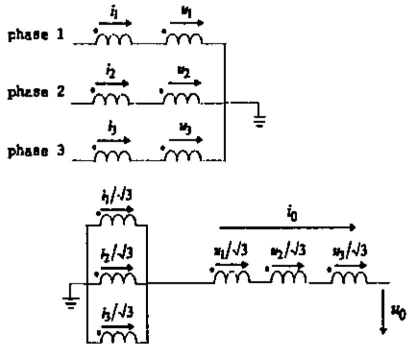

Figure 6 - Link between phases and mode 0

V - DC LINE MODEL

Suppose a dc line with the ground wire already reduced, as shown in Fig. 7.

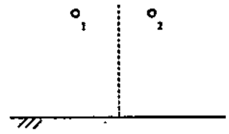

Figure 7 - Schematic representation of a dc line

For the dc line only the media/antimedia (m/a) transformation is necessary to diagonalize the impedance matrix. There is a vertical symmetry and the currents in m/a components can be described as :

$$i_m = \frac{1}{\sqrt{2}}\left(i_1 + i_2\right) \quad (17) \qquad i_a = \frac{1}{\sqrt{2}}\left(i_1 - i_2\right) \quad (18)$$

The impedance matrix in pole, or phase, components and in mode turns in:

$$\left|Z_{ph}\right| = \begin{bmatrix} A & D \\ D & A \end{bmatrix} \quad (19) \qquad \left|Z_{ma}\right| = \begin{bmatrix} z_m & 0 \\ 0 & z_a \end{bmatrix} \quad (20)$$

where

$$z_m = \left(A + D\right) \quad (21) \qquad z_a = \left(A - D\right) \quad (22)$$

Again there will be pole, or phase, and mode elements in the circuit, and the link between them is done by the transformation matrix represented through ideal transformers, in EMTP. The network is simulated in pole or phase domain, representing in the usual way the arresters, switches, sources and so on, while the transmission is represented in mode domain, with one cascade of π-circuit for each mode. They are uncoupled and it is possible to synthesize the frequency dependence through series and parallel resistors and inductors.

VI - DOUBLE THREE PHASE AND SIX PHASE LINE MODEL

6.1 - Double Three Phase Transmission Line

In Fig. 8 it is presented a schematic representation of a double three phase transmission line, with its ground wire already reduced. One line is formed by conductors 2 3 4 and the other by conductors 1 6 5. Again the vertical symmetry plane is respected.

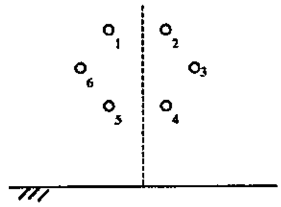

Figure 8 - Schematic representation of a double three phase transmission line

This line is first transformed in two uncoupled "media" and "antimedia" lines and then Clarke's transformation is applied on each new line. The currents in m/a components can be written as:

$$i_{m1} = \frac{1}{\sqrt{2}}(i_1 + i_2) \quad (23); \quad i_{a1} = \frac{1}{\sqrt{2}}(i_1 - i_2) \quad (26)$$

$$i_{m2} = \frac{1}{\sqrt{2}}(i_3 + i_6) \quad (24); \quad i_{a2} = \frac{1}{\sqrt{2}}(i_3 - i_6) \quad (27)$$

$$i_{m3} = \frac{1}{\sqrt{2}}(i_5 + i_4) \quad (25); \quad i_{a3} = \frac{1}{\sqrt{2}}(i_5 - i_4) \quad (28)$$

Note that the new currents are formed by sum and difference of opposite conductors' currents. The impedance matrix, in phase components, can be described as :

$$|Z_{ph}| = \begin{bmatrix} A & D & E & C & G & H \\ D & A & H & G & C & E \\ E & H & B & H & E & C \\ C & G & H & A & D & E \\ G & C & E & D & A & H \\ H & E & C & E & H & B \end{bmatrix} \quad (29)$$

The matrix can be divided in four sub-matrix, and could be rewritten as

$$|Z_{ph}| = \begin{bmatrix} [P] & [Q] \\ [Q] & [P] \end{bmatrix} \quad (30)$$

Applying the m/a transformation:

$$|Z_{ma}| = \begin{bmatrix} [Z_m] & [0] \\ [0] & [Z_a] \end{bmatrix} \quad (31)$$

where

$$|Z_m| = \begin{bmatrix} A+D & H+E & C+G \\ H+E & B+C & E+H \\ C+G & E+H & A+D \end{bmatrix} \quad (32)$$

$$|Z_a| = \begin{bmatrix} A-D & H-E & C-G \\ H-E & B-C & E-H \\ C-G & E-H & A-D \end{bmatrix} \quad (33)$$

The double three phase line has been separated in two uncoupled three phase lines, the "media" and the "antimedia" ones. This has been achieved due to the geometrical properties of the line, that has a vertical symmetry axis, and has nothing to do with line transposition.

Now to represent these "lines" as modes Clarke's transformation has to be applied.

Note that the media impedance matrix could be also written as in (34), which is similar to the three phase line impedance when there is the same vertical symmetry axis.

$$|Z_m| = \begin{bmatrix} B' & D' & F' \\ D' & A' & D' \\ F' & D' & B' \end{bmatrix} \quad (34)$$

Now the "central conductor" of the m/a line is the second one, and not the first one as in the three phase line. Some of Clarke's transformation rows and columns must be shifted.

Applying Clarke to the media impedance there comes :

$$|Zm_{\alpha\beta 0}| = \begin{bmatrix} z_{m\alpha} & 0 & 0 \\ 0 & z_{m\beta} & z_{m\beta 0} \\ 0 & z_{m0\beta} & z_{m0} \end{bmatrix} \quad (35)$$

where

$$z_{m\alpha} = B' - F' \quad (36)$$

$$z_{m\beta} = \frac{1}{3}(2A' + B' - 2D' + F') \quad (37)$$

$$z_{m\beta 0} = z_{m0\beta} = \frac{\sqrt{2}}{3}(A' - B' + D' - F') \quad (38)$$

$$z_{m0} = \frac{1}{3}(A' + 2B' + 4D' + 2F') \quad (39)$$

It can be seen that the component α is a real mode, while β and zero are "quasi-modes", with a coupling element as for the three phase line.

The same transformation can be applied for the antimedia impedance, that has the same characteristics as the media one. Similar $Za_{\alpha\beta 0}$ matrix will result.

If the double three phase line is transposed, then there is no mutual terms in the m/a impedance, and the components are real modes. There will be two pair of equal terms, the $m\alpha$ and $a\beta$, and the $a\alpha$ and $m\beta$. Any linear combination of them results in a mode, for instance : [6]

- six-phase direct and inverse coordinate are real modes, corresponding to the $a\alpha$ and $m\beta$.
- double three phase direct and inverse coordinate are real modes, corresponding to $m\alpha$ and $a\beta$.

For the non-transposed lines the mutual term can be discarded and the components can be treated as "quasi-modes". As only real and frequency independent linear transformation has been used, again the transformation matrix can be represented through ideal transformers. There will be phase domain elements and the transmission line can be modeled in mode domain, with a more accurate representation of the frequency dependence.

6.2 - Six Phase Transmission Line

The six phase transmission line can be modeled in the same way the double three phase line is. The tower configuration is also the same as the double three phase one, shown in Fig. 8. All the assumptions made for the previous line are valid, the six phase line is transformed in two uncoupled three phase line, "media" and "antimedia" ones, and then Clarke's transformation is used to obtain six uncoupled modes[5]. The vertical plane symmetry is also necessary.

VII - THREE PHASE LINE APPLICATION

In Fig. 9 it is presented the data of the three phase line used to illustrate the model.

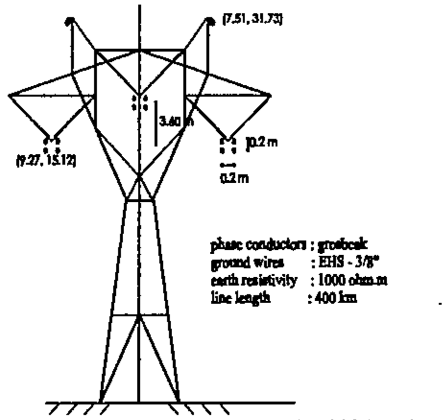

Figure 9 - Schematic representation of the 440 kV three phase line

First the line parameters were calculated in the range of 10 Hz to 10 kHz and then the Clarke's components were obtained, both for transposed and for non transposed line. Then the synthetic circuit was calculated and the line could be modeled in ATP, with cascade of π-circuits 10 km long for each mode.

The line was also represented using Semlyen internal EMTP model. Two tests were then applied :

a) a 1 V step of 1 ms of duration was inputted, for modes α, β and zero. The reception end was opened.
b) the line was energized at one single shot supposing the line opened in the end. The generation equivalent system used was :
   - base power : 170 MVA
   - X (generator + transformer) : 0,3618 pu
   - X/R : 11,4 ; X_1/X_0 : 4,41
   - $U_{generator}$: 0,95 pu

Some results for test a) are presented in Figs 10 to 12 for transposed and non transposed for both synthesized and Semlyen model. For both models, the modes alpha and beta are the same for the transposed lines, as expected. However, for the non transposed line, these modes are not the same. For phenomena affected by a large frequency spectrum, as it occurs with step response and most switching transients, these modes have a resultant similar behavior (although different). This similarity happens for quasi mode model, as it can be seen in Fig. 10, but does not happen for the Semlyen model (Fig. 11). Note that the zero mode step response is also different for the two models (Fig. 12).

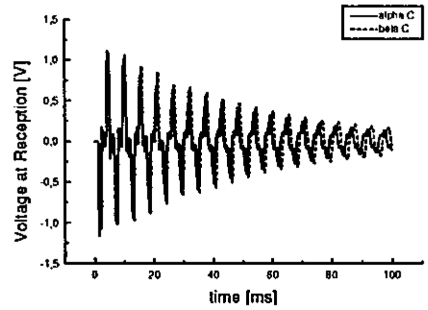

Figure 10 - Step response for mode α and β for Quasi mode model for non transposed line

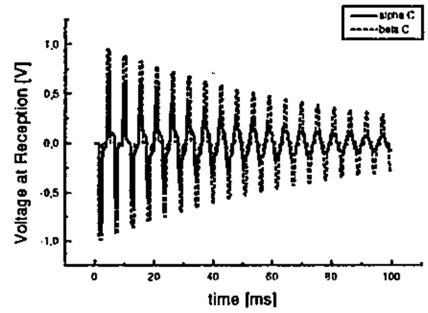

Figure 11 - Step response for mode α and β for Semlyen model for non transposed line

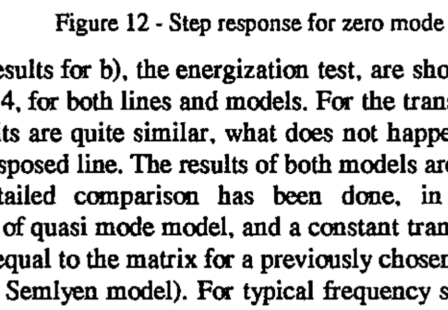

Figure 12 - Step response for zero mode

The results for b), the energization test, are shown in Figs 13 and 14, for both lines and models. For the transposed line the results are quite similar, what does not happen with the non transposed line. The results of both models are different.

A detailed comparison has been done, in frequency domain, of quasi mode model, and a constant transformation matrix, equal to the matrix for a previously chosen frequency (in ATP Semlyen model). For typical frequency spectrum of switching transients, the quasi mode method is a better average, avoiding the "amplification" of a particular frequency behavior, that can happen with a single frequency.

Of course, for some particular conditions, a frequency analysis may be justified, but, anyhow, avoiding one "a priori" and somehow arbitrary choice of a particular frequency.

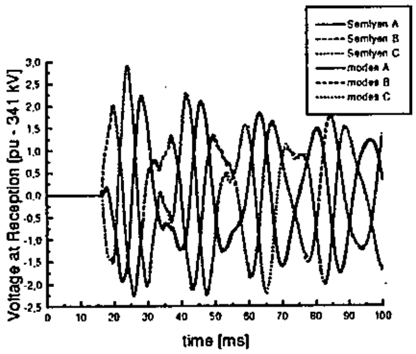

Figure 13 - Energization of the transposed line

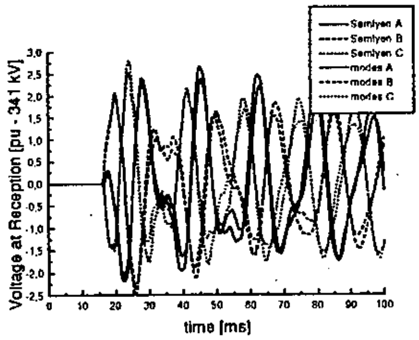

Figure 14 - Energization of the non transposed line

VIII - CONCLUSIONS

This paper presents a new model to represent transmission lines including the frequency dependence of longitudinal parameters. The model uses the natural modes, for ideally transposed lines, and "quasi-modes" for non-transposed lines, and is applied to lines that have a vertical symmetry plane. For ideally transposed three phase lines, even if this plane does not exist, the quasi modes are exact modes and the model can be applied.

The longitudinal impedance is represented in the mode domain through synthetic circuits, and the frequency dependence can be modeled with high accuracy. The line is represented through π-circuits, with one cascade of π-circuit for each mode.

The important contribution of this paper is the representation of the transformation matrix.

For double three phase lines, taking advantage of the line geometry, it is made a former transformation from phase to media/antimedia modes, uncoupling the phases in two groups, with a real, constant transformation matrix. Afterwards, Clarke's type transformation is applied and again the matrix is real and constant, what means that it does not vary with frequency.

For a single three phase line only Clarke transformation is necessary to obtain quasi modes for a non transposed line with a vertical symmetry plane or exact modes for ideally transposed lines, which does not have to respect this symmetry.

It was observed that for the non transposed line with a vertical symmetry plane, for typical switching transients with a large frequency spectrum, in most cases, the use of Clarke transformation matrix has a better result than the use of a single transformation matrix calculated for a single frequency chosen "a priori". This can be explained by the fact that Clarke's transformation matrices conduct to good averages, avoiding the amplification of a particular frequency behavior.

These real transformation matrices can be modeled in the EMTP using ideal transformers.

The model is described for three-phase lines, DC lines, double three-phase lines and six-phase lines.

A 440 kV three-phase transmission line illustrates the model and it is made a comparison with a frequency dependent EMTP line model, the Semlyen one. For the transposed line the two modes, α and β, are the same as could be seen with both models. But, for non transposed line, β is an exact mode and "α" a quasi mode. In examples presented, with quasi modes, the "alpha" and β quasi modes have different, but similar behavior. With Semlyen model, the difference between the two corresponding modes is much higher. The difference between homopolar mode, or zero sequence mode, in proposed model and in Semlyen model, for non transposed lines, is also important. In typical large frequency spectrum transient conditions, for several examples, with non transposed lines, results obtained, with proposed method and with Semlyen method, have shown important differences. For some examples analyzed with detail, the proposed method has shown some advantages.

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