Modelling of Lossy Ground Parameters in the EMTP for Very-Fast Transient Analysis

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Abstract — **A new procedure is developed for the modelling of frequency-dependent line parameters in the EMTP. The modal characteristic impedance matrix and the modal transferring matrix for unidirectional propagation are supplied by the JMARTI subroutine, which computes the ground impedance by using the Carson formulation. New correcting terms in pole-residue form are added to these modal matrices in order to define a line simulation model which is valid in a wide frequency-range, up to nearly 100 MHz, and/or in case of poorly conducting ground. The procedure is applied to the analysis of a three-conductor distribution power line above a lossy ground, both in frequency- and in time-domain. The sensitivity of the simulation model to the ground conductivity is investigated.**

Keywords : Fast Transient Analysis, Line Modelling, Lossy Ground, EMTP.

I. INTRODUCTION

The Electromagnetic Transient Program (EMTP) is a widely-used powerful tool for the time-domain analysis of transients in power networks, having whatever configuration. The network equations are formulated directly in the time-domain and solved by a step-by-step procedure. Therefore, a critical task concerns the analysis of transmission lines having frequency-dependent parameters, simulating skin effect and dissipative phenomena in the conductors and in the ground plane.

The JMARTI subroutine of the EMTP describes the frequency-dependent simulation model of multiconductor overhead lines [1]. In the most general case of untransposed multiconductor line-sections, the punched file of the JMARTI subroutine provides the pole-residue approximation of the modal parameters of the line, together with the coefficients of the modal transformation matrix for the currents and the asymptotic propagation delay. However, the line simulation model computed by the JMARTI subroutine is based on some simplifying hypotheses.

The most argued limitation concerns the assumption that the coefficients of the modal transformation matrices are independent of frequency and assume constant values in the whole frequency-range of interest. This approximation is critical in case of asymmetrical conductor configurations, including frequency-dependent parameters. Nevertheless, several efforts have been made recently in order to remove this limitation [2-4].

A further shortcoming of the frequency-dependent simulation model implemented in the JMARTI subroutine is related to the use of the Carson expression of the per-unit-length (p.u.l.) ground impedance of the line [5]. Actually, several studies have pointed out that the Carson formulation of the ground return parameters cannot be applied in case of very fast transients including frequencies up to 100 MHz and for poorly conducting ground. A rigorous procedure has been developed in order to provide new expressions of the p.u.l. ground impedance and admittance matrices, which are valid in a wide-frequency range and in case of high ground resistivity [6].

The new formulation allows the accurate simulation of the losses in the ground even when the displacement currents are not negligible with respect to the conductive ones. The relevance of the new model on the waveforms of the transient voltage and currents travelling along field-excited power distribution lines has been investigated [6]. A comparative analysis showed that in case of very fast transients due to EMP external source, the peak value and steepness of the line induced voltages computed by using the new model, are respectively 20% and 36% greater than the ones obtained by using the Carson approach, for 0.5 mS/m ground conductivity.

The aim of this paper consists in the implementation of the new ground simulation model in the EMTP. A computational procedure is then developed for the evaluation of the correcting terms to be added to the modal characteristic impedance matrix and the modal transferring matrix for unidirectional propagation provided by the JMARTI subroutine. The correcting terms are expressed in pole-residue form, so that they can be directly included in the EMTP input data file for the transient simulation. The proposed modal matrices can be used in a wide frequency range and/or for poorly conducting ground.

The procedure is then applied to the calculation of the simulation model of an overhead distribution power line for different values of the ground resistivity, both in the frequency- and in the time-domain.

II. LINE SIMULATION MODELS

A. EMTP simulation model for multiconductor lines with frequency-dependent parameters

According to Dommel's basic work, the time-domain solution kernel of the EMTP comes from D'Alambert's solution of the wave equations and Bergeron's model of constant relationship between voltage and current waves travelling along the line. In case of lossless multiconductor lines with constant parameters the solution of the line model can be expressed directly in the time-domain by applying the modal theory.

In the most general case of untransposed multiconductor lossy lines with frequency-dependent parameters, the time-domain solution of the line propagation equations, relating the vectors of the terminal voltages and currents, are expressed as functions of convolution integrals. Under the assumption of constant modal transformation matrices, these convolution products include the transient modal characteristic impedance matrix and the transient modal transmission matrix for unidirectional propagation [1]:

$$\mathbf{z}_{cm}(t) = F^{-1}\left[\mathbf{Z}_{cm}(\omega)\right] \quad , \quad \mathbf{a}_m(t) = F^{-1}\left[\mathbf{A}_m(\omega)\right] \qquad (1)$$

in which F^{1} represents the inverse Fourier transform operator, and the modal matrices $\mathbf{Z}_{cm}(\omega)$ and $\mathbf{A}_m(\omega)$ are defined by the following expressions in the frequency domain:

$$\mathbf{Z}_{cm}(\omega) = \mathbf{N}_t\,\mathbf{Z}_c(\omega)\,\mathbf{N} \quad , \quad \mathbf{A}_m(\omega) = \exp\left[-\mathbf{m}(\omega)\,l\right] \qquad (2)$$

In the previous expressions $\mathbf{Z}_c(\omega)$ is the characteristic impedance matrix of the line, $\mathbf{N}$ and $\mathbf{N}_t$ the current transformation matrix and its transposed, $\mathbf{m}(\omega)$ the diagonal matrix of the modal propagation constants of the line, l the line length.

The JMARTI subroutine of the EMTP provides the constant transformation matrix $\mathbf{N}$ and the pole-residue approximation of the coefficients of the diagonal matrices $\mathbf{Z}_{cm}(\omega)$ and $\mathbf{Q}_m(\omega)$. The j^{th} coefficient of $\mathbf{Q}_m(\omega)$ is the following:

$$Q_{m,j}(\omega) = \exp\left[-m_j(\omega)l\right]\exp\left(j\omega\tau_j\right) \qquad (3)$$

τ_j being the high frequency asymptotic propagation delay of the l-long line for the j^{th} propagation mode, and $m_j(\omega)$ the j^{th} modal propagation constant.

Since the modal quantities are computed by using Carson's formulation of the ground impedance, the JMARTI line simulation model is valid for the analysis of transients involving frequencies up to:

$$f_{max} << \frac{\sigma_g}{2\pi\varepsilon_g} \qquad (4)$$

σ_g and ε_g being the ground conductivity and permittivity.

B. EMTP implementation of the new ground simulation model

In the new simulation model of the line proposed in [6], the p.u.l. ground impedance and admittance matrices, reported in Appendix, are computed basing on a rigorous procedure which allows to remove the constraint (4). The obtained results have highlighted that the attenuation constant of the common propagation mode of the system reaches minimum value in the high frequency range. Moreover, the imaginary parts of the coefficients of the line characteristic impedance matrix assume negative values in the low-frequency range, when the ground medium behaves as a good conductor, but become positive at frequencies higher than 1-10 MHz, due to the prevailing dielectric properties of the ground in the high-frequency range.

In the low-frequency range the new formulations of the ground return parameters reduce to the Carson expressions. It should be noted that both methods predict the same asymptotic behaviour of the modal propagation constants and of the coefficients of the characteristic impedance matrix for frequency approaching infinity.

Actually, if $\hat{\mathbf{Z}}_c(\omega)$ and $\hat{\mathbf{Q}}_m(\omega)$ are the matrices computed by using the new formulation of the p.u.l. ground parameters described in Appendix, it results:

$$\lim_{\omega\to\infty}\mathbf{Z}_c(\omega) = \lim_{\omega\to\infty}\hat{\mathbf{Z}}_c(\omega) = \frac{\eta_0}{2\pi}\mathbf{\Lambda}^{-1} \qquad (5a)$$

$$\lim_{\omega\to\infty}\mathbf{Q}_m(\omega) = \lim_{\omega\to\infty}\hat{\mathbf{Q}}_m(\omega) = 0 \qquad (5b)$$

due to the prevailing effects of the wire lossess for ω approaching infinity. The coefficients of matrix $\mathbf{\Lambda}$ in (5a) are defined by (A.3c) in Appendix, whereas η_0 is the free space wave impedance.

The correcting terms $\Delta\mathbf{Z}_{cm}(\omega)$ and $\Delta\mathbf{Q}_m(\omega)$, to be added to the modal line-quantities supplied by the JMARTI subroutine of the EMTP, are expressed in the following form:

$$\Delta\mathbf{Z}_{cm}(\omega) = \hat{\mathbf{Z}}_{cm}(\omega) - \mathbf{Z}_{cm}(\omega) \qquad (6a)$$

$$\Delta\mathbf{Q}_m(\omega) = \hat{\mathbf{Q}}_m(\omega) - \mathbf{Q}_m(\omega) \qquad (6b)$$

The corresponding pole-residue representation, which is suitable for the direct implementation of the new model in the EMTP, is then computed. A numerical procedure is developed basing on the Model Based Parameter Estimation (MBPE) method [7], in order to evaluate the rational approximation of the coefficients of $\Delta\mathbf{Z}_{cm}(\omega)$, $\Delta\mathbf{Q}_m(\omega)$ in the frequency-domain. Actually, their frequency-spectra are characterized by finite number of resonance peaks, because they do not describe propagation phenomena. Considering that the corresponding transient quantities must be real-functions, an even number of real or complex conjugated poles is enforced to exist. Moreover, stability problems of the time-domain analysis procedure require that all the computed poles must be characterized by negative real part [1-4].

However, a preliminary study highlighted that numerical problems arise in the pole-residue approximation of the coefficients of matrix $\Delta\mathbf{Z}_{cm}(\omega)$, which are characterized by very broad frequency-spectra. The calculation is then performed considering the phase-domain correcting term $\Delta\mathbf{Z}_c(\omega)$ given by:

$$\Delta\mathbf{Z}_c(\omega) = \hat{\mathbf{Z}}_c(\omega) - \mathbf{Z}_c(\omega) \qquad (7)$$

Matrix $\Delta\mathbf{Z}_{cm}$ is finally obtained in the following form:

$$\Delta\mathbf{Z}_{cm}(\omega) = \mathbf{N}_t\,\Delta\mathbf{Z}_c(\omega)\mathbf{N} \qquad (8)$$

Assuming that in the previous expression N, N_t are frequency-independent modal transformation matrices, the correcting terms $\Delta\mathbf{Z}_c(\omega)$ and $\Delta\mathbf{Z}_{cm}(\omega)$ are characterized by same poles but different residues.

III. NUMERICAL RESULTS

The proposed procedure is applied with reference to the overhead three-conductor line having the geometrical configuration shown in Fig.1. The ground conductivity is σ_g = 10 mS/m and the relative ground permittivity ε_{rg} = 1. The length of the line is 1 km.

A. The characteristic impedance matrix

The characteristic impedance matrices $\hat{\mathbf{Z}}_c(\omega)$ and $\mathbf{Z}_c(\omega)$ are computed by using respectively the new and the Carson ground simulation model, taking into account the frequency-dependence of the transformation matrix N. The correcting matrix $\Delta\mathbf{Z}_c(\omega)$, given in (7), is then obtained.

The frequency spectra of the real and imaginary parts of the coefficient $\Delta Z_{c11}(\omega)$ are shown in Fig.2. It is pointed out that nearly overlapping waveforms are obtained for the other coefficients of matrix $\Delta\mathbf{Z}_c(\omega)$. The real part of $\Delta Z_{c11}(\omega)$ is characterized by a null value for the frequency approaching infinity, according to (5a). Therefore, the rational approximation is represented by a strictly proper function. The corresponding four pole expansion is computed by applying the MBPE procedure described in [7]. The obtained poles and residues are reported in Table I.

Successively, the frequency spectra of the real and imaginary parts of the coefficient (1,1) of the characteristic impedance matrix are calculated by using the EMTP. The thin lines in Figs.3a,b are given by the JMARTI subroutine,

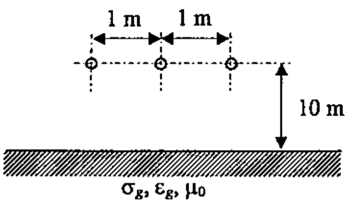


Fig.1 - Geometrical configuration of the overhead distribution line.

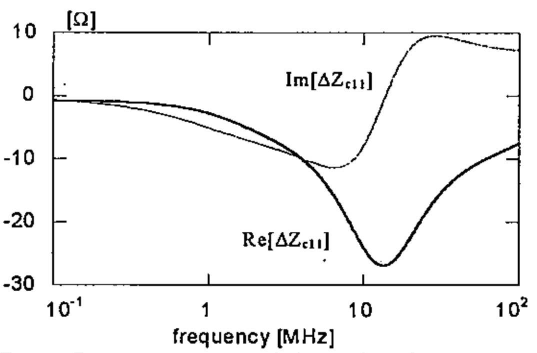


Fig.2 - Frequency-spectra of the real and imaginary parts of $\Delta Z_{c11}(\omega)$ for σ_g=10 mS/m.

TABLE I - *Poles and residues of $\Delta Z_{c11}(\omega)$ for different values of the ground conductivity.*

σ_g [mS/m]	*poles* [s^{-1}]	*residues* [Ω]
10	$-1.03\ 10^7$	$2.31\ 10^7$
	$-5.90\ 10^7+j5.71\ 10^7$	$-3.50\ 10^7-j4.02\ 10^8$
	$-5.90\ 10^7-j5.71\ 10^7$	$-3.50\ 10^7+j4.02\ 10^8$
	$-1.05\ 10^9$	$-3.31\ 10^9$
1	$-2.12\ 10^6$	$1.15\ 10^7$
	$-2.72\ 10^7$	$5.02\ 10^9$
	$-3.41\ 10^7$	$-6.04\ 10^9$
	$-4.77\ 10^8$	$-6.63\ 10^9$

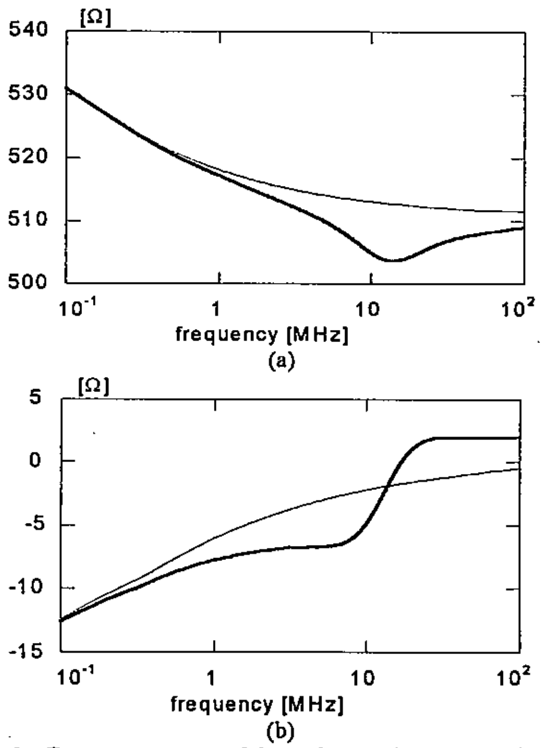


Fig.3 - Frequency-spectra of the real (a) and imaginary (b) parts of the coefficient (1,1) of the characteristic impedance matrix, computed by using the JMARTI subroutine (—) and by adding $\Delta Z_{c11}(\omega)$ (—), for σ_g=10 mS/m.

the thick curves are obtained by adding the contribution of the correcting term $\Delta Z_{c11}(\omega)$. Notice that the spectra are very close in the low frequency-range up to 1-2 MHz, where constraint (4) is satisfied. At the frequency of about 14MHz, the real part of the function computed by including the additional term reaches the minimum value of 503 Ω, whereas the imaginary part becomes positive, due to the resonant behaviour of the ground medium.

The corresponding transient waveforms, obtained by analytical inverse Fourier transform from the frequency-domain, are shown in Fig.4. Notice that the curves calculated by using the JMARTI subroutine and the proposed procedure become overlapping only for late times, greater than 50 ns. For very fast transients, when the time step used in the numerical calculation is smaller than 10 ns,

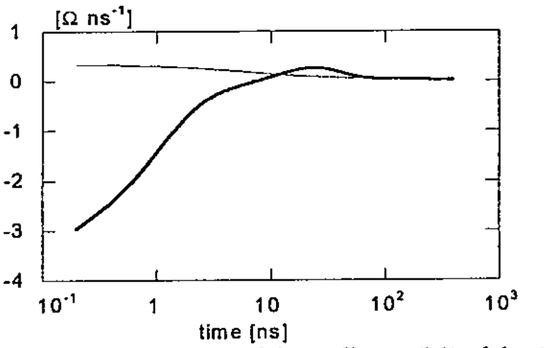


*Fig.4 - Transient waveforms of the coefficient (1,1) of the time-domain characteristic impedance matrix, computed by using the JMARTI subroutine (—) and by adding $\Delta z_{c11}(t)$ (**—**), for σ_g=10 mS/m.*

the two approaches lead to sensibly different results. In fact, according to the new ground simulation model, the attenuation constant of the common propagation mode reaches a minimum in the high frequency-range [6]. Therefore, the highest frequency-components exciting the system are less attenuated during the propagation along the line. As a result, the waveform obtained by adding the contribution of the correcting term is characterized by a negative value at early times and becomes positive after t = 7-8 ns.

Next, the calculation is repeated considering the ground conductivity of 1 mS/m. The curves in Figs.5a,b, representing the real and imaginary parts of the coefficient (1,1) of the characteristic impedance matrix, show that for increasing values of the ground resistivity the contribution of the correcting term $\Delta Z_{c11}(\omega)$ becomes important also in the lower frequency range.

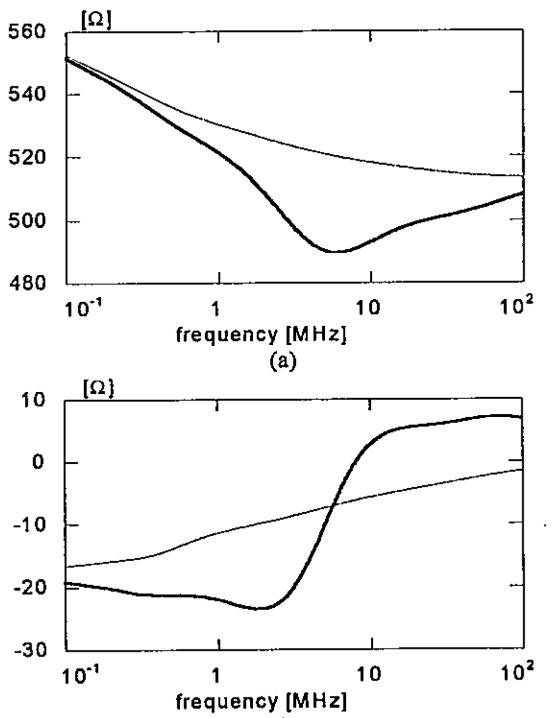


*Fig.5 - Frequency-spectra of the real (a) and imaginary (b) parts of the coefficient (1,1) of the characteristic impedance matrix, computed by using the JMARTI subroutine (—) and by adding $\Delta Z_{c11}(\omega)$ (**—**), for σ_g=1 mS/m.*

The corresponding transient waveforms are shown in Fig.6. In this case, the contribution of the correcting term $\Delta z_{c11}(t)$ cannot be neglected up to 150-200 ns. Actually, the tail of the transient curve is dominated by the poles having the smallest absolute value of the real part. Table I shows that for decreasing values of the ground conductivity the real part absolute value of the poles representing the rational approximation of function $\Delta Z_{c11}(\omega)$ decreases.

The modal characteristic impedance matrix is then computed by using the modal correcting term defined in (8). The corresponding transient expression is derived by analytical inverse Fourier transform from the pole-residue approximation. The waveforms of the coefficient (1,1) of $\mathbf{z}_{cm}(t)$, computed by the JMARTI subroutine and by adding the correcting term $\Delta z_{cm11}(t)$, are shown in Fig.7 for different values of the ground conductivity. The curves have the same shape of the waveforms in Figs.4,6 referring to the phase-domain representation, because for the considered line configuration the contribution of the common propagation mode is dominant.

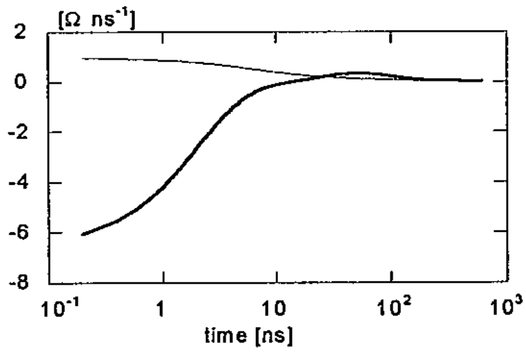


*Fig.6 - Transient waveforms of the coefficient (1,1) of the time-domain characteristic impedance matrix, computed by using the JMARTI subroutine (—) and by adding $\Delta z_{c11}(t)$ (**—**), for σ_g=1 mS/m.*

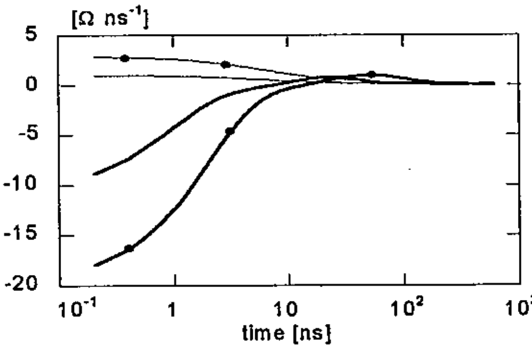


*Fig.7 - Transient waveforms of the coefficient (1,1) of the time-domain modal characteristic impedance matrix, computed by using the JMARTI subroutine (—) and by adding $\Delta z_{cm11}(t)$ (**—**), for σ_g=10 mS/m and σ_g=1 mS/m (•).*

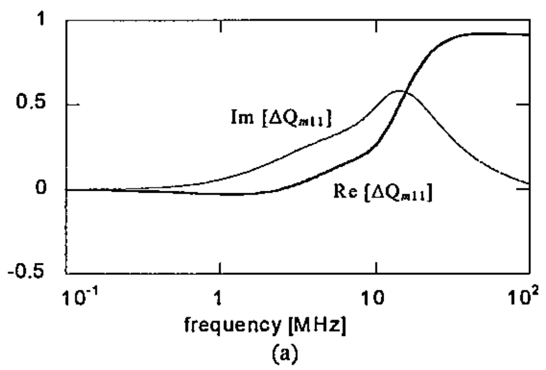
B. The modal Q-matrix

The correcting term $\Delta\mathbf{Q}_m(\omega)$ in (6b) is obtained as difference of matrices $\hat{\mathbf{Q}}_m(\omega)$ and $\mathbf{Q}_m(\omega)$, computed by using the new and the Carson ground models, considering frequency-dependent modal transformation matrices. The frequency-spectra of the real and imaginary parts of the coefficients $\Delta Q_{m11}(\omega)$, $\Delta Q_{m22}(\omega)$ are then obtained for the ground conductivity of 10 mS/m and shown in Figs.8a,b.

The real part of the correcting term $\Delta Q_{m11}(\omega)$ reaches the maximum value of about 0.9 at the frequency of 50-70 MHz; then it decreases to the asymptotic null value for ω approaching infinity. A similar trend characterizes the real part frequency spectrum of $\Delta Q_{m22}(\omega)$, which assumes the maximum value of 0.1 at very high frequency and then decreases to zero. In any case in the frequency range up to 100 MHz, the contribution of both the coefficients $\Delta Q_{m22}(\omega)$ and $\Delta Q_{m33}(\omega)$ is negligible. Actually, they are related to the propagation constants of the differential modes of the system which are weakly affected by the ground return parameters. The computed poles and residues of $\Delta Q_{m11}(\omega)$, $\Delta Q_{m22}(\omega)$, are reported in Table II.

The proper coefficient (1,1) of matrix $\mathbf{Q}_m(\omega)$ is then evaluated by using the JMARTI subroutine and by adding the correcting term $\Delta Q_{m11}(\omega)$, for the ground conductivity of 10 mS/m and 1 mS/m. A comparative analysis of the computed frequency spectra reveals a good agreement between the results of the two methods in the low frequency range, up to about 5 MHz for σ_g = 10 mS/m (Fig.9a) or up to 0.7 MHz for σ_g = 1 mS/m (Fig.9b).

Finally, the corresponding transient coefficients are



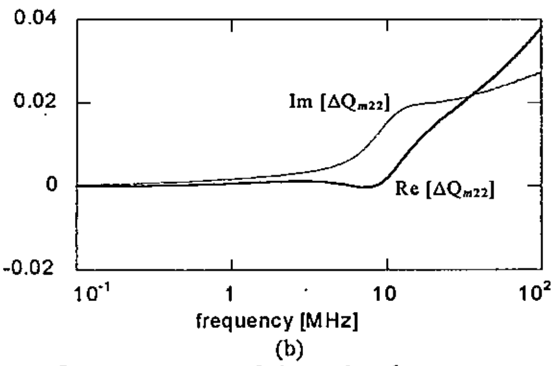
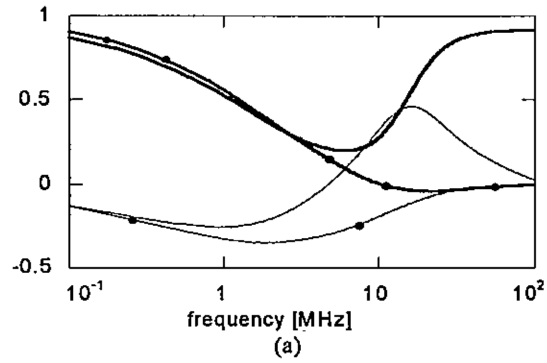


Fig.8 - Frequency-spectra of the real and imaginary parts of $\Delta Q_{m11}(\omega)$ (a) and $\Delta Q_{m22}(\omega)$ (b) for σ_g=10 mS/m.

TABLE II -*Poles and residues of $\Delta Q_{m11}(\omega)$ and $\Delta Q_{m22}(\omega)$ for ground conductivity σ_g = 10 mS/m.*

	poles [s^{-1}]	residues
ΔQ_{m11}	$-4.25\ 10^7$	$-9.51\ 10^6$
	$-7.45\ 10^7+j\,6.91\ 10^7$	$-2.24\ 10^7+j\,3.01\ 10^7$
	$-7.45\ 10^7-j\,6.91\ 10^7$	$-2.24\ 10^7-j\,3.01\ 10^7$
	$-8.87\ 10^9$	$8.18\ 10^9$
ΔQ_{m22}	$-1.10\ 10^8$	$-2.43\ 10^6$
	$-5.85\ 10^8$	$-1.43\ 10^7$
	$-2.43\ 10^9$	$-1.21\ 10^8$
	$-8.03\ 10^{10}$	$7.79\ 10^9$



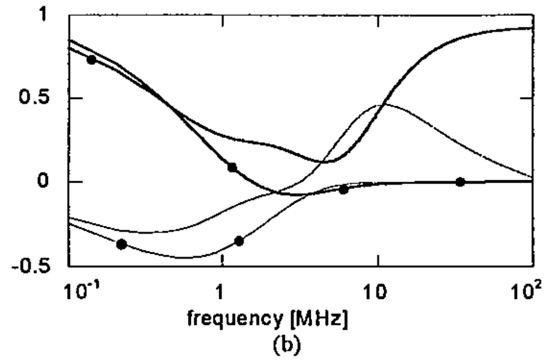


Fig.9 - Frequency-spectra of the real (—) and imaginary (—) parts of the coefficient (1,1) of matrix $Q_m(\omega)$, computed by using the JMARTI subroutine (•) and by adding the term $\Delta Q_{m11}(\omega)$, for σ_g=10 mS/m (a) and σ_g=1 mS/m (b).

computed by analytical inverse Fourier transform. Fig.10 shows that for σ_g = 10 mS/m the two waveforms become overlapping at about 50 ns. For lower ground conductivity (σ_g=1 mS/m), the results of the two approaches are nearly coincident only after 150-200 ns. Moreover, it is interesting to notice that at early times, that means up to 10 ns, the transient curves computed by using the new procedure are weakly affected by the ground conductivity. In fact, the early transient is related to the high frequency behaviour of the system. At frequencies $f \gg \sigma_g/(2\pi\varepsilon_g)$, when the displacement currents in the ground medium prevail on the conductive ones, the propagation characteristics of the system are mainly dependent on the dielectric properties of the earth, rather than on his conductivity.

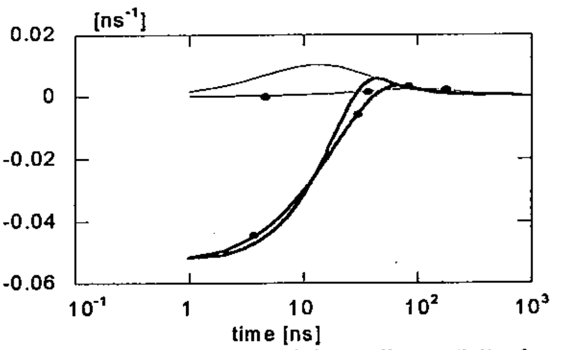


Fig.10 - Transient waveforms of the coefficient (1,1) of matrix $q_m(t)$, computed by using the JMARTI subroutine (—) and by adding the term $\Delta q_{m11}(t)$ (—), for σ_g=10 mS/m and σ_g=1 mS/m (•).

IV. CONCLUSIONS

A calculation procedure is developed for the modelling, in the EMTP, of dissipative multiconductor lines with frequency-dependent parameters in a wide-frequency-range and /or in case of poorly conducting ground. The proposed simulation model, leaving out of the Carson limitations, is based on the new expressions of the p.u.l. ground return parameters, which allows to predict the resonant behaviour of the ground medium in the high frequency range.

The pole/residue expansion of the matrix correcting terms, to be added to the output modal quantities of the JMARTI subroutine, are computed in order to implement the new model in the EMTP.

A sensitivity study is performed with the aim of assessing the relevance of the correcting terms both in the frequency- and in the time-domain, with reference to different values of the ground conductivity.

The obtained results confirm that the use of the new ground simulation model affects mainly the waveforms of the frequency-spectra of the propagation constant and characteristic impedance for the common-mode. Moreover, the coefficients of the characteristic impedance correcting term in the phase-domain are nearly identical.

The proposed procedure has the main advantage of being compatible with the JMARTI subroutine of the EMTP, in the sense that the coefficients of the correcting terms are particularly suitable to be sintetized in rational form. Actually, they represent the resonant phenomenon occurring in the ground medium, which is not accounted for in the Carson model.

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APPENDIX

The p.u.l. series impedance and shunt admittance matrices are defined by the following expressions:

$$\mathbf{Z}' = \mathbf{Z}'_i + \mathbf{Z}'_e + \mathbf{Z}'_g \quad \text{(A.1a)}$$

$$\mathbf{Y}' = \left(\mathbf{Y}'^{-1}_e + \mathbf{Y}'^{-1}_{gs}\right)^{-1} \quad \text{or} \quad \mathbf{Y}' = \mathbf{Y}'_e + \mathbf{Y}'_{gp} \quad \text{(A.1b)}$$

in which $\mathbf{Z}'_i$ is the p.u.l. wire-inner impedance matrix, $\mathbf{Z}'_e$ and $\mathbf{Y}'_e$ the p.u.l. external impedance and admittance matrices.

The p.u.l. ground impedance matrix $\mathbf{Z}'_g$ and the p.u.l. admittance matrices $\mathbf{Y}'_{gs}$, $\mathbf{Y}'_{gp}$ for the series and parallel representation are given respectively by [6]:

$$\mathbf{Z}'_g = \frac{j\omega\mu_0}{\pi}\left(\mathbf{F}_1 - \mathbf{F}_3\right) \quad \text{(A.2a)}$$

$$\mathbf{Y}'_{gs} = j\omega\varepsilon_0\,\pi\left(\mathbf{F}_2 - \mathbf{F}_3\right)^{-1} \quad \text{(A.2b)}$$

$$\mathbf{Y}'_{gp} = j\omega\varepsilon_0 4\pi\Lambda^{-1}\left(\mathbf{F}_3 - \mathbf{F}_2\right)\left[\Lambda - 2\left(\mathbf{F}_3 - \mathbf{F}_2\right)\right]^{-1} \quad \text{(A.2c)}$$

in which the generic coefficients of matrices $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$ and Λ are the following:

$$F_{1ij} = \frac{1}{2}\ln\left(1+\xi_1/D_{ij}\right) \quad , \quad F_{2ij} = \xi_2\ln\left(1+\xi_3/D_{ij}\right) \quad \text{(A.3a)}$$

$$F_{3ij} = \xi_2\ln\left\{1+\xi_3\left[h_i^2+\Delta_{ij}^2\right]^{-1/2}\right\} \quad \text{(A.3b)}$$

$$\Lambda_{ii} = \ln\frac{2h_i}{a_i} \quad , \quad \Lambda_{ij} = \ln\frac{D_{ij}}{d_{ij}} \quad \text{(A.3c)}$$

with:

$$\xi_1 = \frac{2}{\left(k_0^2 - k_g^2\right)^{1/2}} \quad , \quad \xi_2 = \frac{k_0^2}{k_0^2 + k_g^2} \quad , \quad \xi_3 = \frac{\xi_1}{2\xi_2} \quad \text{(A.4a)}$$

$$D_{ij}^2 = \left(h_i + h_j\right)^2 + \Delta_{ij}^2 \quad , \quad d_{ij}^2 = \left(h_i - h_j\right)^2 + \Delta_{ij}^2 \quad \text{(A.4b)}$$

$$k_0 = \omega\left(\mu_0\varepsilon_0\right)^{1/2} \quad , \quad k_g = k_0\left[\varepsilon_{rg} + \sigma_g\left(j\omega\varepsilon_0\right)^{-1}\right]^{1/2} \quad \text{(A.4c)}$$

In the previous expressions a_i, h_i are the radius and height of the i^{th} wire and Δ_{ij} the horizontal distance between conductors i and j.