A Simple and Efficient Method for Including A Frequency-Dependent Effect in A Transmission Line Transient Analysis

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Abstract

This paper proposes a simple and efficient method for calculating a transmission line transient including its frequency-dependent effect. The frequency-dependent distributed-parameter line is modeled by a combination of a distributed-parameter line with constant loss and velocity and an impedance circuit representing the frequency dependence of the original line. The impedance circuit is derived from a four-terminal parameter theory. Calculated examples by the proposed method are compared with the accurate results by a frequency-domain transient program FTP, and the accuracy of the proposed method is confirmed to be satisfactory.

1. Introduction

It is well-known that frequency-dependence of line parameters causes a significant effect on a transmission line transient. A frequency-domain method of a transient calculation has no difficulty to deal with the frequency-dependent effect. A time-domain method such as the EMTP/1/, however, has a difficulty to include the frequency-dependent effect in the transient analysis. Thus, many approaches of its inclusion have been proposed. Principally, the approaches are based on a real-time convolution. A main defect of the real-time convolution is a computation time and memories required for its numerical evaluation. To avoid the defect, a recursive convolution has been proposed/1-4/, and has been widely used to deal with the frequency-dependent effect of a distributed line in its transient calculation. The recursive convolution is very efficient in comparison with the original real-time convolution. It, however, still requires a large amount of memories for a past history of traveling waves on the distributed line. Also, numerical instability sometimes arises during parameter calculations of the recursive convolution.

The present paper proposes a very simple and efficient approach for including the frequency-dependent effect of a distributed-parameter line in its transient calculation. The frequency-dependent impedance of the line is approximated by a simple L-R circuit combined with a distributed line having constant loss and velocity. The theory of the approach is explained in the paper both for singlephase and multiphase lines. It is shown that the approach becomes identical to that of a lumped-resistive modeling of a distributed line developed by Dommel in the EMTP/1/. Calculation examples are demonstrated to show the accuracy and efficiency of the proposed approach for an underground cable.

2. Proposed Model

2.1 Theory of the model

A number of boundary conditions such as towers and groundings in a distributed line system is represented by an equivalent homogeneous line. On the other hand, a homogeneous line is represented by a combination of lumped-parameter circuits (boundary conditions) and distributed lines/5/.

Let's assume that a frequency-dependent distributed-parameter line in Fig.1(a) is represented by an equivalent line in Fig.1(b) which is a combination of lumped impedances Z_n and ideal lossless lines. The four-terminal parameter (F-parameter) of the original line in Fig.1(a) is given by:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh\theta & z_0 \sinh\theta \\ \sinh\theta / z_0 & \cosh\theta \end{bmatrix} \tag{1}$$

where

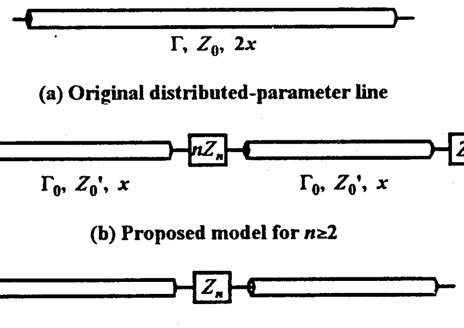
$$\theta = \Gamma \cdot 2x, \ \Gamma : \text{propagation constant} \tag{2}$$

$Z_0 = \sqrt{z/y}$: characteristic impedance

$z = r + j\omega L$: series impedance of a conductor

$y = j\omega C$: shunt admittance of a conductor

$C = 2\pi\varepsilon_0 / \ln(2h/r)$, $2x$: conductor length



(a) Original distributed-parameter line

(b) Proposed model for $n \geq 2$

(c) Proposed model for n=1

Fig.1 A proposed approach to represent a distributed line

The F-parameter of the equivalent line in Fig.1 is given in the following form/5/.

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}$$

$$= \begin{bmatrix} 1 & Z_n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A_0 & B_0 \\ C_0 & A_0 \end{bmatrix} \begin{bmatrix} 1 & nZ_n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A_0 & B_0 \\ C_0 & A_0 \end{bmatrix} \begin{bmatrix} 1 & Z_n \\ 0 & 1 \end{bmatrix} \quad \text{for } n \geq 2$$

$$= \begin{bmatrix} A_0 & B_0 \\ C_0 & A_0 \end{bmatrix} \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A_0 & B_0 \\ C_0 & A_0 \end{bmatrix} \text{for } n = 1 \tag{3}$$

$$A_0 = \cosh\theta_0,\ B_0 = Z_0'\sinh\theta_0,\ C_0 = \sinh\theta_0 / Z_0' \quad (4)$$

where

$\theta_0 = \Gamma_0 \cdot x$,

$\Gamma_0 = \sqrt{z'\cdot y} = j\omega\sqrt{L_0 \cdot C} = j\omega/v_0$

: ideal propagation constant

$Z_0' = \sqrt{z'/y} = 60\ln(2h/r)$

: ideal characteristic impedance

$z' = j\omega L_0 = j\omega(\mu_0/2\pi)\cdot\ln(2h/r)$

L_0 : inductance of an ideal conductor

v_0 : light velocity in free space

n : positive integer

After matrices multiplication, the following result of eq.(3) is obtained.

for $n \geq 2$

$$A' = A_0^2 + (2+n)Z_n A_0 C_0 + B_0 C_0 + n(Z_0 C_0)^2$$

$$B' = Z_n A' + (1+n)Z_n A_0^2 + 2A_0 B_0 + nZ_n^2 A_0 C_0 + Z_n B_0 C_0$$

$$C' = C_0(2A_0 + nZ_n C_0)$$

for $n=1$

$$A' = A_0^2 + (Z_n A_0 + B_0)C_0$$

$$B' = A_0(Z_n A_0 + 2B_0) \quad (5)$$

$$C' = C_0(2A_0 + Z_n C_0)$$

In general, variable θ is very small. Thus, assuming $\theta \ll 1$, eqs.(1) and (4) are approximated by the following equations.

$$\left.\begin{aligned} A &\approx 1 + (z\cdot y\cdot 4x^2)/2 \\ &\approx 1 + (z\cdot 2x)(y\cdot 2x)/2 \approx 1 + Z\cdot Y/2 \\ B &\approx Z,\ C \approx Y \end{aligned}\right\} \quad (6)$$

$$\left.\begin{aligned} A_0 &\approx 1 + (z'\cdot y\cdot x^2)/2 \\ &\approx 1 + (z'\cdot 2x)(y\cdot 2x)/8 \approx 1 + Z'\cdot Y/8 \\ B_0 &\approx Z'/2,\ C' \approx Y/2 \end{aligned}\right\} \quad (7)$$

Substitution of eq.(7) into eq.(5) considering $Z'^2 Y$ and Y^2 are much smaller than Z' gives the following result.

$$\left.\begin{aligned} &A' \approx 1 + Z'\cdot Y/2 + (2+n)Z_n Y/2, \\ &B' \approx Z' + (2+n)Z_n,\ C' \approx Y && \text{for } n \geq 2 \\ &A' \approx 1 + Z'Y/2 + Z_n Y/2, \\ &B' \approx Z' + Z_n,\ C' \approx Y && \text{for } n = 1 \end{aligned}\right\} \quad (8)$$

Because eq.(8) is an approximation of eq.(6), the following relation is obtained for the impedance Z_n.

$$\left.\begin{aligned} Z_n &\approx (Z - Z')/(2+n) && \text{for } n \geq 2 \\ &= Z - Z' && \text{for } n = 1 \end{aligned}\right\} \quad (9)$$

The above equation indicates that the original frequency-dependent line in Fig.1(a) is approximately represented by a combination of two lossless lines with half the length of the original line and lumped impedances Z_n. On the contrary, if the impedance $Z_n(n=1)$ is given as a boundary condition series to the line Z' as illustrated in Fig.1(b), the above equation gives the impedance of an equivalent homogenous line considering the series boundary condition. The approach is the same as a distributed admittance model which gives the admittance of an equivalent homogeneous line considering shunt boundary conditions/5/.

Let's assume that the only the resistance of the line impedance is considered in the equivalent line in Fig.1(b). Then, eq.(9) is rewritten by:

$$\left.\begin{aligned} R_n &= R/(2+n) && \text{for } n \geq 2 \\ &= R && \text{for } n = 1 \end{aligned}\right\} \quad (10)$$

where $R = r\cdot 2x$.

When n is taken to be 2, then

$$R_n = R/4 \quad (11)$$

The above is identical to the lumped-resistive modeling of a distributed line developed by Dommel in the EMTP/1/. Thus, the approach corresponds to the theory behind the lumped-resistive modeling and also a conventional tower model composed of an L-R parallel circuit and a lossless distributed parameter line /6,7/.

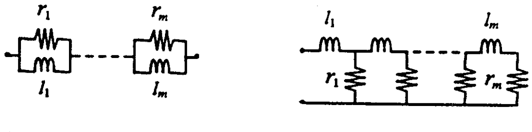
The above analysis concerns the case that a distributed line is devided into 2 sections. If it is required to devide the line into more sections, the same approach is applied though the analysis becomes more complicated. If it is not necessary to devide the line into sections, then it should be clear that the impedance of "$Z_n/2$" is added to the both side of a lossless line with length of "$2x$".

2.2 Modeling of impedance Z_n

The theory explained in the previous section is for a frequency. In the case of a frequency-domain transient program such as the FTP/8/, the proposed line model is at once achieved by eq.(9), i.e. Z_n is evaluated at every frequency and thus, Z_n is frequency-dependent. In the case of a time-domain transient program such as the EMTP, it is necessary to produce a lumped-parameter circuit which represents the impedance Z_n given in eq.(9). A simple circuit equivalent to the frequency-dependent impedance Z_n can be composed either of a series combination of L-R parallel circuits or of a cascaded circuit of lattice circuits based on a reactance theorem as illustrated in Fig.2. The lumped-parameter circuit of Fig.2 can be derived from a rational function approximation of the propagation constant and the characteristic impedance of a given frequency-dependent line/9/. The parameters of an "m" series circuit of L-R parallel and "m" cascaded circuit of lattice circuits are analytically determined if the values of $R(\omega)$ and $L(\omega)$ at "m" different frequencies are given ,where

$$Z_n(\omega) = R(\omega) + j\omega L(\omega) = (Z(\omega) - Z'(\omega))/(2+n)$$

for $n \geq 2$ (12)



(a) Series of parallel circuits **(b) Lattice circuit**

Fig.2 Model circuits of Z_n

When the circuit is composed of 2 series circuits (m=2) as in Fig.2(a), the values of $Z_n(\omega)$ at 2 different frequencies are necessary to determine R_1, L_1, R_2 and L_2 in Fig.2. The frequencies are chosen as the lowest and the highest frequencies involved in a transient. For examples, the power frequency f_p and a dominant transient frequency f_t are chosen in a fault transient.

$$f_{t_s} \approx 1/2\tau \qquad (13)$$

where

$\tau = x/v_0$: propagation time of a line

x : line length

v_0=300 m/μs on an overhead line

=$300/\sqrt{\varepsilon}$ m/μs on a cable

ε : relative permittivity of cable insulator

In the switching surge case, the highest frequency is taken to the following dominant frequency.

$$f_{t_0} \approx 1/4\tau \qquad (14)$$

The lowest frequency is taken to the higher one among the power frequency f_p and the frequency determined by the maximum observation time T_{max}, i.e.

$$f_T = 1/T_{max} \qquad (15)$$

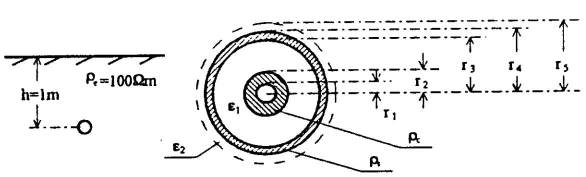
In the multiphase line case, R_1, L_1, R_2 and L_2 in Fig.2 become a square matrix corresponding to the number of phases. One of the advantage of the proposed approach is that the multiphase line is dealt with in a phase domain rather than in a modal domain. Therefore, the frequency-dependent effect of a transformation matrix causes no problem. It should be noted that the propagation constant matrix of a lossless multiphase line is diagonal even in the phase domain, and the transformation matrix is frequency-independent /10/. Thus the proposed approach can avoid the problem of the frequency-dependent transformation matrix even in a transient analysis computer program within a modal framework such as the EMTP.

3. Frequency-Dependent Characteristic of Model Circuit

Fig.3 illustrates configuration of an underground cable with the length of 5.0km. Fig.4 shows a comparison of original accurate impedance $Z_n(\omega)$ for n=1 in eq.(12) and the equivalent impedance of the proposed model circuit in Fig.2(a). The circuit parameters are determined at the following frequencies.

f_1=f_p=60Hz : power frequency

f_2=f_t=8.75kHz : dominant transient frequency of a 5.0km cable



(a) Configuration **(b) Cross-section**

Fig.3 Cable configuration
r_1=10.49[mm], r_2=17.8, r_3=30.75, r_4=32.55, r_5=40.75
ρ_c=1.833×10^{-8}[Ωm], ρ_s=2.8×10^{-8}, ε_1=3.1, ε_2=3.8

Fig.4(a) is the amplitude $|Z_n|$ of the coaxial impedance of the cable per total length, and (b) is the phase angle θ of the impedance, i.e.

$$\left.\begin{array}{l} |Z_n| = \sqrt{R(\omega)^2 + \{\omega L(\omega)\}^2} \\ \theta = \tan^{-1}\{\omega L(\omega)/R(\omega)\} \end{array}\right\} \qquad (16)$$

It is observed from the figure that the impedance of the proposed model circuit agrees satisfactorily with the original impedance for the frequency region $f_1 \le f \le f_2$. The difference of the equivalent impedance becomes greater as frequency increases for $f > f_2$. The difference ,however, is of no significance in a transient calculation, because there exists no frequency component for $f > f_2$ in the transient.

Fig.5 shows the input impedance of the cable when the remote end is grounded. The both ends of the cable sheath are solidly grounded. (a) is the accurate

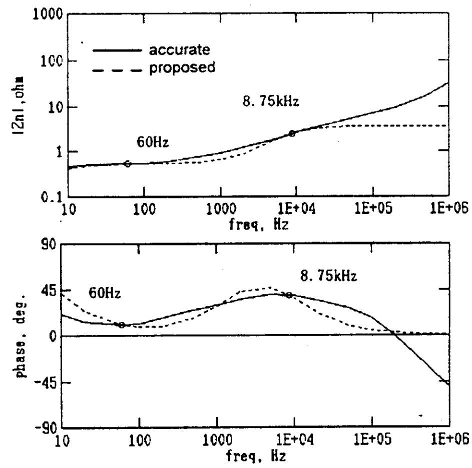


Fig.4. Comparison of accurate impedance Z_n and that of the model circuit in Fig.2.(a)

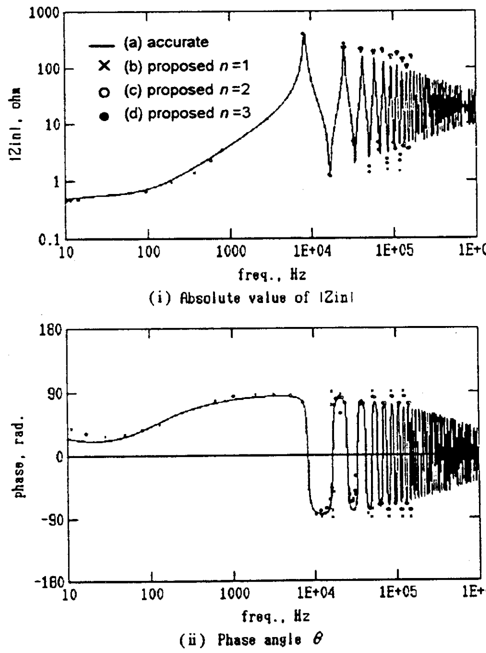


Fig. 5 Effect of "n" on the input impedance of a short-circuited cable with length 5km

impedance of the original cable. (b) to (d) are the input impedance evaluated by the proposed model of a frequency-dependent line. (b) is the case of n=1 in eq.(9) where the original line is represented by the model circuit of Fig.1(c). (b) is the case of n=2 and (c) the case of n=3 in Fig.1(b). It is observed from the figure that the input impedances of all the model circuits agree quite well with the accurate one in the frequency range f_1=60Hz$\le f \le f_2$=8.75kHz. The impedances of the model circuits for $n\ge 2$ agree with the accurate one upto f=20kHz, while that for n=1 agrees upto f=10kHz. The model circuit for n=3 shows a little higher accuracy than that for n=2.

Fig.6 shows the absolute value of the impedance $Z_n(n=1)$ calculated by the lattice circuit in Fig.2(b) for m=2 to 5. r_k and l_k are analytically given in the following equation.

$$\frac{1}{r_k} = \text{real}\{Z_n(\omega_k)\} - \sum_{1}^{k-1} 1/R_i \qquad \text{for}\, k = 1,2,\cdots,m$$

$$l_k = \text{Imag}\{Z_n(\omega_k)\}/\omega_k - \sum_{1}^{k-1} L_i \qquad \omega_m < \omega_k \qquad (17)$$

The figure clearly shows that the accuracy of the model circuit becomes higher as the number of circuits "m" increases, and thus it is necessary to determine an optimum value of "m" in an application of the proposed model circuit. Considering the simplicity of the circuit, m=3 seems to be optimum.

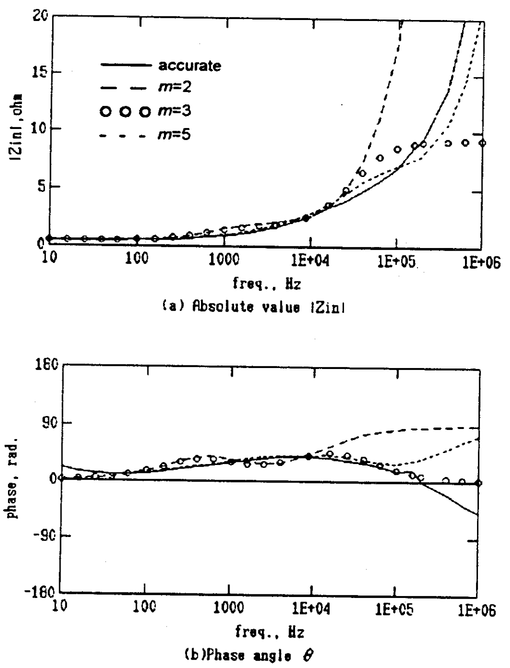


Fig. 6 Effect of "m" of the lattice circuit in Fig. 2(b) on the impedance Z_n

4. Transient Calculations

Fig.7 illustrates an underground cable system of which the cable configuration is given in Fig.3. In the figure, Ls end Rr represent the source-side impedance and the surge impedance of a cable or an overhead line connected to the remote end node r of the cable. Rg is the sheath grounding resistance.

4.1 Effect of "n"

Fig.8 shows calculated results of a switching transient voltage at the remote end for "n"=1 to 3 when Ls=35mH, Rr=∞(open-circuited) and Rg=0(solidly grounded sheath). In the figure, the real line is the exact solution calculated by the frequency transform method/8/. It is observed that there is no significant difference between the cases of n=1 to 3.

Fig.9 shows a transient current at the sending end for n=1 to 3. The accuracy increases as "n" increases, and a difference is observed to be minor.

The above observation agrees with that for the input impedance in Sec.3 and confirms that even the case of n=1 gives a satisfactory result compared with the exact solution.

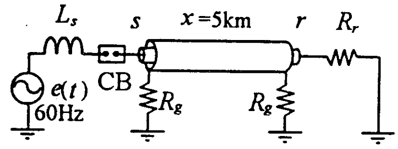


Fig. 7 A cable system

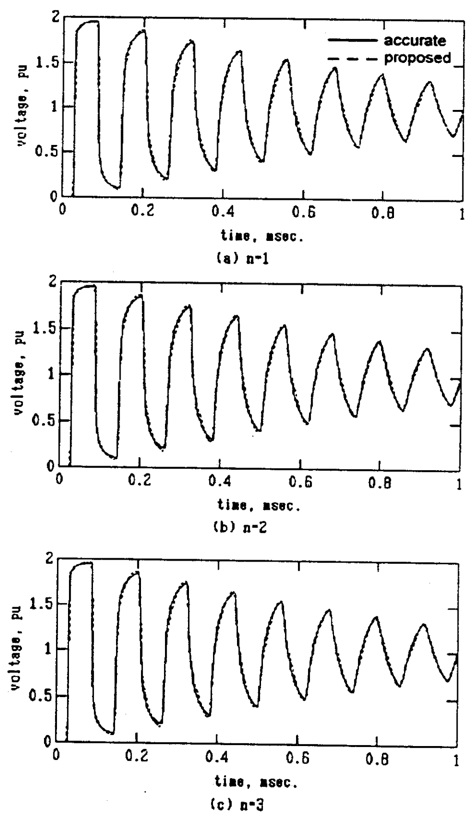


Fig. 8 Calculated results of a switching transient voltage at the remote end

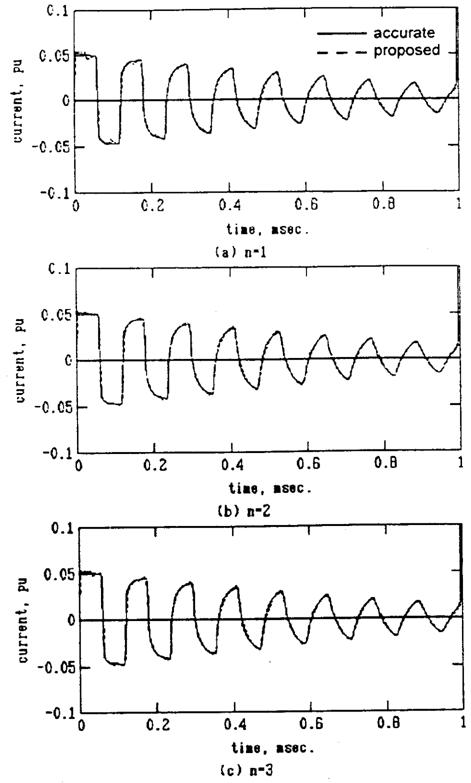


Fig. 9 Calculated results of a switching transient current at the sending end

4.2 Comparison of R-L parallel and Lattice Circuits

Fig.10 shows a comparison of calculated results of a fault surge voltage at the sending end by an *R-L* parallel circuit model and by the lattice circuit model (m=2) when Ls=60mH, Rr=300Ω and Rg=0. A core to ground fault at the remote end initiates at t=4.167ms. It is observed that the parallel circuit model gives a higher peak-voltage than that of the accurate result, while the lattice circuit model gives a lower peak voltage. Otherwise, the accuracy of the both models are not much different. Thus, either model can be used to represent a frequency-dependent line.

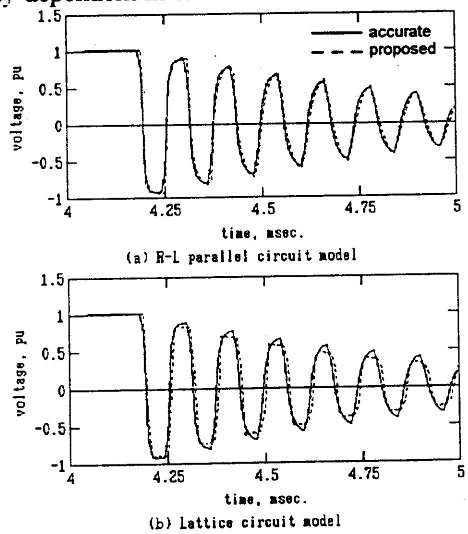


Fig. 10 A fault surge voltage at the sending end

4.3 Effect of the number "m" of the lattice circuits

Fig.11 shows a calculated result of the fault surge same as Sec.4.2 by the lattice circuit model with m=3. A comparison of the figure with Fig.10(b) indicates that the case of m=3 gives a better accuracy than that by the case of m=2. This observation is reasonable and agrees with that made for Fig.6 of the impedance Z_n.

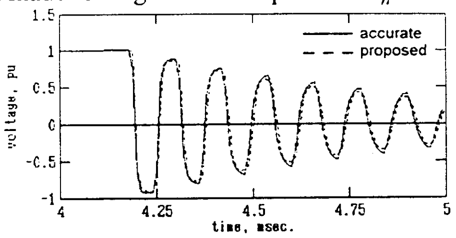


Fig. 11 Effect of "m"

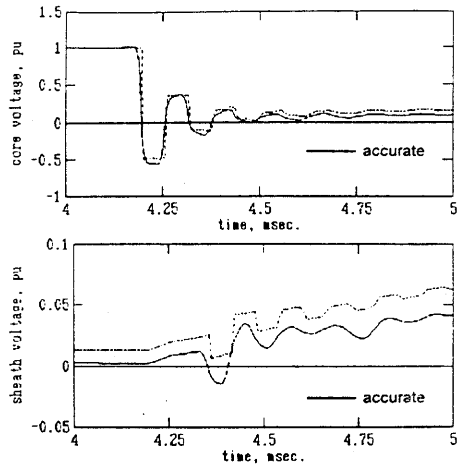
4.4 Comparison with EMTP line models

The electro-magnetic transients program EMTP is the most widely used program for a transient analysis in the world. It includes a frequency-independent line model and a frequency-dependent model (Semlyen or Marti model). Fig.12 shows a comparison of calculated results of a fault surge same as Sec.4.2 except Rg=5Ω in Fig.7.

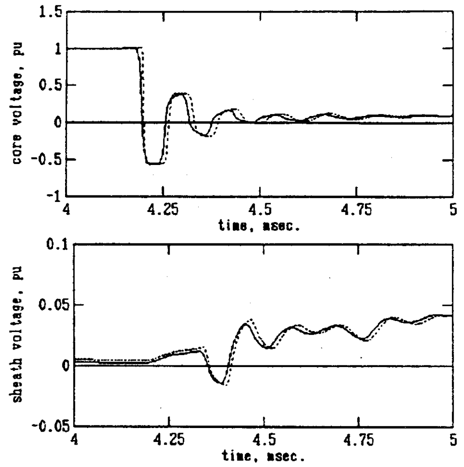
It is clear in Fig.12 that the frequency-independent line gives a poor accuracy as the transient voltage reaches its steady state under the fault condition, and also during the steady state before the fault initiation. The reason for this is that the frequency-independent line model can adopt only one constant transformation matrix at f=8.75kHz, i.e. a dominant transient frequency f_t in Fig.12. Thus, the transformation matrix at f_t largely mismatches the transformation matrix in the steady state corresponding to the power frequency, 60Hz. The frequency-dependent (Semlyen) line model shows a fairly high accuracy in comparison with the accurate result by the frequency transform method. This is reasonable because of the accurate transformation matrices at the power frequency for a steady state and at the dominant transient frequency for a transient state. The accuracy, however, decreases as time increases. This is estimated that the transformation matrices of Semlyen model mismatches the accurate one during the intermediate time period, and thus the numerical error in the past history is accumulated.

The proposed line model shows a fairly good accuracy in the whole time period. This is due to the fact that the frequency-dependent effect of the transformation matrix is included in the model. It, however, needs a better approximation of a oscillating waveform during the transient especially for the sheath voltage.

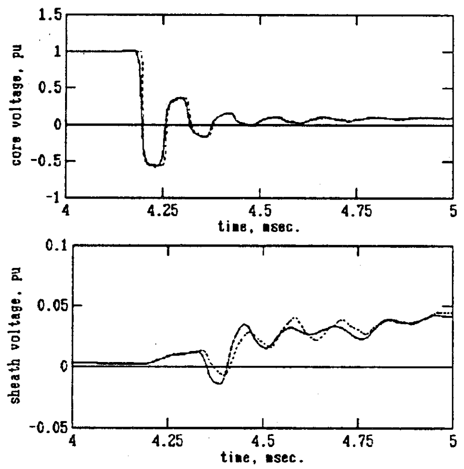
As a total, the proposed line model gives a far better accuracy than that by the EMTP frequency-independent line model. The accuracy of the proposed model is comparable to that by the EMTP frequency-dependent line model. It is estimated to give a better accuracy than the EMTP frequency-dependent model for a longer time period where the frequency dependence of the transformation matrix causes a significant effect.



(a) Frequency-independent line model



(b) Semlyen model



(c) Proposed model

Fig.12 A fault surge voltage at the sending end

5. Conclusions

The present paper has developed a very simple and efficient method of simulating a frequency-dependent line and/or cable in a transient analysis. A frequency-dependent line is represented by a combination of an ideal lossless distributed line and a lumped-parameter circuit Z_n which takes into account the frequency-dependent effect.

The synthesis of the lumped circuit and its optimum condition and parameter have been investigated in the paper. A lattice circuit with 2 or 3 branches seems to be enough to represent the frequency-dependent effect. The lumped circuit Z_n may be subdivided into $Z_n/4$ connected to the sending end, $Z_n/2$ to the center and $Z_n/4$ to the receiving end of the ideal distributed line, or $Z_n/2$ to the sending and receiving ends, or Z_n to the center. The distributed line is not necessary to be an ideal lossless line, but can be a frequency-independent lossless line of which the parameters are given at a very high-frequency f_{max} corresponding to the time step Δt of a computer simulation, i.e. $f_{max}=1/\Delta t$.

Calculated examples of cable transients by the proposed line model agree satisfactorily with the accurate solutions by the frequency transform method. Its accuracy is far better than that of the EMTP frequency-independent line model, and comparable to that of the frequency-dependent line model. The proposed model, however, is observed to be less accurate than Semlyen model for the sheath voltage, and needs a further improvement of its accuracy in the multiphase line case.

References

1) W. Scott-Meyer : EMTP Rule Book(Mode 31),Bonneville Power Administration, Portland, 1982.4.

2) A. Semlyen and A. Dabuleanu : "Fast and accurate switching transient calculation on transmission lines with ground return using recursive convolutions, " IEEE Trans., Vol. PAS-94, pp.561-571, 1975.

3)A. Ametani : "A highly efficient method for calculating transmission line transients", ibid., Vol. PAS-95, pp.1545-1551, 1976.

4)J. R. Marti : "Accurate modelling of frequency-dependent transmission lines in electromagnetic transient simulations", ibid., Vol. PAS-101, pp.147-157, 1982.

5)A. Ametani and Y. Kasai : "Application of an equivalent homogeneous line model to a transient analysis on a distribution system", T. IEE Japan, Vol. 114-B(10), pp.1059-1065, 1994.

6)L. V. Bewley : Traveling Wave on Transmission Systems, Wiley, New York, 1951.

7)M. Ishii et.al. : "Multistory transmission tower model for lightning surge analysis", IEEE Trans., Vol. PWRD-6(3), pp.1327-1335, 1991.

8)N. Nagaoka and A. Ametani : "Development of a generalized frequency-domain program-FTP", ibid., Vol. PWRD-3(4), pp.1996-2004, 1988.

9)S. Kato et.al. : "Frequency-dependent characteristic of a tower", IEE Japan, High Volt. Res. Report HV-92-96, 1992.

10)A. Ametani : Distributed-Parameter Line Theory, Corona Pub. Co., Tokyo, 1992.2.