# PHASE-DOMAIN MULTIPHASE TRANSMISSION LINE MODELS

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*Abstract.* In general, multiphase transmission line and cable models for transient programs, such as the EMTP, base their solution on transformations between the phase and modal domains. The main difficulty with these models is the synthesis of the frequency dependent transformation matrices. This paper presents two line models that circumvent this problem by writing the propagation functions directly in the phase domain and thus avoid the use of modal transformation matrices. The first model (developed in connection with our work on corona modelling) avoids the use of modal transformation matrices by separating the ideal-line travelling effect from the losses effect (resistance and internal inductance). The second proposed model is a full frequency dependent distributed parameter model based on idempotent decomposition. By using idempotents instead of eigenvectors as the basis for modal decomposition, the problems associated with the indeterminacy of the frequency dependent eigenvector functions are eliminated.

Keywords: Transmission line modelling, EMTP simulation, phase-domain modelling, idempotent analysis.

## 1. INTRODUCTION

Overhead transmission lines and underground cables are the basic transmission links across a power system. Wave propagation and distortion along lines and cables affect the waveshapes reaching the system equipment and the associated stresses upon these equipment.

Despite their importance in transient studies, the accurate modelling of transmission lines in time domain simulations is still not fully satisfactorily resolved. Good time-domain frequency dependent transmission line and cable models, based on Wedepohl/Hedman's theory ([3, 4]) of modal decomposition of natural propagation modes (eigenmodes) are available in the EMTP program [1, 2]. The main problem with these models is the frequency dependence of the transformation matrices (matrices of eigenvectors) that connect the modal and phase domains.

This paper introduces two new line models being developed at the University of British Columbia. These models avoid the use of transformation matrices to relate the decoupled eigenmode propagation in the modal domain to the coupled-mode propagation in the phase domain. Since in their final form the equations are solved entirely in phase coordinates, these models can be regarded as phase-domain line models. They differ, however, from other recently proposed phase-domain models (e.g., [11]) in that the frequency dependent wave propagation functions are still first synthesized in the frequency domain, as in [1] and [2].

## Lumped Line Model (zi-line).

The first proposed phase-domain line model (zi-line, for "z-lumped ideal-line") has been developed in connection with our work on a wide-band corona model [5]. This line model follows the basic ideas advanced in [6], except that it is formulated entirely in phase coordinates, thus eliminating the problems associated with modal transformation matrices.

The premise here is that if the nonlinear corona model has to be lumped between relatively short line sections then advantage can be taken of the space discretization. This space discretization can be used to "simplify" the modelling of the distortion part of the propagation function ($[e^{-\gamma(\omega)x}]$). That is, the distortion due to the resistance and the internal inductance can be grouped into one lumped frequency dependent impedance matrix in phase coordinates $[Z^{loss}(\omega)]$ while ideal propagation (at the speed of light in the medium) due to the external magnetic field $[L^{ext}]$ and capacitance $[C]$ can be represented by an ideal-line segment. Since for ideal propagation all modes travel at the speed of light, the equations for this part of the model (ideal line) can be formulated directly in phase coordinates. This model is similar to the constant-parameter line model in the EMTP (cp-line) where the resistance is lumped in the middle and at the end points of the line. As opposed to the cp-line model, however, the proposed zi-line model can fully take into account the frequency dependence of the line parameters (in $[Z^{loss}(\omega)]$) and *exactly* takes into account the asymmetry of any arbitrary line configuration without the need for a transformation matrix. The cp-line model in the EMTP works in modal coordinates and assumes that the transformation matrix relating mode and phase is constant and real and, therefore, it cannot correctly simulate strongly asymmetrical configurations. The only limitation of the proposed zi-model is that the losses are assumed lumped and, therefore, a certain number of line sections are needed to simulate the actual distributed nature of these parameters.

## Idempotent Line Model (id-line)

The second proposed line model (id-line, for "idempotent-line") is a fully frequency dependent distributed-parameter line model. This model is conceptually similar to the current frequency dependent line models in the EMTP (fd-line and q-line, [1] and [2]), except that it overcomes the problems of these models with respect to the frequency dependence of the modal transformation matrices. The main problem in synthesizing

the elements of the transformation matrices as functions of frequency is that the eigenvectors that make up these matrices are only defined up to a complex constant factor. This means, for example, that different scalings ("normalizations") of the eigenvectors will result in different frequency functions for the elements of the transformation matrix. The question of which one is the "right" normalization is very difficult to answer. The problem of multiple-choice eigenvector functions is aggravated in the case of repeated eigenvalues, which occurs in certain regions of the frequency spectrum. The eigenvectors for N repeated eigenvalues can lie anywhere in a given N-dimensional plane.

The proposed id-line model solves these problems by avoiding the use of eigenvectors as base functions for modal decomposition. Instead, the phase-domain propagation function $[e^{-\gamma_{ph}x}]$ (matrix function) is expressed as a linear combination of the *natural* propagation modes $e^{-\gamma_i x}$ (scalar functions) with idempotent coefficient matrices. The idempotent matrices [7] result from column-row products of the matrix of eigenvectors (modal transformation matrix) and its inverse, and are *uniquely defined*, independent of the particular scaling of the eigenvectors or of the multiplicity of the eigenvalues.

## 2. Proposed Lumped Line Model (zi-line)

The modelling of transmission lines is based on the travelling wave equations, which can be expressed, for a given frequency $\omega$ as

$$\frac{d^2 \mathbf{V}}{dx^2} = [\mathbf{ZY}]\mathbf{V} \quad \text{and} \quad \frac{d^2 \mathbf{I}}{dx^2} = [\mathbf{YZ}]\mathbf{I} \qquad (1)$$

where $[\mathbf{ZY}]$ or $[\mathbf{YZ}]$ are full matrices that couple the propagation of voltage and current waves in all the phases. The series impedance matrix $[\mathbf{Z}]$ is a full matrix with elements of the form

$$Z_{ij} = R_{ij} + j\omega(L_{ij}^{ext} + \Delta L_{ij}) \qquad (2)$$

where $R_{ij}$ is the resistance of the conductor plus the correction for the ground return effect, $\omega$ is the frequency, $L_{ij}^{ext}$ is the inductance related to the external flux, and $\Delta L_{ij}$ is the inductance related to the internal flux inside the conductor plus the correction for ground return. Matrix $[\mathbf{Z}]$ can thus be rewritten as

$$[\mathbf{Z}] = [\mathbf{Z}^{loss}] + j\omega[\mathbf{L}^{ext}] \qquad (3)$$

$$[\mathbf{Z}] = [\mathbf{Z}^{loss}] + [\mathbf{Z}^{ext}] \qquad (4)$$

with $\quad Z_{ij}^{loss} = R_{ij} + j\omega\Delta L_{ij} \qquad (5)$

The shunt admittance matrix $[\mathbf{Y}]$ can be written as (assuming conductance $G \approx 0$)

$$[\mathbf{Y}] = j\omega[\mathbf{C}], \quad \text{with} \quad [\mathbf{C}] = [\mathbf{P}]^{-1} \qquad (6)$$

$[\mathbf{P}]$ is the Maxwell coefficients matrix. The $[\mathbf{Y}]$ matrix does not require any corrections for ground return and contains only elements that depend on the capacitances (geometry) of the system. With equations (3) to (6), we can rewrite $[\mathbf{YZ}]$ as

$$[\mathbf{YZ}] = j\omega[\mathbf{C}]([\mathbf{Z}^{loss}] + j\omega[\mathbf{L}^{ext}]) \qquad (7)$$

$$[\mathbf{YZ}] = j\omega[\mathbf{C}][\mathbf{Z}^{loss}] - \omega^2[\mathbf{C}][\mathbf{L}^{ext}] \qquad (8)$$

Equation (8) shows how the product $[\mathbf{YZ}]$ can be expressed as the combination of two main components: the first one related to the losses and ground effect corrections and the second one related to the ideal propagation.

If we assume that the effect of the losses term is small compared with the effect of the ideal component, then we can lump the losses and obtain the model shown in Figure 1. This condition is met when the section length is small. In the general case of frequency dependent parameters the term $[\mathbf{Z}^{loss}(\omega)]$ is a function of both the frequency and the section length.

### *Modelling of the Multiphase Ideal Line*

Once the impedance corrections ($[\mathbf{Z}^{loss}]$) are extracted from the complete line model, the ideal transmission line can be easily modelled in the phase domain. Because the travelling time $\tau$ is the same for all the modes and the solution of the system of travelling wave equations is decoupled, Dommel's ideal line equation [8] for the time-domain single-phase case can be rewritten for the multiphase case, as illustrated in Figure 2. Using circuit theory, the terminal voltage vectors $v_k(t)$ and $v_m(t)$ can be expressed as functions of the terminal current vectors $i_k(t)$ and $i_m(t)$, the characteristic impedance matrix $[\mathbf{Z}_c]$, and the history source vectors $e_{kh}(t)$ and $e_{mh}(t)$. The history source vectors are calculated at each time step as functions of past values, through the expressions

$$e_{kh}(t) = v_m(t - \tau) + [\mathbf{Z}_c] i_m(t - \tau) \qquad (9)$$

$$e_{mh}(t) = v_k(t - \tau) + [\mathbf{Z}_c] i_k(t - \tau) \qquad (10)$$

with $\quad [\mathbf{Z}_c] = [\mathbf{Y}]^{-1}\{[\mathbf{Y}][\mathbf{Z}^{ext}]\}^{1/2} \qquad (11)$

In this case, the matrix $[\mathbf{Z}_c]$ represents the coupled system in the phase domain and is a full matrix. The implementation of this model in the EMTP is similar, with small modifications, to the standard coupled R-L model [9]. Like in the case of the single-phase ideal line model in the EMTP, the multiphase ideal line model is independent of the integration rule, and represents an exact



Corrections for losses and internal flux    Ideal line, only external flux

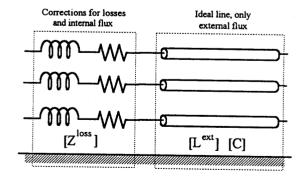$[\mathbf{Z}^{loss}]$    $[\mathbf{L}^{ext}]$  $[\mathbf{C}]$

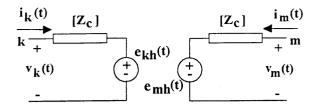Figure 1. Separation of basic effects in the zi-line model.

Figure 2. Multiphase ideal transmission line
model in the time domain.

solution of the line equations. If nodes are numbered sequentially at the terminals of the line, the resultant conductance matrix [G] for the line will be a block diagonal matrix, as shown in Figure 3.

## Modelling of the Lumped Section

For the general case of frequency dependent line parameters, the elements of the matrix $[Z^{loss}(\omega)]$ are frequency dependent functions, and they can be synthesized by rational functions approximations, as in the frequency dependent line models in the EMTP [1, 2]. Here, in order to illustrate the basic properties of the proposed model, the case of line parameters evaluated at a single frequency will be explained and the model will be used in comparisons with the EMTP's constant parameter and frequency dependent line models.

The matrix $[Z^{loss}]$ represents, mathematically, a resistance and an inductance in series. In the physical phenomenon, however, it is difficult to separate the "series" and "parallel" effects. Actually, in a minimal solution (one $R$ and one $L$) a parallel R-L combination around a given frequency point represents the skin effect much better than a series R-L (which represents no skin effect at all). A minimum realization of the proposed model for the losses is shown in Figure 4, for a two-phase example. A series resistance $R_o$ is added to the $R^{par}L^{par}$ parallel combination to satisfy the dc condition ($R_o = R_{dc}$). In the full frequency dependent implementation, additional R-L parallel blocks, resulting from the asymptotic fitting procedure of [1, 2], provide a full-accuracy synthesis over the entire frequency range.

As indicated, each element of the $[Z^{loss}]$ matrix from line constants is replaced by an equivalent circuit through the equations:

$$R^{par} = \frac{(\omega L)^2}{R - R_o} + (R - R_o) \quad \text{and} \quad L^{par} = \frac{(R - R_o)^2}{\omega^2 L} + L \quad (12)$$
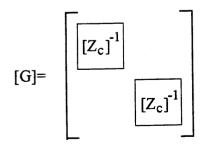


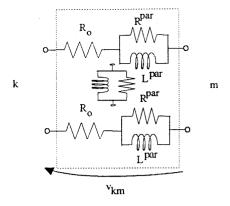Figure 3. Conductance matrix for the ideal multiphase transmission line



Figure 4. Equivalent constant parameter circuit for losses.

where $R$ and $L$ are the real and imaginary parts of the corresponding element of $[Z^{loss}]$.

The multiphase $[Z^{loss}]$ equivalent can be implemented in an EMTP's time domain type solution using the procedure explained in [9] for coupled R-L elements. The basic equations, with the trapezoidal rule of integration, for the equivalent resistance matrix $[R^{eq}]$ and the history current-source vector $h_{km}$ are

$$[R^{eq}] = [R_o] + [R^{eqpar}] \quad (13)$$

with $[R^{eqpar}]$ the equivalent multiphase resistance for the discretized parallel $R^{par}$ - $L^{par}$ circuit.

$$h_{km}(t) = [A] v_{km}(t - \Delta t) + [B] h(t - \Delta t) \quad (14)$$

$$\text{with } [A] = 2[R^{eq}]^{-1}[R^{eqpar}]\left[\frac{2L^{par}}{\Delta t}\right]^{-1}[R^{eqpar}][R^{eq}]^{-1}$$

$$\text{and } [B] = [R^{eq}]^{-1}\{[R^{eq}] - 2[R_o][R^{eqpar}]\left[\frac{2L^{par}}{\Delta t}\right]^{-1}\} \quad (15)$$

## Short-Section Transmission Line Solution

The phase domain solutions of the ideal line section and the $[Z^{loss}]$ section can be combined to simulate a short section of a transmission line. For the short-section model, the part corresponding to $[Z^{loss}]$ is divided into two sections. Each half section of $[Z^{loss}]$ is added at the ends of the ideal line section, as shown in Figure 5.

A set of simulations were done to study the relation between the section length of the zi-line model and the frequency of the study. The transmission line used for the testing was a typical flat-configuration three-phase 230 kV line. The base case is shown in Figure 6 and consists of one sinusoidal source attached to node a and different resistance terminations at the other ends. The source frequency was set equal to the frequency used for the calculation of the line parameters. The time-domain
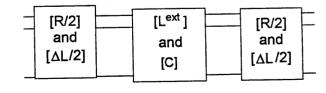


Figure 5 - Proposed line-section for the zi-line model.

response of the zi-line model was compared against the solution obtained with the constant-parameter model of the EMTP. By repeating this process for several frequencies an empirical relation between the length of the line-section and the frequency of the analysis was established. The results are shown in Figure 7 and they give the recommended maximum length of the zi-line section that could be used for a study at a given dominant frequency.

## Full-Length Transmission Line Solution

The solution for a complete transmission line is accomplished by connecting several of the zi-line short-section models in cascade. With an appropriate ordering of the nodes, the resultant conductance matrix for the full transmission line is a block diagonal matrix. The resultant blocks are combinations of the matrices $[Z_c]$ from the ideal line (Fig. 3) and $[R^{eq}]$ for the loss network (equation 13).

The implementation of the final zi-line model was done in a stand-alone program written in ADA-95 and several cases were tested. The results of these cases were compared against the constant parameter (cp-line) and the frequency dependent (fd-line) models in the EMTP. Figure 8 shows the results when a unit step voltage is injected at t = 0 into node a in the system of Fig. 6. For comparison purposes, the line was assumed balanced (the cp-line model and the fd-line model are theoretically exact only for balanced lines). The line parameters for the cp-model and for the proposed zi-model were calculated at 5000 Hz. The total line length is 100 km; sections of 5 km were used for the zi-model and of 10 km for the cp-model. The time step was 1.6 μs.

Figure 8(a) shows the induced voltage at node c, while Figure 8(b) shows the receiving-end voltage at node d. These results illustrate how, in general, the response of the proposed zi-model is closer to the response of the frequency dependent model than that of the constant parameter model. As can be observed, both, the shape and the magnitude of the waveforms produced by the zi-model follow reasonably well the behaviour of the fd-model. This is particularly noticeable in the first reflections of the transient. It should be emphasized that in the case of constant parameters used in these simulations,
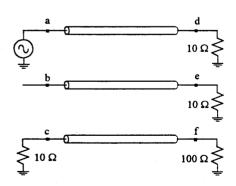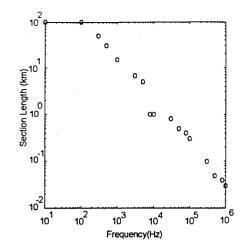


Figure 7. Proposed zi-line section length vs frequency of analysis.
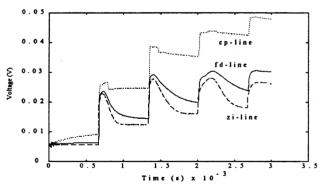
despite its much better performance, the zi-model requires basically the same computational cost as the cp-line model.

## 3. IDEMPOTENT LINE MODEL (ID-LINE)

The frequency dependent line and cable models of [1, 2] are based on solving the line travelling wave equations in the "modal domain", using diagonalizing transformation matrices. That is, the original full-matrix propagation equations in phase coordinates,

$$\frac{d^2 V_{ph}}{dx^2} = [Z_{ph} Y_{ph}] V_{ph} \quad \text{and} \quad \frac{d^2 I_{ph}}{dx^2} = [Y_{ph} Z_{ph}] I_{ph} \quad (16)$$

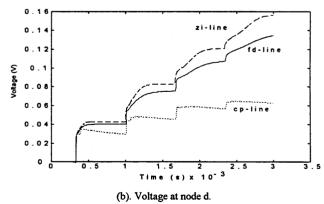are transformed into diagonal-matrix equations in



(a). Voltage at node c.



(b). Voltage at node d.

Figure 8. Comparison between the zi-line model, the constant-parameter EMTP model and the frequency-dependent EMTP model.



Figure 6. Base case for analysis of the zi-line model.

modal coordinates

$$\frac{d^2\mathbf{V}_m}{dx^2} = [\mathbf{Z}_m\mathbf{Y}_m]\mathbf{V}_m \quad \text{and} \quad \frac{d^2\mathbf{I}_m}{dx^2} = [\mathbf{Y}_m\mathbf{Z}_m]\mathbf{I}_m \quad (17)$$

by the transformations

$$[\mathbf{P}] = \text{eigenvectors of } [\mathbf{Z}_{ph}\mathbf{Y}_{ph}] \text{ and} \quad (18)$$

$$[\mathbf{Q}] = \text{eigenvectors of } [\mathbf{Y}_{ph}\mathbf{Z}_{ph}] \quad (19)$$

The line model of [1] makes the simplification of assuming constant (frequency independent) eigenvector matrices [P] and [Q], while the cable model of [2] synthesizes the frequency-dependent elements of these matrices with rational functions.

A fundamental problem of the eigenvector description (modal transformation matrices) is that eigenvectors are not uniquely defined. In the case of distinct eigenvalues individual eigenvectors can be multiplied by arbitrary complex-number scaling factors and the new [P] or [Q] matrices will still be valid diagonalizing matrices. In the case of N repeated eigenvalues the corresponding eigenvectors can be anywhere in an N-dimensional space. This freedom in the location of eigenvectors may lead to serious mathematical complications in formulations based on synthesis by continuous frequency-domain functions (for example, by rational-function approximations).

The problems related to the indeterminacy of the eigenvector solution can be avoided if the line modelling problem is formulated not in terms of eigenvector decomposition but in terms of *idempotent* decomposition.

A full idempotent-based transmission line model is currently under development by our research group. Due to space limitations in this paper, however, only the fundamental principles of this model will be introduce here.

*Idempotent-Based Propagation Equations*

The concepts and properties of idempotents used here are taken from reference [7]. Basically an idempotent matrix $[\mathbf{I}_i]$ is a matrix that has the property

$$[\mathbf{I}_i]^2 = [\mathbf{I}_i] \quad (20)$$

and, of course, $[\mathbf{I}_i]^n = [\mathbf{I}_i]$ for any integer n. From this apparently innocuous property one can intuit that a matrix function response expressed as a power expansion based on idempotents will bring out the basic constituent responses of the system (eigenmodes). The application of this concept to the transmission line propagation equations is discussed next.

Consider, as in [1, 2], the line propagation function for each decoupled line mode

$$A_i = e^{-\gamma_i l} = e^{-\gamma_{li} l} e^{-j\omega\tau_i} \quad (21)$$

where $\tau_i$ is the time-delay (phase velocity) of component mode i. In the frequency dependent line and cable models of [1, 2], the distortion and attenuation function $e^{-\gamma_{li} l}$ is synthesized by a minimum-phase rational function $p_i(\omega)$, and $A_i$ is expressed as

$$A_i(\omega) = p_i(\omega)e^{-j\omega\tau_i} \quad (22)$$

Equation (22) defines (in a synthesis form suitable for modelling) each *natural* propagation mode of the line. In the conventional approach, the component modes are combined into a coupled-mode propagation in the phase domain using the transformation matrix of eigenvectors [Q]:

$$[\mathbf{A}_{ph}] = [\mathbf{Q}][\mathbf{A}_m][\mathbf{Q}]^{-1} \quad (23)$$

where $[\mathbf{A}_m]$ is the diagonal matrix with the natural component modes.

For a frequency dependent $[\mathbf{Q}(\omega)]$, equation (23) leads to convolutions in the time domain. To avoid these numerically very expensive convolutions, reference [2] synthesizes the elements of $[\mathbf{Q}(\omega)]$ by rational functions. Difficulties encountered with this approach in the modelling of overhead lines seem to be related to the indeterminacy of the frequency functions in $[\mathbf{Q}(\omega)]$, particularly in frequency regions with repeated eigenvalues. (An interesting theoretical analysis of problems related to repeated eigenvalues can be found in [10].)

As indicated earlier, indeterminacy problems can be eliminated when the formulation is done in terms of idempotents instead of eigenvectors. The break-up of (23) in terms of idempotents can be done as follows. (A three-phase line is used to explain the procedure, which applies equally to an N-phase line.)

1) Write [Q] in terms of its constituent columns:

$$[\mathbf{Q}] = [\mathbf{C}_1 \ \mathbf{C}_2 \ \mathbf{C}_3] \quad (24)$$

where $\mathbf{C}_i$ are the eigenvectors.

2) Write $[\mathbf{Q}]^{-1}$ in terms of its constituent rows:

$$[\mathbf{Q}]^{-1} = \begin{bmatrix} \mathbf{R}_1 \\ \mathbf{R}_2 \\ \mathbf{R}_3 \end{bmatrix} \quad (25)$$

Even though not necessary in the derivation, it is interesting to mention that when the eigenvectors of [P] and [Q] in (18) and (19) are normalized according to the Euclidean norm (vector length equal to one), then $[\mathbf{Q}]^{-1} = [\mathbf{P}]^t$ and the row vectors in (25) correspond to the eigenvectors of [P].

3) Write (23) in terms of the column and row partitions in (24) and (25)

$$[\mathbf{A}_{ph}] = [\mathbf{Q}][\mathbf{A}_m][\mathbf{Q}]^{-1} \quad (26)$$

$$[\mathbf{A}_{ph}] = [\mathbf{C}_1 \ \mathbf{C}_2 \ \mathbf{C}_3] \begin{bmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{bmatrix} \begin{bmatrix} \mathbf{R}_1 \\ \mathbf{R}_2 \\ \mathbf{R}_3 \end{bmatrix} \quad (27)$$

thus giving

$$[\mathbf{A}_{ph}] = [\mathbf{C}_1\mathbf{R}_1]A_1 + [\mathbf{C}_2\mathbf{R}_2]A_2 + [\mathbf{C}_3\mathbf{R}_3]A_3 \quad (28)$$

In (28),

$$[\mathbf{C}_i\mathbf{R}_i] = [3x3] \text{ idempotent coefficient matrix i} \quad (29)$$
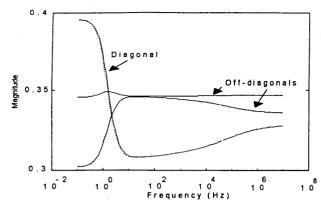
Figure 9. Fitting of idempotent elements.

and the $A_i$ terms are as in (22).

Equation (28) gives the phase-domain propagation function $[\mathbf{A}_{ph}(\omega)]$ in terms of the natural (modal) propagation modes $A_i = e^{-\gamma_i l}$, where the $\gamma_i$'s are the eigenvalues of $[\mathbf{YZ}]^{1/2}$. Equation (28) then retains the natural eigenvalues of the problem while expressing the result in coupled phase coordinates. Since the idempotent coefficient matrices $[\mathbf{C}_i\mathbf{R}_i]$ are the product of a column of the eigenvectors matrix $[\mathbf{Q}]$ times a row of its inverse $[\mathbf{Q}]^{-1}$, the result is independent of the scaling factors used in the eigenvectors. The elements of the idempotent matrices as functions of frequency are ,therefore, *uniquely defined* and the problems of frequency continuity of the eigenvector functions are eliminated.

For the time domain implementation of the model, the idempotent matrices are synthesized by rational function approximations. The functions to be synthesized are generally smooth and simple to fit using the asymptotic tracing technique of [1, 2]. Figure 9 shows an example of this synthesis for a diagonal and two off-diagonal elements of the idempotent coefficient matrix for mode 3 of a single-circuit transmission line (The idempotent functions been fitted were obtained with the help of the Newton-Raphson eigenvector tracing technique of [12]).

## 4. CONCLUSIONS

The major stumbling block in modal domain modelling for time domain simulations has been the frequency dependence of the transformation matrices that relate modal and phase coordinates. Two line models are presented in this paper that avoid the use of transformation matrices. The first model separates the losses (resistance and internal inductance) from the propagation channel (external inductance and capacitance). Propagation is then simulated as ideal (at the speed of light in the medium) and equal for all modes. The delays and shaping effects of the losses are simulated by lumped coupled-impedance matrices. This model provides an inexpensive alternative for full frequency dependent line modelling. Its only limitation is that the line has to be sectionalized into a reasonable number of segments so as to simulate the actual distributed nature of the losses. In this regard, it is particularly suited for corona modelling where the corona phenomenon also needs to be

sectionalized.

The second proposed model adopts the "opposite" philosophy of the first one. Instead of trying to idealize the propagation characteristics, it tries to *exactly maintain* the individuality of the natural component modes of propagation. This is achieved by using idempotents theory to express the coupled phase-domain propagation function as a linear combination (with matrix coefficients) of the independent modal-domain propagation modes.

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