Computation of Power Systems Transients by Using Sets of Algebraic and of State Space Equations

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Abstract – **This paper presents a method for time-domain transient computation based on the state space equations for the power system elements and a modified algebraic nodal equation for the elements connecting. The well known line models with distributed and frequency-dependent parameters can be incorporated easily in the algorithm. Non-linear characteristics of the elements and time step size changes do not lead to additional iterations in the solution process.**

Keywords: Transient Analysis, Power System Modeling

I. INTRODUCTION

Usually, transients in power systems are computed on the basis of a system of linearized algebraic equations. These linearized algebraic equations are normally derived from the differential equations of the network branches by applying the trapezoidal rule. The method has gained universal acceptance under the name Difference Conductance Method (DCM) [1].

The advantage of modeling with algebraic equations can be seen primarily in a simple algorithm for the connection of the branch equations to the network nodal equations without using any topological matrices. On the other hand time consuming iteration steps are required if nonlinearities are considered and/or network structure changes (due to faults) or changes in the time step size occur.

The power system modeling with state space equations or algebraic-differential equation systems are other solution methods that can be used in place of the linearized algebraic equation system [2]. Both the formulation and solution of the complete system of state space equations are sophisticated and require topological aids (i.e. definition of trees). Because the eigenspectrum of the system state matrices is very wide, the solution of the complete state space equation system requires time consuming implicit integration methods. Changes in the network topology, due to faults, as well as certain non-linear characteristics of the elements, make a new formulation of the state space equations necessary.

This paper develops a new power system modeling method based on both a set of linear algebraic equations and a set of state space ones. From the algebraic equations the nodal voltages are obtained at each step in a similar manner as in the DCM algorithm. The advantage of the Kirchhoff's node law for the coupling of the elements is used, too. The state space equations of the elements keep their form and are not linearized. They are solved separately by a conform integration rule and time step size for each type of elements. This has further advantages as the multi time-scale character of the power system [3] can be used for an effective solving process, and the model yields a parallel structure. Elements with non-linear characteristics, such as electrical machines, can be integrated by explicit integration rules avoiding iteration steps.

II. MODELING OF POWER SYSTEM ELEMENTS

The power system elements (PSE) are normally modeled by equivalent circuits consisting of combinations of resistive, inductive and capacitive branches as well as voltage or current sources. For lines, special models with distributed parameters taking into account their frequency dependence have been developed [4].

The PSE models can generally be divided into models with lumped parameters (LPM) and those with distributed parameters (DPM), as shown in Fig. 1.

The LPMs consist of R-, L- and C-elements and voltage or current sources. According to terminal characteristics the LPMs can further be classified as inductive models (LM), capacitive models (CM) and resistive models (RM). A LM begins with an inductivity seen from the terminals. Therefore the terminal voltages are independent variables (or input variables) and the terminal currents are state variables. CMs are characterized by a capacity as the first element at the terminals. Consequently the terminal currents are input variables and the terminal voltages become state variables. RMs begin with a resistivity and have no terminal state variables. Input variables can be the terminal currents or voltages. In the following, the terminal currents are preferred as input variables for RMs.

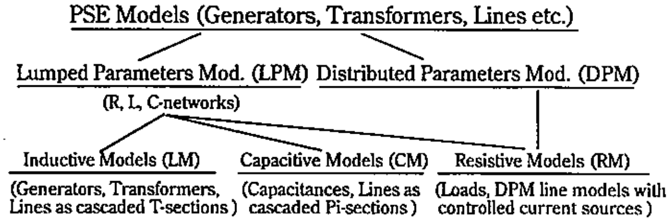

Fig. 1 Classification of PSE models

The general implicit form of the state space equations for the LMs and CMs in matrix notation is:

$$\begin{bmatrix} F_{ee} & F_{ei} \\ F_{ie} & F_{ii} \end{bmatrix}\begin{bmatrix} \dot{x}_e \\ \dot{x}_i \end{bmatrix} + \begin{bmatrix} H_{ee} & H_{ei} \\ H_{ie} & H_{ii} \end{bmatrix}\begin{bmatrix} x_e \\ x_i \end{bmatrix} = \begin{bmatrix} y_e \\ y_i \end{bmatrix} \quad (1)$$

Equation (1) is partitioned in terminal or external (e) and internal (i) state and input variables.

From (1) only the external variables x_e and y_e are needed for the connection with other elements. Therefore it is useful to write the first row of (1) in the following form:

$$F_{ee}\dot{x}_e + H_{ee}x_e + q_e = y_e \quad (2)$$

in which the influence of the internal state variables is expressed by a controlled source vector

$$q_e = F_{ei}\dot{x}_i + H_{ei}x_i \quad (3)$$

From the Tab. 1 we can see the specific form of (2) for the three kinds of LPMs. Fig. 2 shows the corresponding equivalent circuits with the typical elements seen from the terminals and controlled sources. Note that each inductivity in the LM-circuit has a resistivity in series and each capacity in the CM-circuit is in parallel with a conductance.

TABLE I
Meaning of LPM Terminal Quantities

Model	$\dot{x}_e$	x_e	y_e	q_e	F_{ee}	H_{ee}
Capacitive (CM)	$\dot{u}_e$	u_e	i_e	i_{qCe}	C_{ee}	G_{Cee}
Inductive (LM)	$\dot{i}_e$	i_e	u_e	u_{qe}	L_{ee}	R_{ee}
Resistive (RM)	-	u_e	i_e	i_{qRe}	-	G_{Ree}

In original abc-coordinates all conductors are coupled so that R_{ee}, L_{ee}, G_{Cee}, G_{Ree} and C_{ee} are usually full 3x3 matrices.

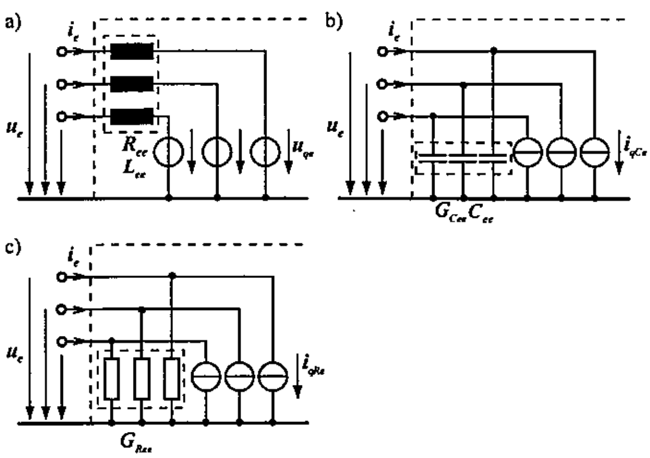

Fig.2 Three-phase equivalent circuits of a) LM b) CM and c) RM

Example 1. Inductive LPM for a synchronous generator

For a generator being symmetrical in the subtransient state ($L_q''=L_d''=L''$), (1) has the following detailed form in phase-coordinates:

$$\begin{bmatrix} L_s & L_g & L_g \\ L_g & L_s & L_g \\ L_g & L_g & L_s \end{bmatrix}\begin{bmatrix} \dot{i}_a \\ \dot{i}_b \\ \dot{i}_c \end{bmatrix} + \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} u_M \\ u_M \\ u_M \end{bmatrix} + \begin{bmatrix} u_a'' \\ u_b'' \\ u_c'' \end{bmatrix} = \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} \quad (4)$$

Substituting

$$u_M = R_M i_M + L_M \dot{i}_M = R_M(i_a + i_b + i_c) + L_M(\dot{i}_a + \dot{i}_b + \dot{i}_c) \quad (5)$$

in (4) we get:

$$\begin{bmatrix} L_m & L_n & L_n \\ L_n & L_m & L_n \\ L_n & L_n & L_m \end{bmatrix}\begin{bmatrix} \dot{i}_a \\ \dot{i}_b \\ \dot{i}_c \end{bmatrix} + \begin{bmatrix} R_m & R_n & R_n \\ R_n & R_m & R_n \\ R_n & R_n & R_m \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} u_a'' \\ u_b'' \\ u_c'' \end{bmatrix} = \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} \quad (6)$$

or in matrix notation, in agreement with (2):

$$L_{ee}\dot{i}_e + R_{ee}i_e + u_{qe} = u_e \quad (6a)$$

The matrix elements and the equation for the computation of the subtransient voltages as a function of the internal state variables are given in the appendix. In this equation all nonlinearities of the generator are included. The special case that the neutral point is not grounded will be discussed below (see example 2).

For the connection of the elements the explicit state space equation form of the inductive LPMs is necessary. To obtain it, (2) written in special terms of LMs is multiplied from the left by L_{ee}^{-1} getting:

$$\dot{i}_e = -L_{ee}^{-1}R_{ee}i_e - L_{ee}^{-1}u_{qe} + L_{ee}^{-1}u_e = A_{ee}i_e - B_{ee}u_{qe} + B_{ee}u_e \quad (7)$$

After expanding with $1/\omega_0$ on the left and rigth side, (7) is changing to a node equation again:

$$\frac{1}{\omega_0}\dot{i}_e = i'_e = Y_{ee}u_e + i_{qLe} \quad (8)$$

with:

$$Y_{ee} = \frac{1}{\omega_0}B_{ee} = \frac{1}{\omega_0}L_{ee}^{-1} \quad (9)$$

and:

$$i_{qLe} = \frac{1}{\omega_0}A_{ee}i_e - Y_{ee}u_{qe} \quad (10)$$

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Example 2. Explicit form of generator LM

Starting from (6) the diagonal elements of $\boldsymbol{Y}_{ee}$ become:

$$Y_m = \frac{1}{\omega_0} B_m = \frac{1}{3\omega_0}\left(\frac{2}{L_1} + \frac{1}{L_0'}\right) = \frac{1}{3}\left(\frac{2}{X_1} + \frac{1}{X_0'}\right) = \frac{1}{3}(2Y_1 + Y_0') \quad (11)$$

The off-diagonal elements of $\boldsymbol{Y}_{ee}$ are:

$$Y_n = \frac{1}{\omega_0} B_n = -\frac{1}{3\omega_0}\left(\frac{1}{L_1} - \frac{1}{L_0'}\right) = -\frac{1}{3}\left(\frac{1}{X_1} - \frac{1}{X_0'}\right) = -\frac{1}{3}(Y_1 - Y_0') \quad (12)$$

For the elements of $\boldsymbol{A}_{ee}$ we find:

$$A_m = -\frac{1}{3}\omega_0(2Y_1R_1 + Y_0'R_0'), \quad A_n = \frac{1}{3}\omega_0(Y_1R_1 - Y_0'R_0') \quad (13)$$

If the neutral-point is not connected with the earth the elements L_M and R_M are infinite and the matrices $\boldsymbol{L}_{ee}$ and $\boldsymbol{R}_{ee}$ becomes irregular. In this case, we assume first that $L_M \neq \infty$ and obtain the above given expressions for $\boldsymbol{A}_{ee}$ and $\boldsymbol{Y}_{ee}$. Then we set $Y_0 = 0$ obtaining the matrices $\boldsymbol{A}'_{ee}$ and $\boldsymbol{Y}'_{ee}$ with the elements:

$$Y_m' = \frac{2}{3}Y_1, \quad Y_n' = -\frac{1}{3}Y_1, \quad A_m' = -\frac{2}{3}\omega_0 Y_1 R_1, \quad A_n' = \frac{1}{3}\omega_0 Y_1 R_1 \quad (14)$$

From (14) we can see, that the $\boldsymbol{A}'_{ee}$ and $\boldsymbol{Y}'_{ee}$ become also irregular. This means, that concerning $i_a + i_b + i_c = 0$, only two currents are state variables, and $\boldsymbol{A}'_{ee}$ has a zero-eigenvalue. The singularity of $\boldsymbol{A}'_{ee}$ and $\boldsymbol{Y}'_{ee}$ does not create difficulties in the solution process of the explicite state space equation. Only one of the three differential equations is integrated unnecessary.

For the next steps the element equations are summarized in the following matrix form:

$$\begin{bmatrix} \boldsymbol{i}_L' \\ \boldsymbol{i}_R \\ \boldsymbol{i}_C \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_L & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{G}_R & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{G}_C \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_L \\ \boldsymbol{u}_R \\ \boldsymbol{u}_C \end{bmatrix} + \begin{bmatrix} \boldsymbol{i}_{qL} \\ \boldsymbol{i}_{qR} \\ \boldsymbol{C}\dot{\boldsymbol{u}}_C \end{bmatrix} \quad (15)$$

III. THE ALGEBRAIC NODE EQUATIONS

According to the kinds of elements which are connected at the corresponding node we define 3 different types of nodes. The node designations are obvious from Tab.2.

TABLE II
Designation of the Node Types

Node Type	Elements at the Node
Inductive (LN)	LM only
Resistive (RN)	RM only or RM and LM
Capacitive (CN)	CM only or CM and RM and/or LM

So the designation LN means, only inductive elements are connected with this node type. At a RN, both the resistive and inductive elements or only resistive elements are connected. The CN is the common node which connects all 3 kinds of elements. The special cases, only CMs or only CMs and RMs or only CMs and LMs are allowed.

For the connection of the elements at the various types of nodes, Kirchhoff's current law is written by means of the nodal matrix (basic incidence matrix) in the following partitioned form:

$$\begin{array}{c} \\ \text{LN} \\ \text{LR} \\ \text{LC} \end{array} \begin{array}{c} \begin{array}{ccc} \text{LM} & \text{RM} & \text{CM} \end{array} \\ \begin{bmatrix} \boldsymbol{K}_{LL} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{K}_{RL} & \boldsymbol{K}_{RR} & \boldsymbol{0} \\ \boldsymbol{K}_{CL} & \boldsymbol{K}_{CR} & \boldsymbol{K}_{CC} \end{bmatrix} \end{array} \begin{bmatrix} \boldsymbol{i}_L \\ \boldsymbol{i}_R \\ \boldsymbol{i}_C \end{bmatrix} = \mathbf{o} \quad (16)$$

The zero sub-matrices in the first row of the nodal matrix result from the fact that only LMs are connected at a LN. The zero sub-matrix in the second line indicates that no CMs exist at a RN.

Equation (16) is transformed to:

$$\begin{bmatrix} \boldsymbol{K}_{LL} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{K}_{RR} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{K}_{CR} & \boldsymbol{K}_{CC} \end{bmatrix} \begin{bmatrix} \boldsymbol{i}_L \\ \boldsymbol{i}_R \\ \boldsymbol{i}_C \end{bmatrix} = \begin{bmatrix} \mathbf{o} \\ -\boldsymbol{K}_{RL}\boldsymbol{i}_L \\ -\boldsymbol{K}_{CL}\boldsymbol{i}_L \end{bmatrix} \quad (17)$$

with the modified nodal matrix $\boldsymbol{K}'$.

Now we differentiate the first row of (17) noting that the currents of the LMs are state variables and therefore differentiable. After multiplying the first row by $1/\omega_0$ we get:

$$\begin{bmatrix} \boldsymbol{K}_{LL} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{K}_{RR} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{K}_{CR} & \boldsymbol{K}_{CC} \end{bmatrix} \begin{bmatrix} \boldsymbol{i}_L' \\ \boldsymbol{i}_R \\ \boldsymbol{i}_C \end{bmatrix} = \begin{bmatrix} \mathbf{o} \\ -\boldsymbol{K}_{RL}\boldsymbol{i}_L \\ -\boldsymbol{K}_{CL}\boldsymbol{i}_L \end{bmatrix} \quad (18)$$

remember that (see (8)):

$$\boldsymbol{i}_L' = \frac{1}{\omega_0}\dot{\boldsymbol{i}}_L$$

The terminal voltages of the elements can be expressed in terms of the nodal voltages by means of the transposed nodal matrix $\boldsymbol{K}'$ from (16):

$$\begin{bmatrix} \boldsymbol{u}_L \\ \boldsymbol{u}_R \\ \boldsymbol{u}_C \end{bmatrix} = \begin{bmatrix} \boldsymbol{K}_{LL}^{\mathrm{T}} & \boldsymbol{K}_{RL}^{\mathrm{T}} & \boldsymbol{K}_{CL}^{\mathrm{T}} \\ \boldsymbol{0} & \boldsymbol{K}_{RR}^{\mathrm{T}} & \boldsymbol{K}_{CR}^{\mathrm{T}} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{K}_{CC}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_{NL} \\ \boldsymbol{u}_{NR} \\ \boldsymbol{u}_{NC} \end{bmatrix} \quad (19)$$

Substituting (19) in (15) and substituting the thus modified (15) in (18) yields:

$$\begin{bmatrix} K_{LL} & 0 & 0 \\ 0 & K_{RR} & 0 \\ 0 & K_{CR} & K_{CC} \end{bmatrix} \begin{bmatrix} Y_L & 0 & 0 \\ 0 & G_R & 0 \\ 0 & 0 & G_C \end{bmatrix} \begin{bmatrix} K_{LL}^T & K_{RL}^T & K_{CL}^T \\ 0 & K_{RR}^T & K_{CR}^T \\ 0 & 0 & K_{CC}^T \end{bmatrix} \begin{bmatrix} u_{NL} \\ u_{NR} \\ u_{NC} \end{bmatrix} =$$

$$= \begin{bmatrix} -K_{LL} i_{qL} \\ -K_{RL} i_L - K_{RR} i_{qR} \\ -K_{CL} i_L - K_{CR} i_{qR} - K_{CC} i_{qC} - K_{CC} C \dot{u}_C \end{bmatrix} \tag{20}$$

The multiplication of matrices is resulting in:

$$\begin{bmatrix} K_{LL} Y_L K_{LL}^T & K_{LL} Y_L K_{RL}^T & K_{LL} Y_L K_{CL}^T \\ 0 & K_{RR} G_R K_{RR}^T & K_{RR} G_R K_{CR}^T \\ 0 & K_{CR} G_R K_{RR}^T & K_{CC} G_C K_{CC}^T + K_{CR} G_R K_{CR}^T \end{bmatrix} \begin{bmatrix} u_{NL} \\ u_{NR} \\ u_{NC} \end{bmatrix} =$$

$$= \begin{bmatrix} -K_{LL} i_{qL} \\ -K_{RL} i_L - K_{RR} i_{qR} \\ -K_{CL} i_L - K_{CR} i_{qR} - K_{CC} i_{qC} - K_{CC} C \dot{u}_C \end{bmatrix} \tag{21}$$

or more briefly:

$$\left[\begin{array}{cc:c} Y_{LL} & Y_{LR} & Y_{LC} \\ 0 & G_{RR} & G_{RC} \\ \hdashline 0 & G_{CR} & G_{CC} \end{array}\right] \begin{bmatrix} u_{NL} \\ u_{NR} \\ \hdashline u_{NC} \end{bmatrix} = \begin{bmatrix} i_{NL} \\ i_{NR} \\ \hdashline i_{NC} \end{bmatrix} \tag{21a}$$

IV. SOLUTION PROCESS

The basic solution process is as follows (see also Fig.3). First, the YG-matrix of the algebraic equation (21) is set up directly from the network structure following to a great extent the standard algorithm for the formation of the admittance matrix in steady-state analysis. Since the voltages u_{NC} are state variables and therefore well-known at each time step, (21) can be reduced to:

$$\begin{bmatrix} Y_{LL} & Y_{LR} \\ 0 & G_{RR} \end{bmatrix} \begin{bmatrix} u_{NL} \\ u_{NR} \end{bmatrix} = \begin{bmatrix} i_{NL} \\ i_{NR} \end{bmatrix} - \begin{bmatrix} Y_{LC} \\ G_{RC} \end{bmatrix} u_{NC} \tag{22}$$

Next, (22) is solved using ordered triangular factorization and exploiting the matrix sparsity. The special form of the YG-matrix, which contains a upper zero sub-matrix, accelerates the factorization procedure. Knowing the node voltages u_{NL} and u_{NR}, the terminal voltages u_L and u_R of the inductive and resistive elements are also known from (19). Now the LMs and RMs currents are calculated from the first and second row of (15).

For solution of the inductive state space equations an explicit integration rule can be used. Finally, using currents known, the source values and the node voltages u_{NC} at the capacitive nodes are updated.

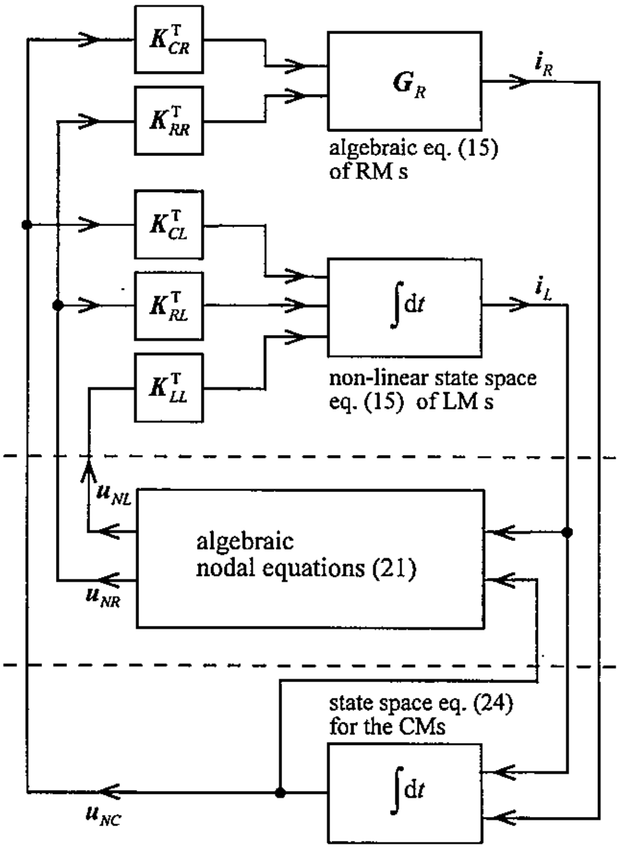

Fig.3 Structure of the set of algebraic and of state space equations

To calculate u_{NC}, the eliminated last row of (21) is available:

$$G_{CR} u_{NR} + G_{CC} u_{NC} = i_{NC} = -K_{CL} i_L - K_{CR} i_{qR} - K_{CC} i_{qC} - K_{CC} C \dot{u}_C \tag{23}$$

and with $\dot{u}_C = K_{CC}^T \dot{u}_{NC}$ according to (19):

$$C_{CC} \dot{u}_{NC} = -G_{CC} u_{NC} - K_{CL} i_L - K_{CR} i_{qR} - K_{CC} i_{qC} - G_{CR} u_{NR} \tag{24}$$

The matrix $C_{CC} = K_{CC} C K_{CC}^T$ is the capacitive nodal matrix. It is formed in the same way as the nodal admittance matrix. This is from the fact that in a pure capacitive network the node voltages form a state variable vector.

From Fig. 3 we can see the separate integration of the elements state space equations and the parallel structure of the model, too. As the element equations are kept in their conservative form, an object and parallel oriented computer program structure is indicated.

V. ONE BASIC EXAMPLE

As a simple example, let us consider the system shown in Fig. 4. The line 1 should be representated by a LPM and the line 2 by a DPM. Then the model consists of 4 LMs (E1, E2, E3 and E2), 2 RMs (E5 and E7) and 1 CM (E6) with 4 nodes (N1 as a LN, N2 as a RN and the nodes N3 and N4 as CNs).

To simplify the notation and for more clearness, all elements are described by single-phase models. Additionally, the LPM of line 1 should be consist of only one π-section (being a CM), and the damping in the DPM of the line 2 is omitted.

The state space equations in the explicit form for the inductive elements are:

$$\text{E1: } i_1' = \tfrac{1}{\omega_0} \dot{i}_1 = Y_1 u_1 + i_{q1}$$
$$\text{E2: } i_2' = \tfrac{1}{\omega_0} \dot{i}_2 = Y_2 u_2 + i_{q2}$$
$$\text{E3: } i_3' = \tfrac{1}{\omega_0} \dot{i}_3 = Y_3 u_3 + i_{q3}$$
$$\text{E4: } i_4' = \tfrac{1}{\omega_0} \dot{i}_4 = Y_4 u_4 + i_{q4}$$

The algebraic equations for the resistive elements are:

$$\text{E5: } \quad i_5 = G_5 u_5$$
$$\text{E7A: } i_{7A} = G_7 u_{7A} + i_{q7A}$$
$$\text{E7B: } i_{7B} = G_7 u_{7B} + i_{q7B}$$

The state space equations for the capacitive elements have the form:

$$i_{6A} = i_{q6A} + G_6 u_{6A} + C_6 \dot{u}_{6A}$$
$$i_{6B} = i_{q6B} + G_6 u_{6B} + C_6 \dot{u}_{6B}$$

For the computation of the source quantities we have:

$$u_{q1} = \hat{u}_{q1} \cos(\omega_0 t + \varphi_1)$$
$$u_{q4} = \hat{u}_{q4} \cos(\omega_0 t + \varphi_4)$$
$$i_{q1} = \tfrac{1}{\omega_0} A_1 i_1 - Y_1 u_{q1}$$
$$i_{q2} = \tfrac{1}{\omega_0} A_2 i_2$$
$$i_{q3} = \tfrac{1}{\omega_0} A_3 i_3$$
$$i_{q4} = \tfrac{1}{\omega_0} A_4 i_4 - Y_4 u_{q4}$$
$$i_{q7A} = -G_7 u_{7B}(t-\tau_7) - i_{7B}(t-\tau_7)$$
$$i_{q7B} = -G_7 u_{7A}(t-\tau_7) - i_{7A}(t-\tau_7)$$
$$L_6 \dot{i}_{q6A} + R_6 i_{q6A} = u_{6A} - u_{6B}$$
$$i_{q6B} = -i_{q6A}$$

where τ_7 is the traveling time of the line 2. The Y- and G-matices from (15) become:

$$\boldsymbol{Y}_L = \operatorname{diag}(Y_1\ Y_2\ Y_3\ Y_4)$$
$$\boldsymbol{G}_R = \operatorname{diag}(G_5\ G_7\ G_7)$$
$$\boldsymbol{G}_C = \operatorname{diag}(G_6\ G_6)$$

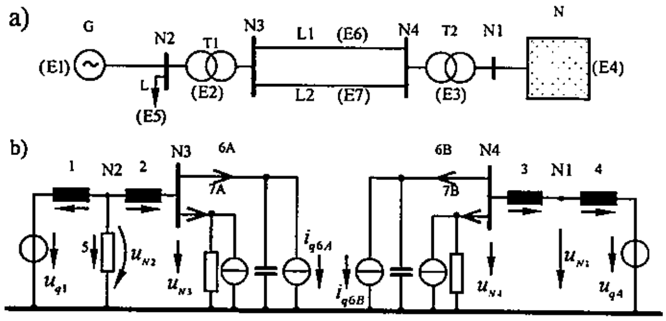

Fig.4 Example for the formulation of element and node equations
a) system configuration b) equivalent circuits

The partitioned nodal matrix from (16) has the form:

$$\boldsymbol{K} = \begin{array}{r} \text{L-Node} \\ \text{R-Node} \\ \text{C-Node} \\ \text{C-Node} \end{array} \overset{\text{inductive elements} \quad \text{resistive el.} \quad \text{cap. el.}}{\left[\begin{array}{cccc:ccc:cc} 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ \hdashline 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \hdashline 0 & -1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \end{array}\right]}$$

The modified nodal matrix $\boldsymbol{K}'$ from (17) differ from $\boldsymbol{K}$ in zero elements instead the bold marked elements. But the matrices $\boldsymbol{K}$ and $\boldsymbol{K}'$ have theoretical importance only, because the algebraic equation (21) can assembled from the network scheme directly following the nodal admittance building rule. By doing so, we find from Fig. 4b:

$$\left[\begin{array}{c:c:cc} Y_3+Y_4 & 0 & 0 & -Y_3 \\ \hdashline 0 & G_5 & 0 & 0 \\ \hdashline 0 & 0 & G_6+G_7 & 0 \\ 0 & 0 & 0 & G_6+G_7 \end{array}\right] \left[\begin{array}{c} u_{N1} \\ \hdashline u_{N2} \\ \hdashline u_{N3} \\ u_{N4} \end{array}\right] =$$
$$= \left[\begin{array}{c} i_{q3} - i_{q4} \\ \hdashline -i_1 - i_2 \\ \hdashline i_2 - i_{q7A} - i_{q6A} - C_6 \dot{u}_{6A} \\ -i_3 - i_{q7B} - i_{q6B} - C_6 \dot{u}_{6B} \end{array}\right]$$

Note that the Y-elements of the LMs do not appear in the G-matrices and the LMs currents, being state variables, are represent in the right side terms respectively. The capacitive state space equation (24) has the form:

$$\begin{bmatrix} C_6 & 0 \\ 0 & C_6 \end{bmatrix} \begin{bmatrix} \dot{u}_{N3} \\ \dot{u}_{N4} \end{bmatrix} = -\begin{bmatrix} G_6+G_7 & 0 \\ 0 & G_6+G_7 \end{bmatrix} \begin{bmatrix} u_{N3} \\ u_{N4} \end{bmatrix} - \begin{bmatrix} -i_2 + i_{q6A} + i_{q7A} \\ i_3 + i_{q6B} + i_{q7B} \end{bmatrix}$$

VI. CONCLUSIONS

A new method for power system modeling by an algebraic nodal equation and the state space equations of power system elements has been described. The advantages of the proposed approach are:

1) The non-linear state space equations of the elements are not linearized. They are not stiff and can be therefore solved by explicit integration rules, avoiding time-consuming iteration steps.

2) The algebraic nodal matrix equation (21) can be formulated directly from the network structure without any topological tools. Its YG-matrix has a form which is similar to an upper triangular matrix.

3) At each time step, the nodal voltages are available as output variables.

4) The model structure is suitable for object oriented programming and parallel algorithms (see Fig. 3).

VII. REFERENCES

[1] H. W. Dommel, *Electromagnetic transients program reference manual (EMTP Theory book),* Bonneville Power Administration, August 1986.

[2] G. Fodor, *Nodal analysis of electrical networks*, Akademiai Kiado, Budapest, 1988.

[3] J. H. Chow, *Time-scale modelling of dynamic networks with applications to power systems*, Springer-Verlag, Berlin, Heidelberg, New York, 1982.

[4] W. S. Meyer, H. W. Dommel, "Numerical modelling of frequence-dependent transmission line parameters in an electromagnetic transient program", *IEEE Trans. on Power Apparatus and Systems*, vol. 93, 1974, pp. 1401-1409.

APPENDIX

Elements of $\boldsymbol{L}_{ee}$:

$$L_m = L_s + L_M = \frac{1}{3}\left(L_0 + 3L_M + 2L_1\right) = \frac{1}{3}\left(L_0' + 2L_1\right)$$

$$L_n = L_g + L_M = \frac{1}{3}\left(L_0 + 3L_M - L_1\right) = \frac{1}{3}\left(L_0' - L_1\right)$$

where:

$$L_s = \frac{1}{3}\left(L_0 + 2L_1\right), \quad L_g = \frac{1}{3}\left(L_0 - L_1\right) \text{ and } L_0' = L_0 + 3L_M$$

when L_0 and $L_1 = L''$ are the zero- and positive sequence inductivities. The elements of $\boldsymbol{R}_{ee}$ are analogue terms. The subtransient voltages are calculated from:

$$\boldsymbol{u}'' = \dot{\boldsymbol{K}}_{SL}\boldsymbol{\psi}_L + \boldsymbol{K}_{SL}\dot{\boldsymbol{\psi}}_L = \begin{bmatrix} u_a'' & u_b'' & u_c'' \end{bmatrix}^{\mathrm{T}}$$

$$\dot{\boldsymbol{\psi}}_L + \boldsymbol{R}_L\boldsymbol{L}_{LL}^{-1}\boldsymbol{\psi}_L - \boldsymbol{R}_L\boldsymbol{K}_{LS}\boldsymbol{i}_e = \boldsymbol{u}_L = \begin{bmatrix} u_F & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{\psi}_L = \begin{bmatrix} \psi_F & \psi_D & \psi_Q \end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{R}_L = \mathrm{diag}\left(R_F\ R_D\ R_Q\right)$$

$$\boldsymbol{K}_{SL} = \begin{bmatrix} k_F\cos\vartheta_a & k_D\cos\vartheta_a - k_Q\sin\vartheta_a \\ k_F\cos\vartheta_b & k_D\cos\vartheta_b - k_Q\sin\vartheta_b \\ k_F\cos\vartheta_c & k_D\cos\vartheta_c - k_Q\sin\vartheta_c \end{bmatrix} = \frac{2}{3}\boldsymbol{K}_{LS}^{\mathrm{T}}$$

with

$$\vartheta_a = \omega t + \vartheta_0, \quad \vartheta_b = \vartheta_a - 2\pi/3, \quad \vartheta_c = \vartheta_a + 2\pi/3$$

and the coupling factors k_F, k_D and k_Q:

$$k_F = \frac{L_{hd}}{L_{hd}(L_{\sigma F} + L_{\sigma D}) + L_{\sigma F}L_{\sigma D}}L_{\sigma D}$$

$$k_D = \frac{L_{hd}}{L_{hd}(L_{\sigma F} + L_{\sigma D}) + L_{\sigma F}L_{\sigma D}}L_{\sigma F}$$

$$k_Q = \frac{L_{hq}}{L_Q}$$

The elements of $\boldsymbol{L}_{LL}$ are the rotor winding self- and mutual inductivities:

$$\boldsymbol{L}_{LL} = \begin{bmatrix} L_{hd} + L_{\sigma f} & L_{hd} & 0 \\ L_{hd} & L_{hd} + L_{\sigma D} & 0 \\ 0 & 0 & L_{hq} + L_{\sigma Q} \end{bmatrix}$$