A New Method to Determine Frequency Characteristics of a Power System Including Nonlinear Effects

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Abstract — **Existing methods of determining the frequency characteristics of electrical networks often neglect the nonlinear effects of magnetic saturation and the switching of power electronic (FACTS) devices. Linearizing the system in this way may result in an apparent frequency response whose resonant frequencies and magnitudes are in error. This, in turn, may lead to the misapplication of filter banks, controls, or protections. A time domain simulation method to determine the frequency characteristics of a network containing nonlinearities is proposed. Results using this large-signal approach are compared to those obtained using the conventional, small-signal, approach. It is shown that the response of the nonlinear system may be markedly different from that of the linearized system. Nonlinear dynamical behaviors such as bifurcations are also investigated. It is recommended that the relative effect of system nonlinearities be established before relying on a linearized frequency scan.**

Keywords: Frequency Response, Nonlinear Frequency Characteristics, Harmonic Analysis, Power System Nonlinearities

I. INTRODUCTION

Power systems contain a large number of nonlinear devices. These devices can degrade power quality since they can inject or augment harmonics. To mitigate harmonics in power system, filters or control strategies are used. An accurate frequency representation of the power system is necessary to design these filters or controls. This paper presents a novel time-domain approach to determine the frequency characteristics of a power system that includes nonlinear elements.

A. *Power System Nonlinearities*

The nonlinearity inherent in power transformers and iron core reactors is saturation of the magnetic core. The power electronics contained in FACTS devices can provide two types of nonlinearities: switching nonlinearities and/or device nonlinearities. For longer lines, the transmission line's frequency dependence is also a factor.

B. *Why is the Frequency Characteristic Important?*

Many power electronic devices inject harmonics into the power system. Two examples are HVDC converter stations and thyristor controlled reactors in SVCs. Recently, the installation of shunt capacitor banks has increased dramatically due to constraints on the construction of new transmission and generation facilities. New shunt capacitor bank installations can shift the power system frequency characteristic so that a parallel resonance occurs at a frequency of an injected harmonic, producing voltage distortion. At the same time, processes used in modern manufacturing are demanding high power quality. To avoid power quality problems, an accurate system representation is needed to design harmonic filters and controls for mitigating harmonic currents.

C. *Traditional Method to Calculate the Response*

A small-signal approach is commonly used to determine the system's frequency characteristic. In this approach the of the system nonlinearities are linearized and phasor calculations are performed at discrete frequencies. This method is used the traditional method used in EMTP analyses.

II. BACKGROUND

This work centered around the use of the Electromagnetic Transients Program (EMTP) to determine a system's frequency characteristic. Other methods, such as harmonic power flow programs, were not considered.

A. *Basic EMTP Method*

Both the EPRI/DCG and ATP versions of EMTP have a FREQUENCY SCAN feature [1, 2]. This subprogram calculates the phasor bus voltages at discrete frequencies. When using this method, the frequency of all the sinusoidal (Type-14) sources is incremented using either linear or logarithmic spacing. The driving point impedance is:

$$Z(\omega) = \frac{V(\omega)}{I(\omega)} \tag{1}$$

where $I(\omega)$ is the injected current phasor value and $V(\omega)$ is the resulting phasor voltage for a given frequency. To obtain a plot of driving point impedance, a one ampere source can be used, as shown in Fig. 1. The output voltage for that node is then numerically equal to the driving point impedance. The drawback to this method is that it requires that the system be linearized, therefore nonlinear effects are lost.

B. *Kizilcay Improvements*

An improvement to the FREQUENCY SCAN method was outlined by Kizilcay at the 1994 European EMTP Meeting in

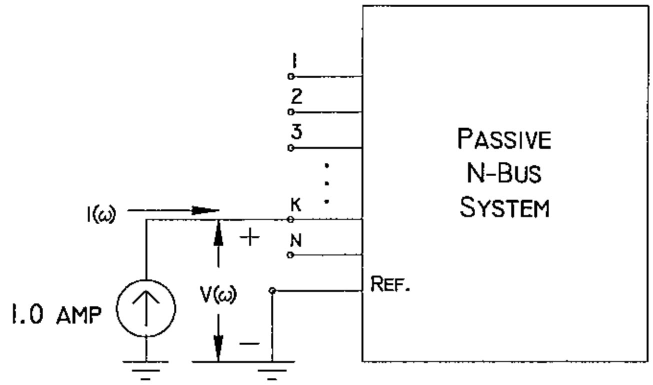

Fig. 1. Traditional method to calculate driving point impedance using EMTP

Denmark [3]. This improvement involves incorporating frequency-dependent admittance matrices into the FREQUENCY SCAN computation. For example, if a long transmission line was included in the model, several matrices could be entered and the phasor calculation would shift from one matrix to the next as the frequency was incremented. This improvement does not address the voltage dependent or time switching nonlinearities.

C. Jiang and Gole Method

Jiang and Gole suggested using Fast Fourier Transform (FFT) techniques to obtain the frequency characteristics of the power system near an HVDC converter terminal [4]. This method is able to detect harmonic interactions between the converter terminal and AC system that linearized methods could not detect. The drawback to this method is that it relies on superposition, and therefore, it is limited to only small signal analysis.

III. A NEW METHOD FOR NONLINEAR NETWORKS

The basic EMTP method is small signal, and is inadequate for systems with large nonlinearities. It does not accurately represent the frequency-dependent nature of transmission lines. The addition of frequency-dependent admittance matrices still does not address the magnetic saturation inherent in power transformers. FFT methods, while implemented in the time domain, still rely on the superposition theorem. Superposition is not valid for nonlinear systems.

A. Time Domain Method

The proposed time domain method to obtain the frequency characteristics of the power system is to retain the normal steady-state 60-Hz or 50-Hz sources and to perturb the system with a slowing varying injected current. In a time domain simulation, EMTP is able to represent both the frequency-dependent nature of transmission lines and the magnetic saturation of power transformers and iron-core reactors. Since the system response changes both with frequency and excitation level, three control parameters are possible: frequency of the injected current, injected current magnitude, and nominal system voltage.

The proposed time domain solution method incorporates either a varying frequency or magnitude current source in the simulation. The term "Frequency Sweep" was chosen to describe a simulation where the injected current's frequency was used as the control parameter. This type of simulation provides a means to compare the new method to traditional methods. The term "Current Sweep" was chosen for a simulation where the injected current's magnitude was ramped. These simulations are appropriate if the frequency of concern is already known, as is the case for many harmonic sources.

A slowly varying control variable is needed to investigate nonlinear dynamics such as bifurcations [5,6]. A bifurcation is when the system response "changes qualitatively." Two example bifurcations are: 1) when the system response jumps from one steady-state mode to another, or 2) when the response shifts from a "period one" to a "period two" response. A period one response is defined as a response that has the same period as the input. A period two response is defined as a response that takes two periods of the input to repeat. Bifurcations are the main reason that linear methods are inappropriate for systems with nonlinearities. With linear frequency domain methods, one can never be sure if the response calculated is the only true response.

The single-phase circuit shown in Fig. 2 was used to demonstrate the Frequency Sweep method in an earlier paper [7]. Two current sources were used in these simulations. The first was a 60-Hz, 155-A current steady-state source which produces the nominal operating voltage. The second was a ramped-frequency, constant-magnitude current source to perturb the system. The capacitance value in the circuit was chosen to approximate one phase of a three-phase, 30 MVAR, shunt bank at 69-kV. The series R and L component values were chosen to represent a short line segment. The nonlinear inductance, representing a saturable line reactor, was chosen so that the circuit had a resonant point close to 60 Hz. The linearized version of the circuit used a constant value of 300 mH for the line reactor; the saturable line reactor model

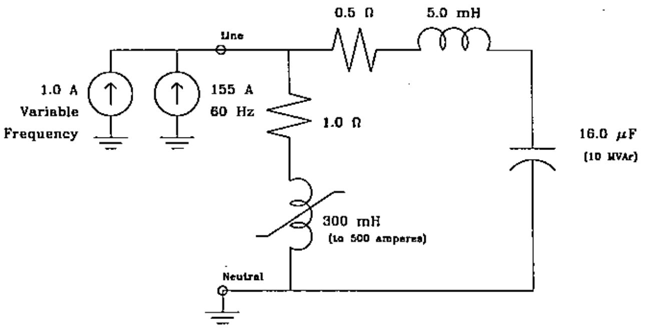

Fig. 2. First circuit used to obtain time-domain frequency characteristics

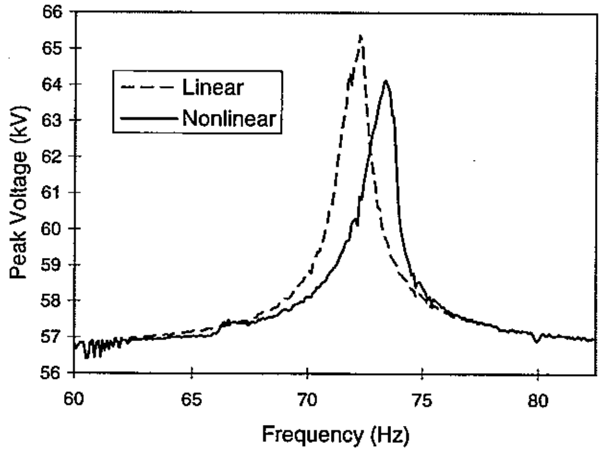

Fig. 3. Results of Frequency Sweep for linear and nonlinear versions of the example system

in the nonlinear version of the circuit had λ-i breakpoints of (500 A, 150 V-s), (1000 A, 250 V-s), and (1500 A, 350 V-s).

The linearized version of this circuit is parallel-resonant at 72.6 Hz. While sources for 72.6 Hz currents are not common in power systems, sources for 3rd and 5th harmonic currents are. One reason to obtain the frequency response of the power system is to determine if resonance occurs at these frequencies. The voltages caused by resonances at the 3rd and 5th harmonic will produce power quality problems.

The results of a Frequency Sweep of the actual and linearized version of the circuit is shown in Fig. 3. As shown in the figure, the peak is shifted and attenuated. This result was expected since saturation of the iron-core reactor will cause the incremental inductance to decrease and produce a soft-limiting of the voltage. The output from this early implementation of the Frequency Scan method is not a smooth as expected. This was due to errors in sampling the waveforms that will be discussed later.

B. Extension to Practical Systems

The systems of interest include transformers, distributed-parameter transmission lines, generators, and shunt reactive devices such as shunt capacitor banks [8]. Therefore, the next step in demonstrating this new method was to develop cases that included all of these elements. In addition, it is not practical to model every bus in the system. Therefore, an equivalent for the system must be included in the cases to represent busses electrically distant from the point of interest.

IV. IMPLEMENTATION IN EMTP

The implementation of this method in EMTP requires a controlled current source and sampling of the output waveform. Both of these functions require logical branching. Outputting a portion of the waveform also requires control over I/O functions. At the time this work was done, the only option available for this work was the MODELS feature of ATP. Since then, Dr. Scott Meyer has been working on complied TACS. Although it has not been tested, this new feature should provide the same functionality as MODELS and much shorter run times.

A. Controlled Injected Current Source

The MODELS feature of ATP was used to provide a source of the variable frequency or peak magnitude current. One caveat to be aware of when modeling a source with a time-dependent frequency is the definition of frequency. Ramping the frequency causes the instantaneous frequency to be twice that of the requested value. This is because the argument of the cosine function now has a t squared term, and the definition of instantaneous frequency is the derivative with respect to time of the cosine's argument. The equation for the injected current, i_{inj}, is then:

$$i_{inj} = i_p \cos\left[(2f_0 + rt)\pi t\right], \qquad (2)$$

where i_p is the requested peak current in amperes, f_0 is the initial frequency in Hz, and r is the desired frequency rate of change in Hz/s. Ramping the current magnitude does not involve any such modifications.

When ramping the control variable, care must be taken not to drive the system into oscillatory behavior or numerical instability. To verify that the system remains quasi-steady-state, a short run near a linear resonant point can be used or the output waveform can be sampled. One indication that the rate of ramping is too great is the development of "beat frequency" oscillations. Similar to the rule of thumb for simulation time step size, the ramping rate should be divided by two and the simulation rerun if this problem occurs.

One drawback to this method is the execution time. One run can take several hours. The simulation time step is limited by the transmission line length and other system parameters. The shorter the transmission line, in general, the smaller the time step to avoid a non-integer number of time steps in the travel time of the line. As discussed in [9], to avoid interpolation errors the travel time of the line should be an integer multiple of the simulation time step. This limitation, along with ramping rate limitations, leads to long simulations. In addition, these simulations can produce extremely large data files (100's of MB) if the results are stored for each time step.

B. Output Waveform Sampling

Since this is a nonlinear system, the methods pioneered in recent years in nonlinear dynamics can be applied to gain a better understanding of the simulation results. Also, as eluded to earlier, storing the entire simulation waveform is not practical.

One of the methods developed for nonlinear systems is Poincaré sampling. This method involves the sampling of the output once every period of the forcing function. The trigger for sampling can be fixed in time or based on the phase angle of the source. One potential problem with this method is determining a useful sampling rate when two sources with different frequencies are used. This problem is evident in the raggedness of the results from the preliminary work, shown in Fig. 3.

A solution purposed by Moon [10] is a *double Poincaré section.* Using this method, a sample of the output is taken when the triggers for both inputs are coincident. A fixed time interval trigger cannot be used for the variable frequency input, so a fixed phase angle was used instead. This was implemented by triggering on a positive-going zero crossing of the input. A small interval, 1.4 times the simulation timestep, on either side of the fixed time interval trigger for the fixed-frequency input was used to check for coincidence.

Another way to measure the system response is through a peak voltage envelope. Since the system is nonlinear, subharmonics are possible in the response. Therefore, two or more periods of the nominal system frequency are sometimes necessary to obtain one point on the peak voltage envelope. This was accomplished by using a variable length window to obtain the local minimum and maximum. The window length was equal to the period of the variable frequency source when the frequency of the source was less than one-third of the fixed-frequency source. When the frequency of the variable-frequency source was greater than or equal to one-third of the fixed frequency source the window length was three periods of the fixed-frequency source.

As stated earlier, a very large data file would be required to retain the entire simulation waveform. Tracking only the minimum and the maximum, and Poincaré sampling greatly reduce the output file size. To illustrate, the number of points required for a typical Frequency Scan simulation are given in the table below.

Table I. Waveform Storage Example	
Simulation Time	200 seconds
Maximum Injected Frequency	1000 Hz
Store Entire Waveform	
Minimum Sampling Rate	10,000 samples per second
Number of points required	2,000,000
Peak Voltage Envelope	
Fixed-Frequency Source	60 Hz
Number of points required	8,000

One possible reason to keep the entire waveform would be to confirm that the simulation results had remained quasi-steady-state. In lieu of retaining the entire waveform, four small sections (equal to the peak voltage window) of the output waveform are written to an ASCII file at user-defined frequencies or simulation times. Another reason to retain the entire waveform is simulation run times. The simulations run several times faster when the MODELS code is removed. A post-processing program, such as MATLAB™, could then be used to generate the Poincaré map and peak voltage envelope. This is only a viable option when the user has virtually unlimited disk space.

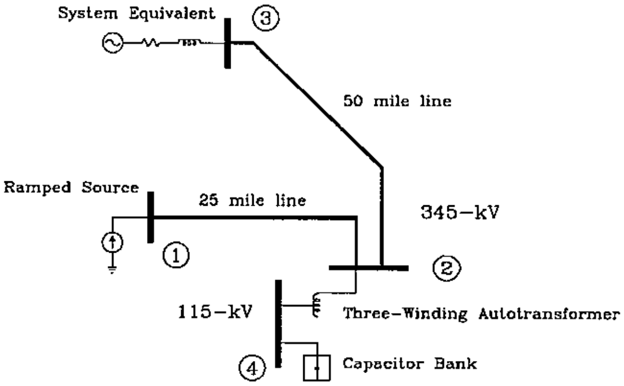

Fig. 4. Simplified, single-phase model for testing

V. SIMULATION RESULTS

A simplified version of the high voltage AC system in the northwest quadrant of the metropolitan Minneapolis, Minnesota, area shown in Fig. 4, was chosen to demonstrate the new method. The actual system includes an HVDC link, two large generators, and a new capacitor bank installation. For the initial demonstration, the following simplifications were made:

- only one phase was modeled,
- the two generators were included in system equivalent, and
- the HVDC station was replaced by the injected current source.

To verify the method, a simulation using only a one ampere injected current (without any steady-state excitation) was compared to a linear frequency scan. With a small injected current, the system will still be in the linear operating region, so the results should be the same. As shown in Fig. 5, the results from the two methods are almost identical.

When the steady-state 60-Hz source was included in the model, the complete set of results was much more complicated than a traditional Z(ω) plot. The Z(ω) plot assumes only one independent variable: the frequency. In these simulations there are three independent variables: frequency, injected current magnitude, and nominal system voltage. Since impedance, Z, is not defined for a nonlinear system, a new measure must be found. In this study, plots of the Poincaré samples, called bifurcation diagrams, and the peak voltage envelope, were used as measures.

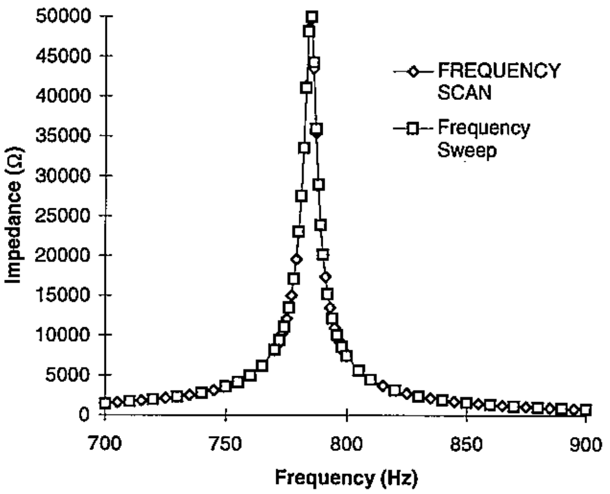

Fig. 5. Comparison of new Frequency Sweep method to traditional FREQUENCY SCAN method

The results from the simulations when the steady-state source was included show that the frequency at which the maximum of the peak voltage envelope occurs (analogous to the peak resonance point in a linear system) is shifted when the magnitude of the injected current is increased. This is shown in Fig. 6. These results are similar to those obtained for the preliminary circuit. The raggedness in the characteristics has been eliminated by using the proper sampling as proposed by Moon and discussed earlier. Also note that if a linearized system had been used, a five-fold increase in the input, for example, from 10 A to 50 A, would have resulted in a five-fold increase in the output. Comparing the results for 10 A and 50 A, one can see there is only a four- fold increase. This reduction in peak voltage magnitude is due to magnetic saturation. This behavior is observable in operating power systems, so the reduction of the peak resonant point was predicted. The shift in frequency of the peak resonant point can be explained by the shift in incremental inductance as the λ-i characteristic is traversed.

Unlike a linear system, the response of a nonlinear system can vary depending on the direction that the control variable is changed. In the Frequency Sweep cases, one can either start at the initial frequency, for example DC, and increase the frequency; or start at the maximum desired frequency, for example 1000 Hz, and decrease the frequency. As shown in Fig. 8, the response of the system for the same input, such as an injected current at 795 Hz, can be very different. This demonstrates the Achilles Heel of linearized analysis — all the possible responses cannot be calculated.

As stated previously, there are three independent variables in these simulations. To better visualize the results, three-dimensional plots with frequency for the x-axis and injected current magnitude for the y-axis were produced. Several possible z-axis variables were considered: the local maxima, local minima, and Poincaré sample. The maximum voltage is important for filter design and equipment specification. The local minimum is important if a bifurcation occurs. When a bifurcation occurs the absolute values of local minimum and maximum may be different. The Poincaré sample of the output voltage is important to track bifurcations and other nonlinear dynamics.

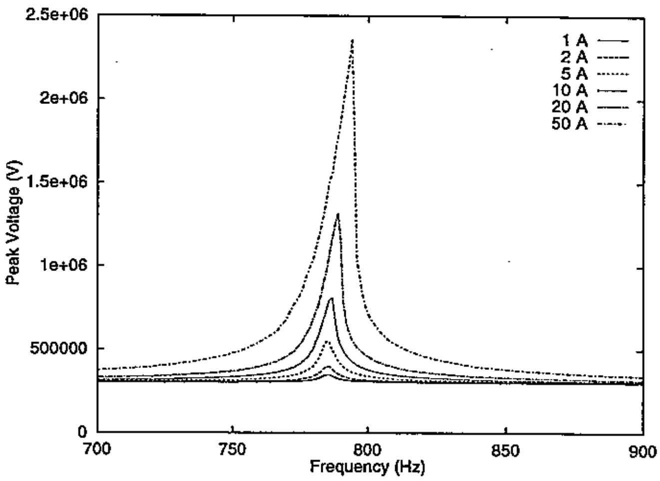

Fig. 6. Shifting of peak "resonance" for an increase in the injected current magnitude

To obtain these three-dimensional plots, either a collection of frequencies, a collection of injected current magnitudes, or a combination of both can be used. The most efficient method found to obtain these three-dimensional plots is to run a Frequency Sweep simulation to determine the frequency range near resonance and then run Current Sweeps in that range. This method was used to obtain the plot in Fig. 7. In a coupled nonlinear system there is always the possibility of

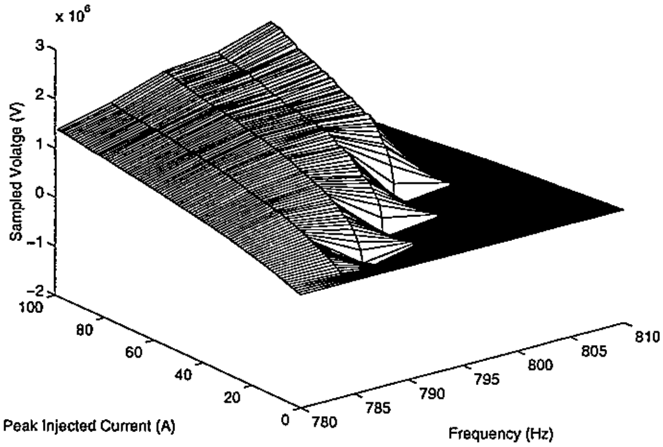

Fig. 7 A family of Current Sweeps downward in the frequency range found by a Frequency Sweep

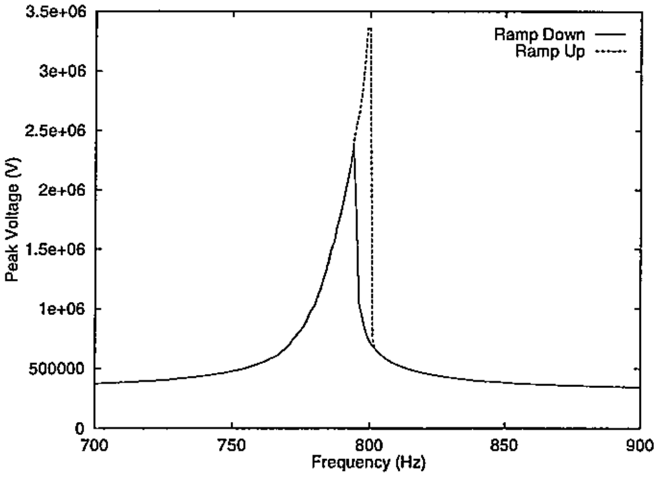

Fig. 8. Hysteresis in the response for downward and upward ramping of the injected current frequency

bifurcations and more complex resonances, nonlinear dynamics, chaos, etc. This plot shows that the response is dependent on the peak magnitude of the injected current.

VI. CONCLUSIONS

The results demonstrate that the nonlinear nature of the circuit components should not be excluded from system frequency response calculations. The results show that errors will occur when operating above the first breakpoint of a nonlinear inductance. These simulations also show that these errors become more pronounced as the nonlinear inductance goes further into saturation (the shifting shown in Fig. 6).

In addition to finding nonlinear dynamics that would not have be seen using linear methods, the method proposed has several benefits, among which are:

- providing a more complete range of system frequency characteristics for harmonic filter designers, and
- producing waveforms that can be compared to event recorder waveforms, to verify possible system resonances.

This work is being extended to obtain the response of larger, three-phase systems operating at varying levels of saturation. Since the results are dependent upon the nominal system voltage, a family of similar three-dimensional plots can be produced. Improved methods of interpreting the time-domain simulation results (i.e. developing a time-domain mathematical measure comparable to impedance in the frequency domain) are also being investigated. A promising candidate for this is the peak voltage normalized by magnitude of the injected current.

VII. ACKNOWLEDGMENTS

The authors from Michigan Technological University gratefully acknowledge the help and support from Northern States Power Company for this research.

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